# Amplitude Modulation

AM modulation is a technique that is used for broadcast transmission in the 150 KHz to 20 MHz range. This covers the broadcast in long wave (range: 150—300 KHz), medium wave (MW, range: 500-1500 KHz), and short wave (SW, range: 6—20 MHz). Table 1 shows the classification of electromagnetics (EM) waves in term of their frequency (and wave length). EM waves travel close to the speed of light and for all practical purposes they are assumed to be travelling at the speed of light. The frequency $f$ and the wavelength $\lambda$ of an EM wave are related by the following equation:

$$f = C\lambda \qquad\qquad (1)$$

where $C$ is the speed of light (3 X $10^8$ meters/second).

Table 1: Radio frequency classification

| Classification | Frequency ($f$) | Wavelength ($\lambda$) |
|---|---|---|
| Low frequency (LF) | 30 KHz—300 KHz | 10 km—1km |
| Medium frequency (MF) | 300 KHz—3 MHz | 1km—100 m |
| High frequency (HF) | 3 MHz—30 MHz | 100 m—10 m |
| Very high frequency (VHF) | 30 MHz—300 MHz | 10 m—1 m |
| Ultra high frequency (UHF) | 300 MHz—3GHz | 1 m—10 cm |
| Super high frequency (SHF) | 3 GHz—30 GHz | 10 cm—1 cm |
| Extremely high frequency (EHF) | 30 GHz—300 GHz | 1 cm—1 mm |

EM waves with wavelengths 30 cm or larger are referred to as *radio waves*. EM waves with wavelengths less than 30 cm but more than one cm are referred to as *microwaves*. EM waves with wavelength smaller than one cm are referred to as *millimeter waves*. EM waves exhibit different behaviors depending upon the frequency (or the wavelength) of the wave. EM waves propagate through one or more of the three main mechanisms: (1) direct wave, in which wave travels straight, (2) reflected wave, it is formed when a direct wave cannot penetrate a medium and is reflected back or in a different direction, and (3) surface wave, in which wave induces currents at the surface of the earth and because of that it bends towards the surface of the earth.

Long waves largely propagate through surface waves. As the frequency of the radio waves increases (wavelength decreases) they are not able to induce currents on the surface of the earth. Consequently, they no longer remain able to travel through the surface wave. Furthermore, the radio waves up to the shortwave range are not able to penetrate upper layer of atmosphere called

ionosphere and are reflected back to earth. The ionosphere spread somewhere from 50 to 200 miles above sea level and is filled with ions.

There are three layers of ionosphere: D, E, and F layers. The D layer is closest to earth at about 60 miles above sea level. Long waves are not able penetrate the D layer and any portion of the long wave directed towards sky is reflected back to earth at the D layer. The next layer of the ionosphere is called E layer, which is approximately 80 miles above the sea level. Radio waves in the medium wave range are able to penetrate the D layer but are not able to penetrate the E layer and are reflected back to earth at the E layer. Radio waves in the short wave range are able to penetrate both the D and E layers; however, they are not able to penetrate the final layer of the ionosphere (F layer, which is about 150 miles above the sea level) and are reflected back to earth at that layer.

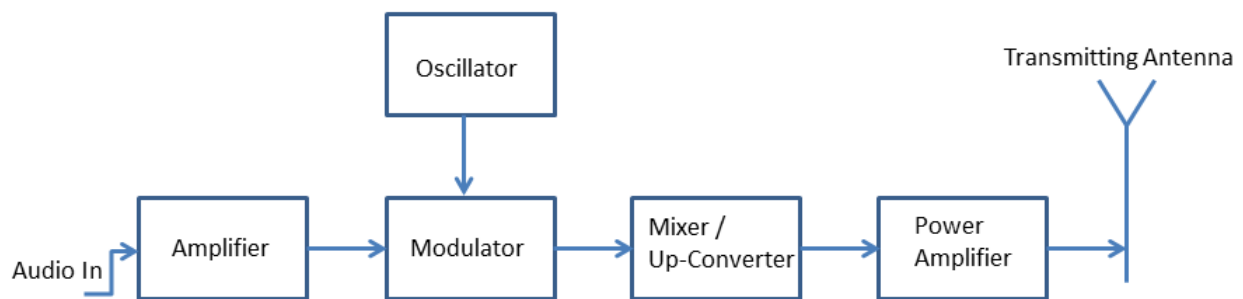Figure 1 shows a functional block diagram of an AM transmitter. The AM transmitter works as follows:



Figure 1: Functional block diagram of an AM transmitter.

The audio signal coming from the microphone (or other sources) is amplified and fed to the amplitude modulator. An AM channel has a bandwidth of 10 KHz. A local oscillator provides the modulator with a high frequency carrier signal that could also be the broadcast frequency of the AM transmitter. In other instances, the AM signal is converted to a higher frequency through a *mixer*. Modulated signal from the mixer is fed to RF coupler and is finally transmitted in the air.

Figure 2 shows a functional block diagram of an AM receiver. The AM receiver works as follows:

