Question: why we need to prove Lorent'z Transformation over Galilian Transformation?

Ans

we need to prove Lorent'z Transformation because Galilian transformation failed to solve the problem of constancy of speed of light but Lorent'z transformation satisfy the 2nd postulate of Special theory of relativity.

8 Lorent'z Transformation:

$$x' = \gamma (x - vt)$$

$$y' = y$$

$$z' = z$$

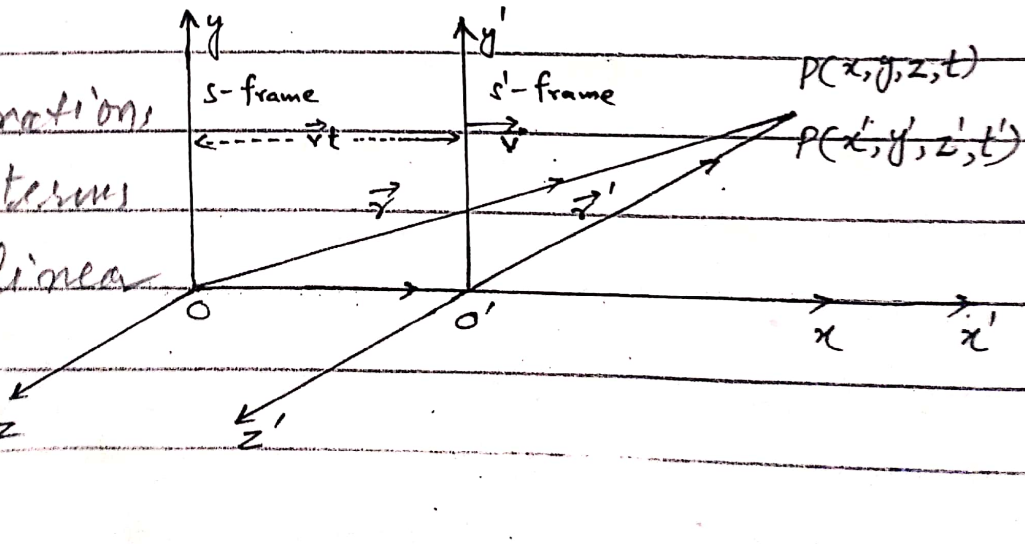
$$t' = \gamma \left(t - \frac{v}{c^2} x\right)$$

where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ called Lorent'z factor.

proof:

Consider two frames S & S'. Initialy these frames coincides so $t' = t = 0$. If S' frame moving along x-axis with velocity $\vec{v}$. Then axis remains parallel to each other.

The trans formations (x', y', z') in terms of (x, y, z) is linear



where K is a Scalar factor to be determined

(15)

Since $S \parallel S'$

Therefore

if $\vec{a} \parallel \vec{b}$
$\Rightarrow \vec{a} = \lambda \vec{b}$

$$x' = k(x - vt) \quad \text{---} \quad (1)$$

$$x = k(x' + vt') \quad \text{---} \quad (2)$$

$\because$ G.T
$x' = x - vt$
$x = x' + vt'$
where
$t' = t$

According to the Lorentz $t \neq t'$

For value of k.

Consider a ray of light instead of particle

Then $(S = vt)$

$x = ct$ in S-frame

$x' = ct'$ in S'-frame

Here $\boxed{c' = c}$

put in (1) & (2), we get:

$(1) \Rightarrow \quad ct' = k(ct - vt)$

$$ct' = k(c - v)t \quad \text{---} \quad (3)$$

$(2) \Rightarrow \quad ct = k(ct' + vt')$

$$ct = k(c + v)t' \quad \text{---} \quad (4)$$

Multiplying (3) & (4)

$$ct' \times ct = k(c - v)t \times k(c + v)t'$$

$$c^2 \cancel{tt'} = k^2 (c - v)(c + v) \cancel{tt'}$$

$$c^2 = k^2 (c^2 - v^2)$$

$$\frac{c^2}{c^2 - v^2} = k^2$$

$$\Rightarrow k^2 = \frac{c^2}{c^2\left(1 - v^2/c^2\right)} \Rightarrow k^2 = \frac{1}{1 - v^2/c^2}$$

Taking Square-root.