The received RF signal first goes through as RF amplifier followed by a band-pass filter (BPF). It is then fed to a mixer to down-convert the RF frequencies to intermediate frequencies (IF, if needed). Next step involves BPF and amplification of the IF signals followed by AM detection (demodulation). Finally, the demodulated signal goes though low-pass filter (LPF) and amplification before being fed to the speaker. In this course, you will be involved in the design

and analysis of oscillators, modulators, demodulators (both AM and FM that will be discussed later), mixers (down/up frequency converters), and filters.
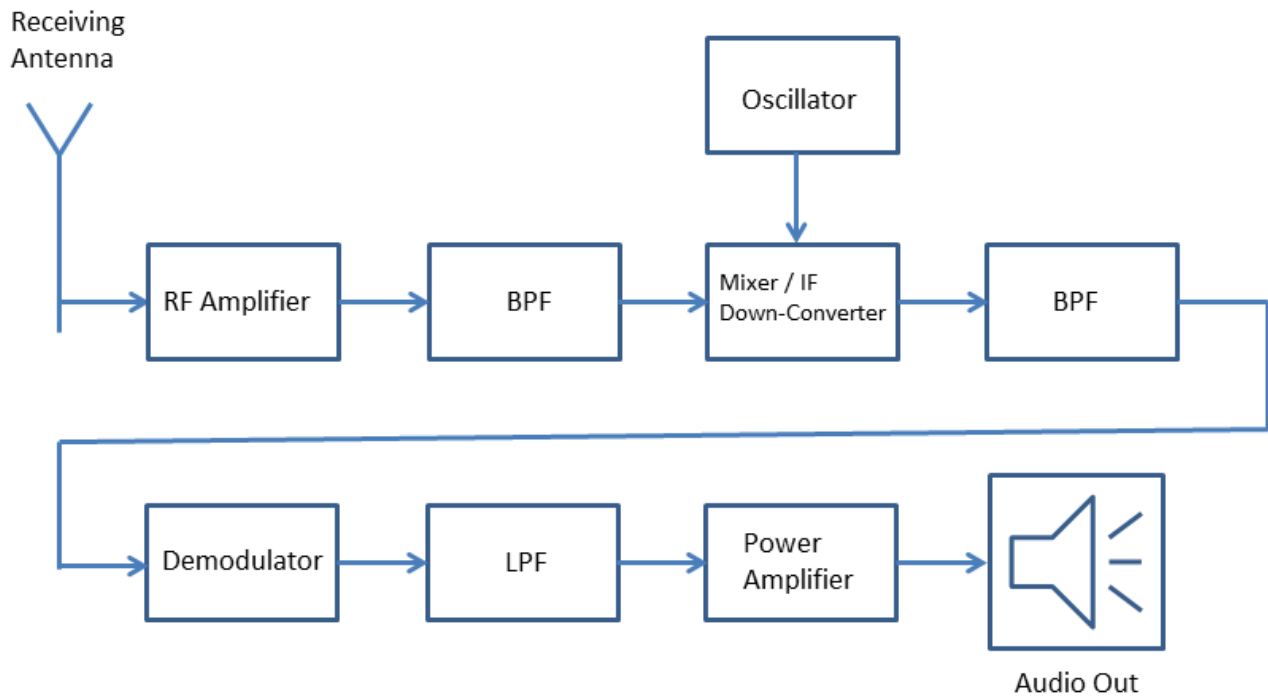


Figure 2: Functional block diagram of an AM receiver.

*Mathematical Foundation of Amplitude Modulation and Demodulation*

In amplitude modulation technique the amplitude of a high frequency carrier is changed with variations in the low frequency message signal. Let us define m(t) and c(t) as the massage and carrier signals, respectively. The amplitude modulated signal, $m_c(t)$ is then given by

$$m_c(t) = m(t)c(t) \qquad (2)$$

Figure 3 shows an amplitude modulated signal along with the original message signal. It can be seen that the amplitude of the high frequency carrier is changing with the message signal. In essence, the message signal has *wrapped* itself around the carrier and can be retrieved through the AM demodulation. A full explanation of amplitude modulation requires an understanding of the frequency domain representation of electrical signals through Fourier transform. Electrical signal can be represented in terms of their behavior with respect to time (time-domain representation) as well as with respect to frequency (frequency-domain representation). We'll represent signals in time-domain in lower case letters and signals in frequency domain in upper case letters. For example,
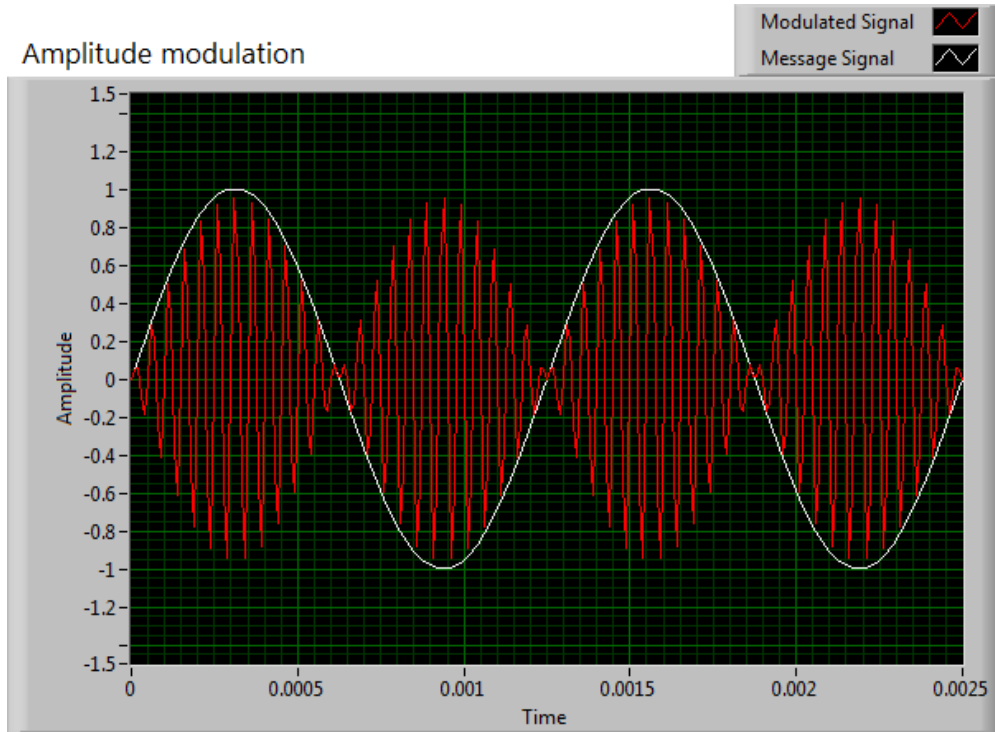
Figure 3: Amplitude modulated signal.

v(t) = voltage in time domain

V(f) = voltage in frequency domain

A frequency domain representation of a time domain signal is found by computing Fourier transform of that signal. The Fourier transform of a time domain signal v(t) is defined as

$$F[v(t)] = V(f) = \int_{-\infty}^{\infty} v(t)e^{-j2\pi ft}\, dt. \qquad (3)$$

The inverse Fourier transform of a frequency domain signal V(f) is defined as

$$F^{-1}[V(f)] = v(t) = \int_{-\infty}^{\infty} V(f)e^{j2\pi ft}\, df. \qquad (4)$$

In the analysis that follows we'll represent a time domain signal and its Fourier transform (FT) as a pair separated by a double arrow with the time domain signal on the left hand side and its FT on the right hand side. That is,

v(t) ↔V(f)

*Properties of the Fourier transform:*

1. It's a complex function—it has a magnitude and a phase (angle).

2.

$$V(0) = \int_{-\infty}^{\infty} v(t)\, dt \qquad (5)$$

3.

$$V(-f) = V^*(f) \qquad (6)$$

Magnitude exhibits even symmetry; that is

$$|V(-f)| = |V(f)| \qquad (7)$$

Phase exhibits odd symmetry; that is

$$\emptyset_{V(-f)} = -\emptyset_{V(f)} \qquad (8)$$

4. Superposition

If v(t) = a₁ v₁(t) + a₂ v₂(t) then

$$V(f) = F[v(t)] = a_1 F[v_1(t)] + a_2 F[v_2(t)] \qquad (9)$$

or

$$V(f) = a_1 V_1(f) + a_2 V_2(f) \qquad (10)$$

5. Time delay

$$v(t - t_d) \leftrightarrow V(f) e^{-j2\pi f t_d} \qquad (11)$$

6. Scaling

$$v(at) \leftrightarrow \frac{1}{a} V\left(\frac{f}{a}\right) \dots \dots . a \neq 0 \qquad (12)$$

7. Modulation

$$v(t) e^{j2\pi f_o t} \leftrightarrow V(f - f_o) \qquad (13)$$

Recall that the amplitude modulated signal, $m_c(t)$ is given by

$$m_c(t) = m(t)c(t) \tag{14}$$

If the carrier is a sinusoidal signal such as $V_{cp}\cos\omega_o t$, the Fourier transform of the AM signal can be found as follows:

First note that

$$\cos\omega_o t = \frac{1}{2}\left(e^{j\omega_o t} + e^{-j\omega_o t}\right) \tag{15}$$

Therefore, AM signal can be expressed as

$$m_c(t) = m(t)c(t) = m(t)V_{cp}\cos\omega_o t = \frac{V_{cp}m(t)}{2}e^{j\omega_o t} + \frac{V_{cp}m(t)}{2}e^{-j\omega_o t} \tag{16}$$

The Fourier transform of the AM signal is

$$m_c(t) \leftrightarrow M_c(f) = \frac{V_{cp}}{2}M(f - f_o) + \frac{V_{cp}}{2}M(f + f_o) \tag{17}$$

It can be seen from Eq. (17) that the FT of the AM signal is the FT of the message signal (with some amplitude scaling) shifted to the frequency, $f_o$, of the sinusoidal carrier c(t). Figure 4 shows the FT of the message signal as well as the modulated signal. Note that the bandwidth of the original message signal is $f_m$; however, the modulated signal has a bandwidth of $2f_m$ as it is made of two sideband of m(t). This kind of AM modulation is known as double sideband AM (DSB AM). In reality, only one sideband is needed for demodulation because any single sideband contains all the information needed to retrieve original message signal. Therefore, in many practical applications one sideband is removed and only a single sideband is transmitted in the AM signal to reduce the bandwidth. This kind of AM modulation is known as the single sideband (SSB) AM.

If the AM signal given by the Eq. (17) is multiplies by the carrier signal again the FT of the new signal is given by

$$m_c(t)c(t) = g_c(t) \leftrightarrow G_c(f) = \left[\frac{V_{cp}}{2}M_c(f - f_o) + \frac{V_{cp}}{2}M_c(f + f_o)\right]. \tag{18}$$

Substituting $M_c(f)$ from Eq. (17) into Eq. (18) we get

$$G_c(f) = \frac{V_{cp}}{2}\left[\frac{V_{cp}}{2}M(f - f_o - f_o) + \frac{V_{cp}}{2}M(f - f_o + f_o)\right]$$
$$+ \frac{V_{cp}}{2}\left[\frac{V_{cp}}{2}M(f + f_o - f_o) + \frac{V_{cp}}{2}M(f + f_o + f_o)\right]$$