$$k = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma \quad (\text{say}) \quad \text{---} \quad (5)$$

if $\beta = \frac{v}{c}$ (speed parameter) $\Rightarrow \gamma = \frac{1}{\sqrt{1 - \beta^2}}$

using equation 5 in 1

$$\boxed{x' = \gamma(x - vt)}$$

Now for value of t'

$$② \Rightarrow x = k(x' + vt')$$

$$\frac{x}{k} = x' + vt'$$

$$\frac{x}{k} - x' = vt'$$

$$vt' = \frac{x}{k} - x'$$

$$t' = \frac{x - kx'}{kv} \quad ⑥$$

put $x' = k(x - vt)$ in equ ⑥

$$t' = \frac{x - k[k(x - vt)]}{kv}$$

$$t' = \frac{x - k^2x + k^2vt}{kv}$$

$$t' = k\left[\frac{k^2vt - (k^2-1)x}{k^2v}\right]$$

$$t' = k\left[\frac{k^2vt - (k^2-1)x}{k^2v}\right]$$

$$t' = k\left[\frac{k^2vt}{k^2v} - \frac{(k^2-1)}{k^2v}x\right]$$

$$t' = k\left[t - \left(\frac{k^2-1}{k^2}\right)\frac{x}{v}\right] \quad ⑦$$

Since

$$k = \frac{1}{\sqrt{1 - v^2/c^2}} \Rightarrow k^2 = \frac{1}{1 - v^2/c^2}$$

$$k^2 - 1 = \frac{1}{1 - v^2/c^2} - 1$$

$$k^2 - 1 = \frac{1 - 1 + v^2/c^2}{(1 - v^2/c^2)}$$

$$\frac{k^2-1}{k^2} = \left(\frac{v^2/c^2}{1 - v^2/c^2}\right) \Big/ \left(\frac{1}{1 - v^2/c^2}\right) = \frac{k^2-1}{k^2} = \frac{v^2}{c^2} \quad ⑧$$

using eqn (3) in (4)

$$t' = k\left[t - \frac{v^2}{c^2}\frac{x}{v}\right]$$

$$\boxed{t' = \gamma\left(t - \frac{v}{c^2}x\right)} \qquad \because k = \gamma$$

Hence proved the Lorentz transformation are

$x' = \gamma(x - vt)$ where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

$y' = y$ & $\beta = v/c$

$z' = z$

$t' = \gamma\left(t - \frac{v}{c^2}x\right)$

Question:

Show that Lorent'z Transform satisfy the 2nd postulate of Special theory of relativity? (Principle of constancy of speed of light).

Solution:

According to the Lorent'z Transformation we have

$$x' = \gamma(x - vt)$$

Differentiate w.r.t t'

$$\frac{dx'}{dt'} = \gamma\frac{d}{dt'}(x - vt) \qquad \text{where } t' \neq t$$

$$\frac{dx'}{dt'} = \gamma\frac{d}{dt}(x - vt)\frac{dt}{dt'} \qquad \text{(By chain rule)}$$

$$\frac{dx'}{dt'} = \gamma\left[\frac{dx}{dt} - v\right]\frac{dt}{dt'}$$

By considering the light ray instead of particle So speed of light $= c$

$$c' = \gamma(c - v)\frac{dt}{dt'} \quad \text{--- (1)}$$

Again by using Lorentz Transformation.

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

Differentiate w.r.t t

$$\frac{dt'}{dt} = \gamma \frac{d}{dt}\left(t - \frac{v}{c^2}x\right)$$

$$\frac{dt'}{dt} = \gamma\left(\frac{dt}{dt} - \frac{v}{c^2}\frac{dx}{dt}\right)$$

$\because \frac{dx}{dt} = c$ speed of light

$$\frac{dt'}{dt} = \gamma\left(1 - \frac{v}{c^2}(c)\right)$$

$$\frac{dt'}{dt} = \gamma\left(\frac{c - v}{c}\right)$$

$$\frac{dt'}{dt} = \frac{\gamma(c-v)}{c}$$

(Inverting equ)

$$\frac{dt}{dt'} = \frac{c}{\gamma(c-v)} \quad \text{--- (2)}$$

using (2) in (1)

$$c' = \cancel{\gamma(c-v)} \cdot \frac{c}{\cancel{\gamma(c-v)}}$$

$$\boxed{c' = c}$$

Hence the Lorentz transformation satisfies the 2nd postulate of S.T.R that speed of light remain constant in all Intertial frames.

x———————x