or

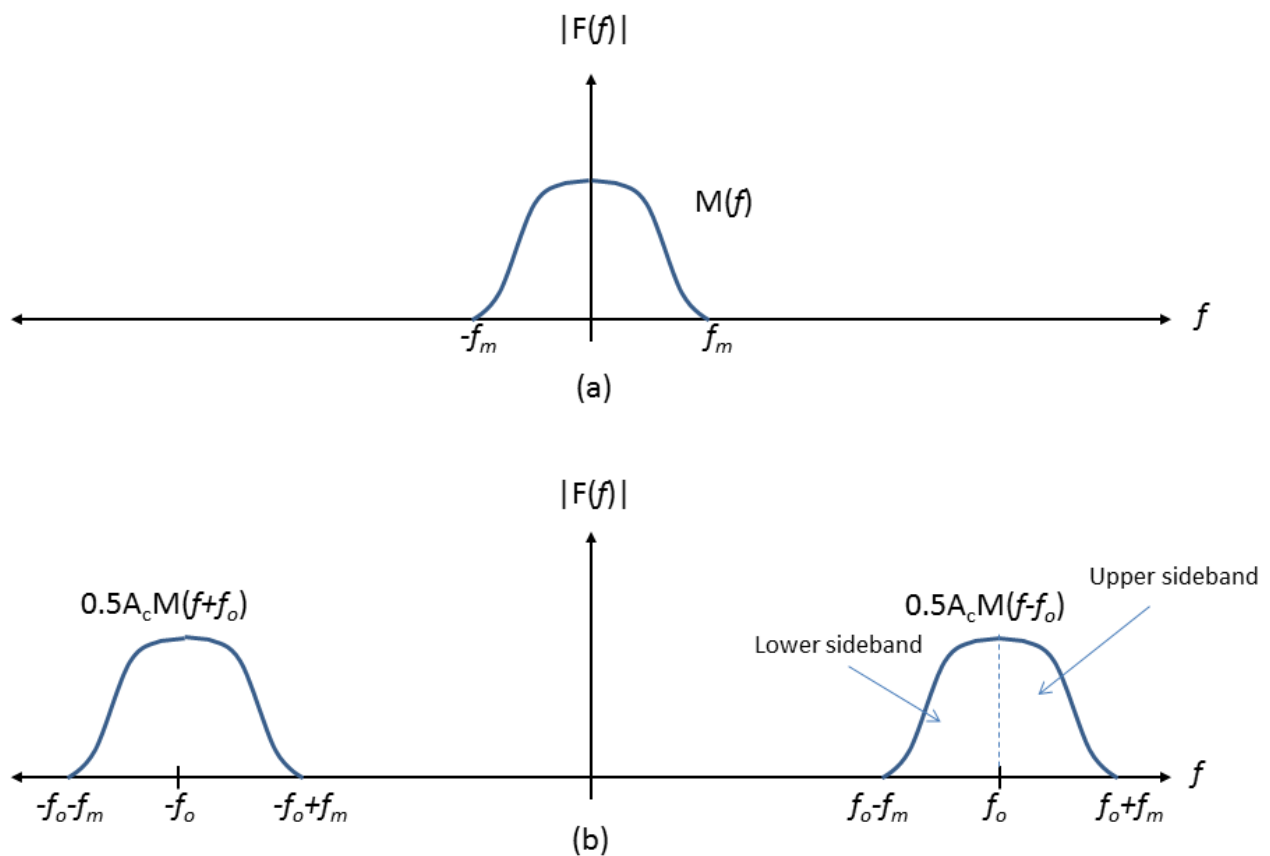$$G_c(f) = \frac{V_{cp}V_{cp}}{4}[M(f - 2f_o) + M(f) + M(f) + M(f + 2f_o)]$$



(a)



(b)

Figure 4: (a) Fourier transform of the message signal m(t). (b) Fourier transform of the modulated signal.

or

$$G_c(f) = \frac{K_c}{2}M(f) + \frac{K_c}{4}[M(f - 2f_o) + M(f + 2f_o)] \qquad (19)$$

where $K_c = V_{cp}V_{cp}$.

Eq. (19) shows that multiplying the modulated signal with the carrier results in shifting the spectrum of the message signal to frequencies of $2f_o$ and $-2f_o$ as well as to the origin. Figure 5 shows the FT of Eq. (19). Therefore, multiplication of the AM signal with carrier results in demodulation of the message signal. At this point, a low-pass filter can be used to retrieve the message signal. This method of demodulation is known as *product detection*. This seemingly simple method of demodulation, however, is not easy to implement because it requires the generation or retrieval of the carrier signal at the receiver that must be in-phase with the original carrier signal, which is not a trivial task. In practice, a simple circuit knows as *envelop detector* followed by a low-pass filter is used for AM demodulation.
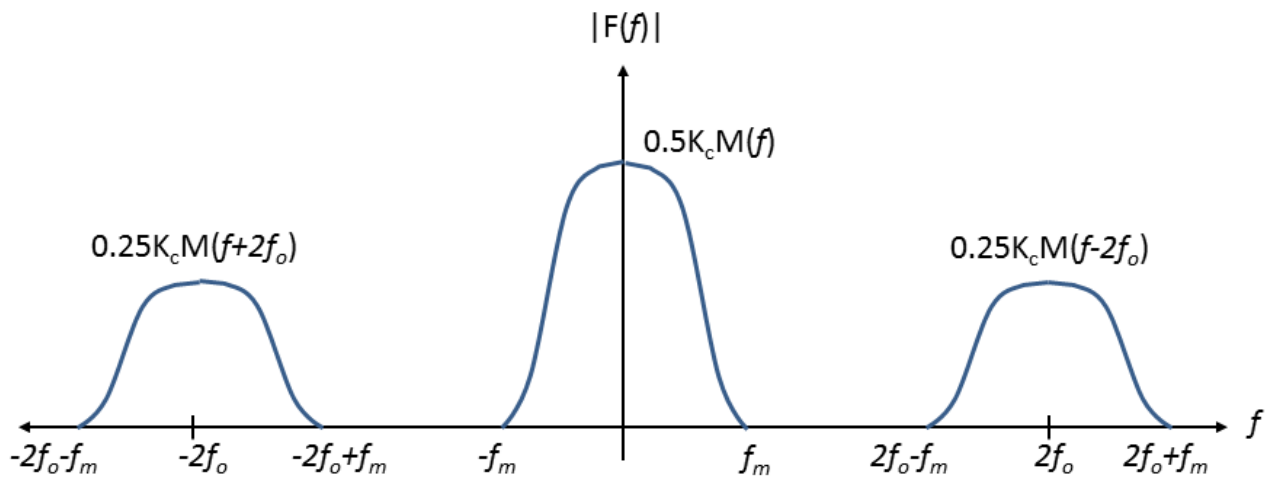


Figure 5: Fourier transform of the product of the AM signal and the carrier.

Note that the modulated signal shown in Fig. 1 has the message signal (and it's mirror image, which $180°$ out of phase) riding the carrier both in positive as well as negative halves. Demodulation of this signal is not possible through a simple envelope detector. The reason for this is that the envelope detector follows amplitude variations in the modulated signal only in the positive half and, therefore, the signal retrieved by the envelope detector is erroneous.

In practice, the AM signal is generated such that the message signal rides the carrier in the positive half and its mirror image rides the message signal in the negative half. This is achieved by adding the carrier $c(t)$ to the modulated signal $m_c(t)$ and the resultant signal $m_{AM}(t)$ is given by

$$m_{AM(t)} = m_c(t) + c(t) = \mu\, m(t)c(t) + c(t)$$
$$= c(t)\big(1 + \mu\, m(t)\big). \tag{19}$$

The AM signal generated by Eq. (19) is shown in Fig. 6 with μ = 1. It can be seen that the envelope representing the message signal is positive all the time and a simple envelope detector can be used to retrieve the message signal. μ is called the modulation index. For μ > 1 the envelop crosses the horizontal axis and goes into the negative half and, therefore, an envelope detector can no longer can be used to retrieved the message signal. This state of AM modulation is called *over-modulation* (Fig. 7).
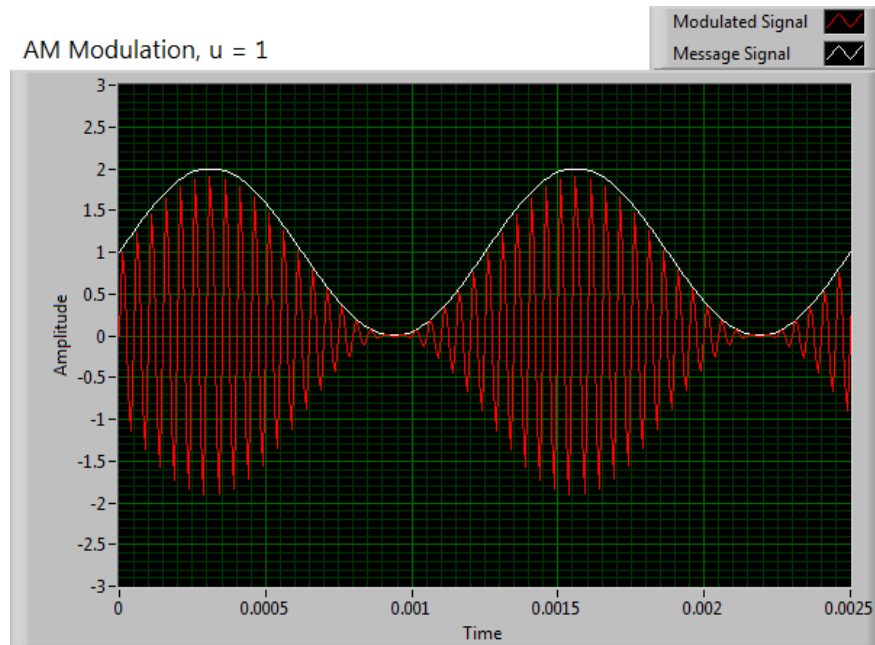


Figure 6: AM modulation with μ = 1.

On the other hand, when μ < 1, the envelope always stay in the positive half and an envelope detector can be used to retrieve the message signal. However, when there is no message present, the transmitted signal contains the carrier with amplitude inversely proportional to μ. This state of AM modulation is called *under-modulation* (Fig. 8). Under-modulation results in some waste of power when there is no message present; however, it makes the demodulation much simpler.

The modulation index, μ, can also be computed from the waveform of the AM signal and is given by (see Fig. 8)

$$\mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}. \tag{20}$$

Also, the peak amplitude of the carrier, $V_{cp}$, and the message signal, $V_{mp}$, can be computed as:

$$V_{cp} = \frac{V_{max} + V_{min}}{2} \tag{21}$$
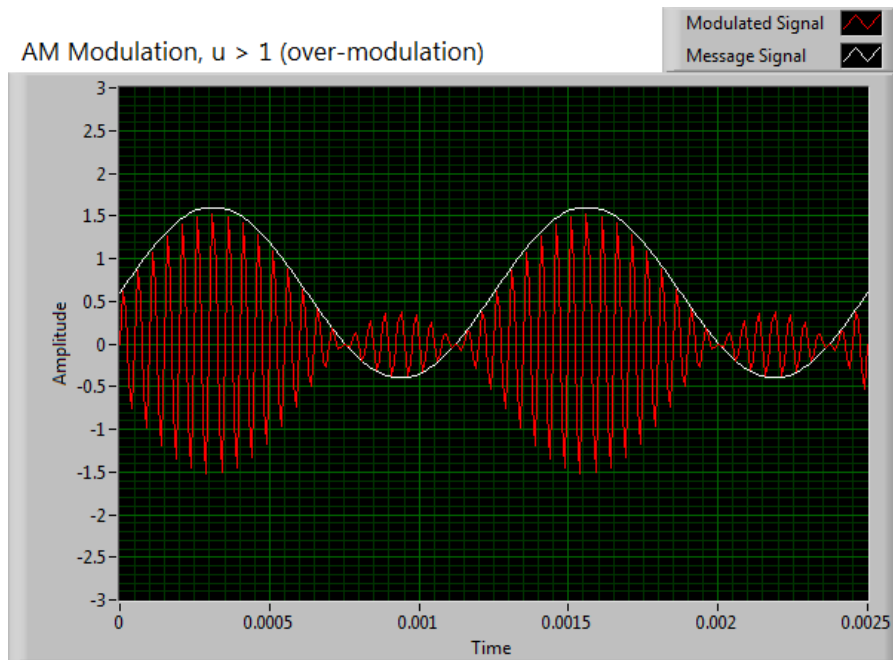
and

$$V_{mp} = \frac{V_{max} - V_{min}}{2}.$$
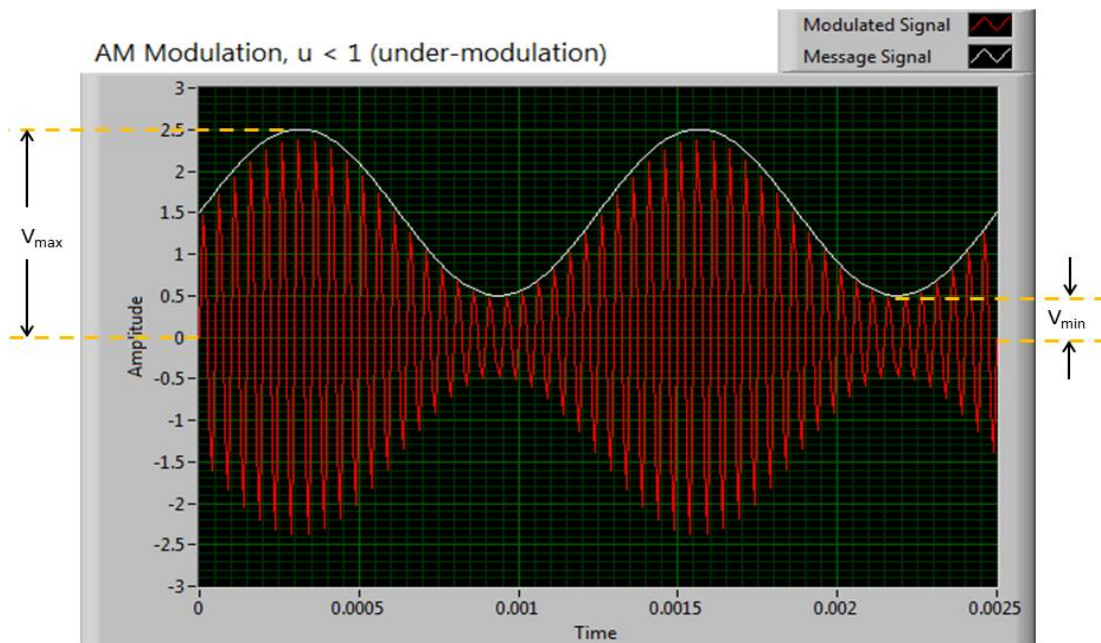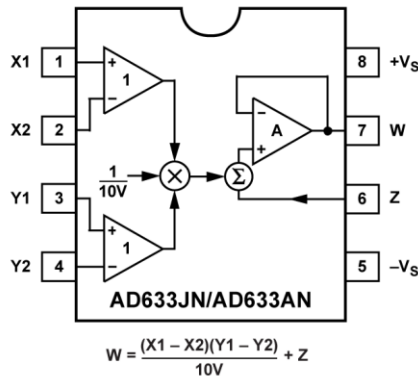
(22)



Figure 7: AM modulation with μ > 1.

Figure 8: AM modulation with μ < 1.

In this set of experiments, you'll use an analog multiplier chip, AD633, to build amplitude modulator and a product detector for demodulation. Figure 9 shows the functional diagram and pin configuration of AD633 chip.



$$W = \frac{(X1 - X2)(Y1 - Y2)}{10V} + Z$$

### Lead PDIP Pin Function Descriptions

| Pin No. | Mnemonic | Description |
|---------|----------|-------------|
| 1 | X1 | X Multiplicand Noninverting Input |
| 2 | X2 | X Multiplicand Inverting Input |
| 3 | Y1 | Y Multiplicand Noninverting Input |
| 4 | Y2 | Y Multiplicand Inverting Input |
| 5 | −$V_S$ | Negative Supply Rail |
| 6 | Z | Summing Input |
| 7 | W | Product Output |
| 8 | +$V_S$ | Positive Supply Rail |

Figure 9: Functional diagram and pin configuration of AD633 chip.

**Assignment 3.1:** Build the amplitude modulator shown in Figure 10. Use the following values:

Frequency of the message signal $=f_m = 1$ KHz
Amplitude of the message signal $= 12$ V p-p
Frequency of the message signal $=f_c = 200$ KHz
Amplitude of the message signal $= 4$ V p-p
$V_{SS} + = +15$V
$V_{SS}- = -15$V
$R_1 = 1$ KΩ

$R_f = 1\ K\Omega$

$C_1 = C_{ps} = 0.1\ \mu F$

$C_f = 0.05\ \mu F$

    (a) Record the waveforms of the signal at the designated points in the circuit (Figures 11 through 13 show the expected output signal).

    (b) Remove the diode $D_1$ and repeat part (a).

    (c) Now reconnect the diode $D_1$ and connect the pin 6 of AD633 chip to ground and repeat part (a).
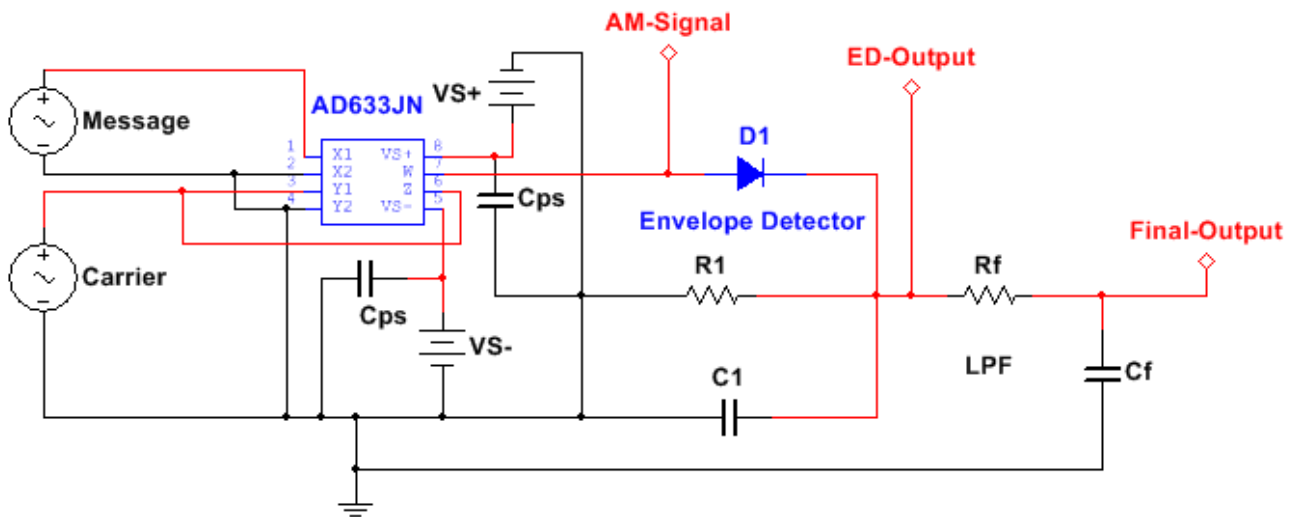
Explain your results.



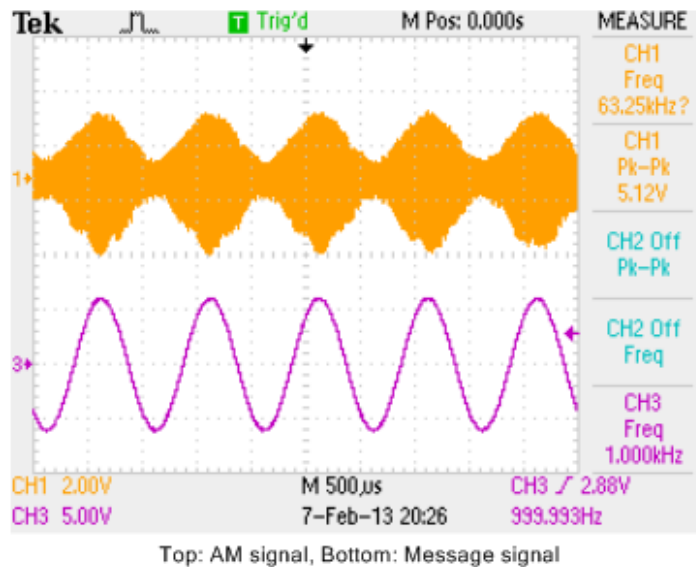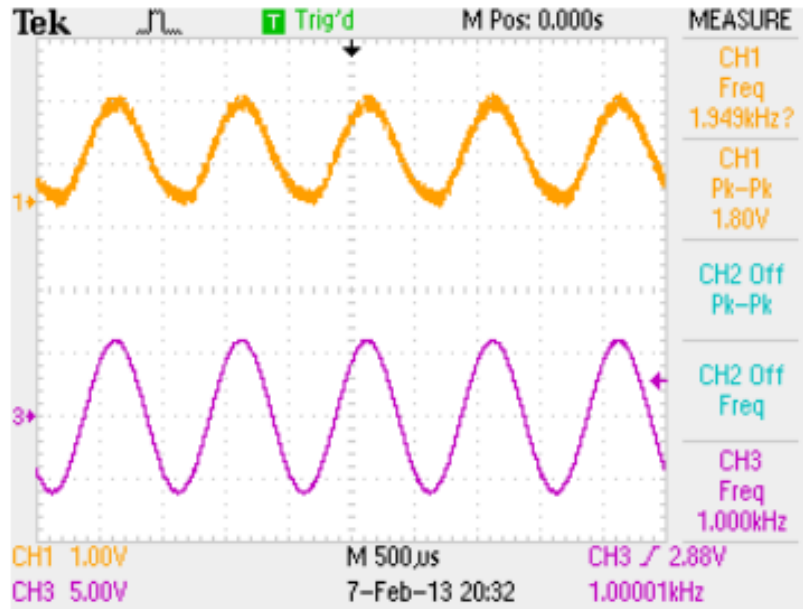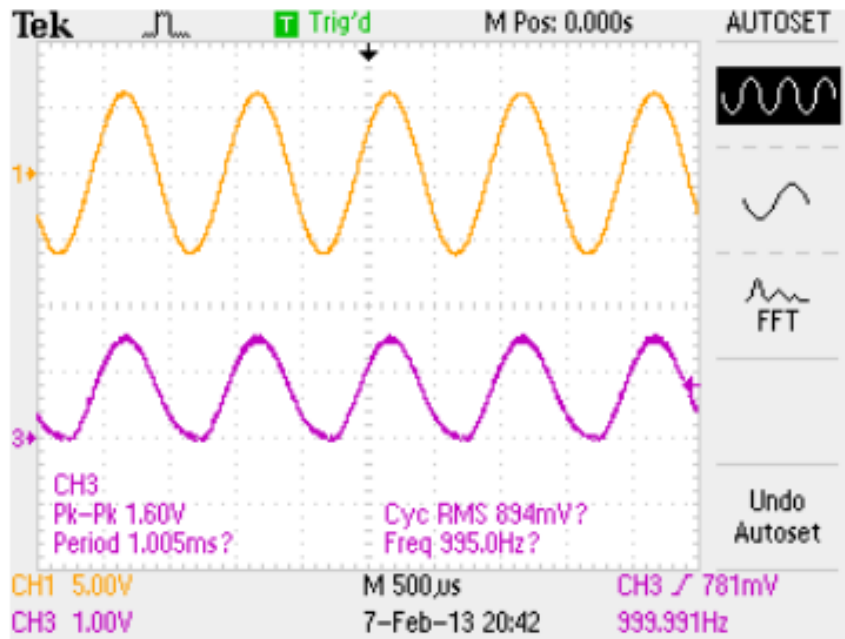Figure 10: Amplitude modulator with demodulation through envelope detection.



Top: AM signal, Bottom: Message signal

Figure 11: Modulated signal; carrier at 200 KHz and message at 1 KHz.

Figure 12: Top: Output of the envelope detector. Bottom: Original message signal.

Figure 13: Top: Original message signal. Bottom: Output of the low-pass filter.

**Assignment 3.2:** Build the amplitude modulator shown in Figure 14. Use the values from the assignment 3.1. Use the LP filter from Figure 10 instead of the op-amp based LP filter and the amplifier shown in Figure 14. Record the waveforms of the signal at the designated points in the circuit (Figures 15—16 show the expected output waveforms). Explain your results.
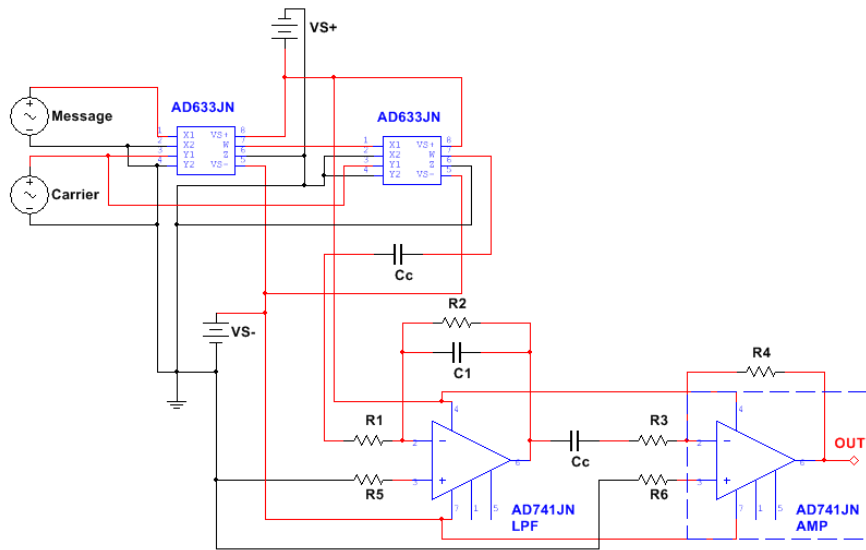


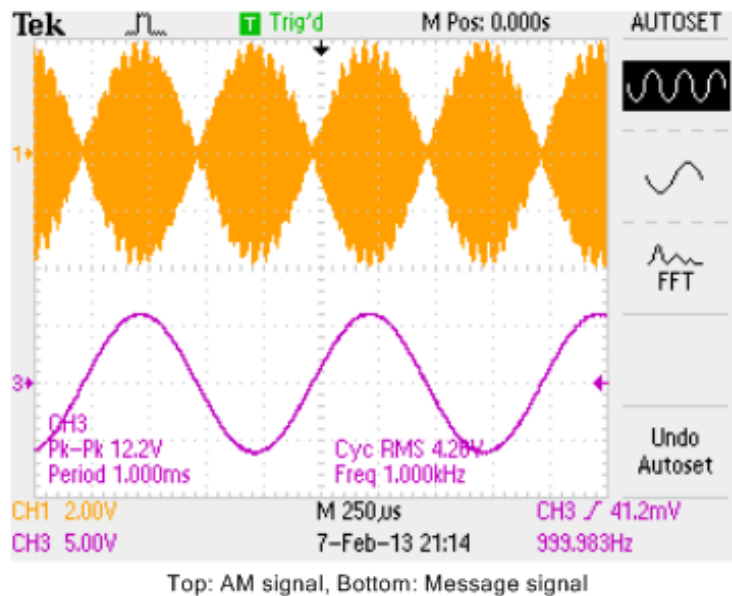Figure 14: Amplitude modulator with demodulation through product detection.



Top: AM signal, Bottom: Message signal

Figure 15: Modulated signal; carrier at 200 KHz and message at 1 KHz.
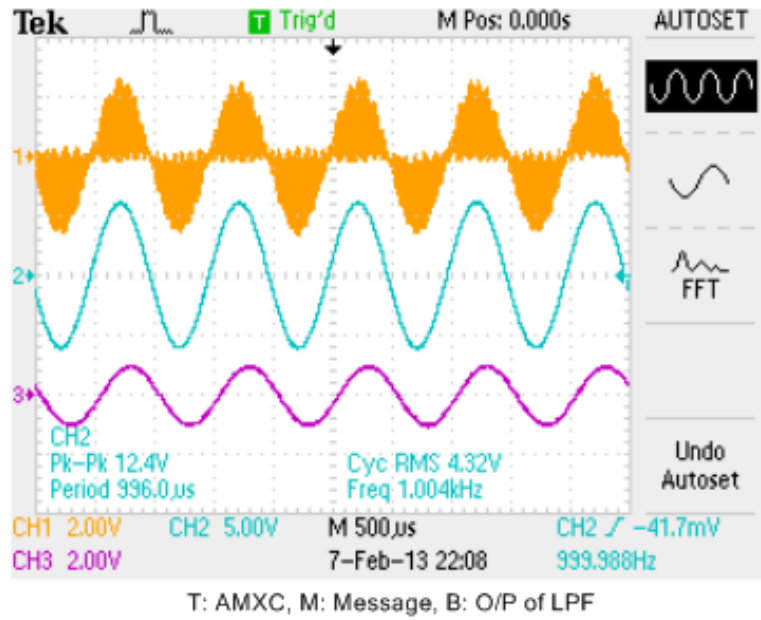


T: AMXC, M: Message, B: O/P of LPF

Figure 16: Top: Product signal. Bottom: Output of the low-pass filter.