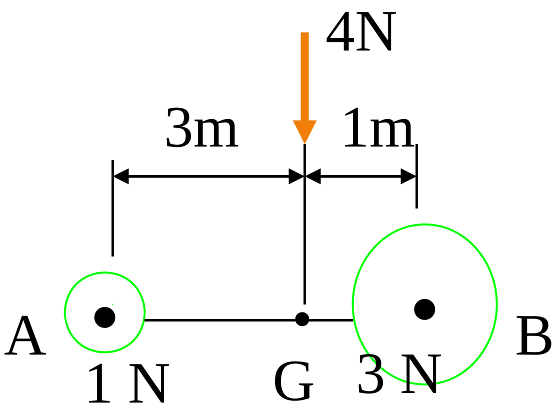
Chapter 9

CENTER OF GRAVITY, CENTER OF MASS AND CENTROID FOR A BODY



CONCEPT OF Center of Gravity

& Center of Mass

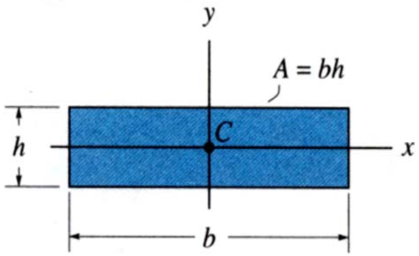
The center of gravity (G) is a point which locates the resultant weight of a system of particles or body.

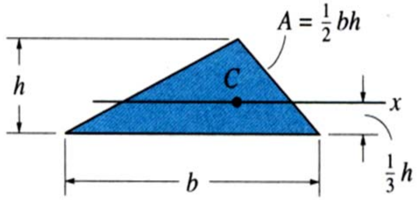
From the definition of a resultant force, the sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at G. For the figure above, try taking moments about A and B.

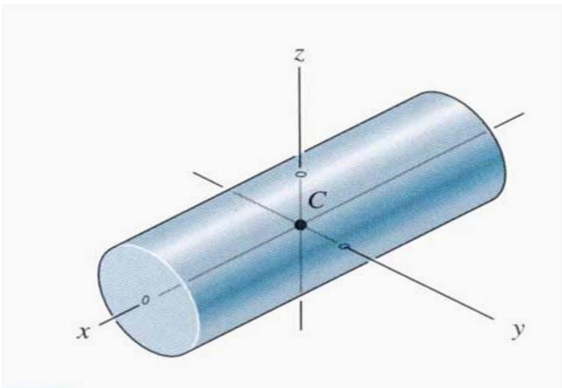
Also, note that the sum of moments due to the individual particle's weights about point G is equal to zero.

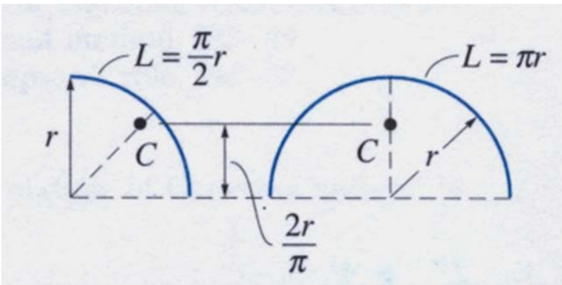
Similarly, the center of mass is a point which locates the resultant mass of a system of particles or body. Generally, its location is the same as that of G.

CONCEPT OF CENTROID









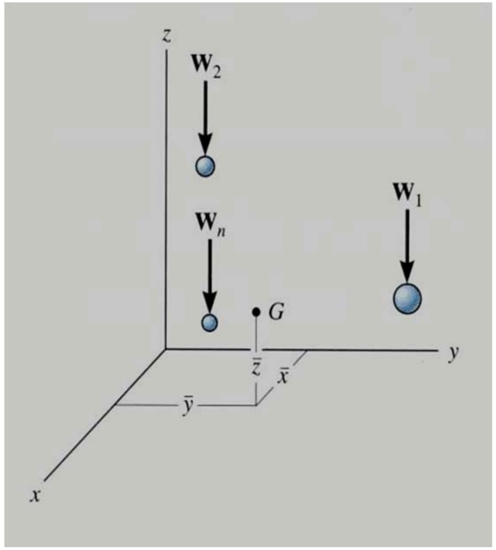
The centroid C is a point which defines the geometric center of an object.

The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogenous (density or specific weight is constant throughout the body).

If an object has an axis of symmetry, then the centroid of object lies on that axis.

In some cases, the centroid is not located on the object.

9.1 CG / CM FOR A SYSTEM OF PARTICLES



Consider a system of *n* particles as shown in the figure. The net or the resultant weight is given as $W_R = \sum W$.

Summing the moments about the y-axis, we get

$$\bar{x}\, W_R = \tilde{x}_1 W_1 + \tilde{x}_2 W_2 + \ldots\ldots\ldots\ldots + \tilde{x}_n W_n$$

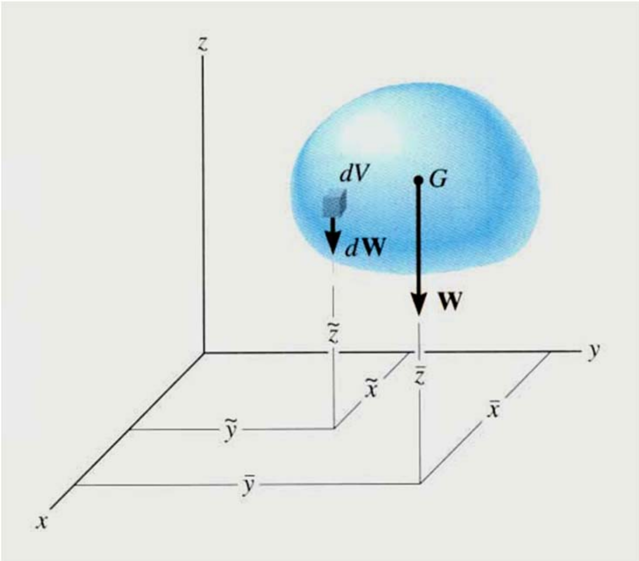
where $\tilde{x}_1$ represents x coordinate of W_1, etc..

Similarly, we can sum moments about the x- and z-axes to find the coordinates of G.

$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} \qquad \bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W} \qquad \bar{z} = \frac{\Sigma \tilde{z} W}{\Sigma W}$$

By replacing the W with a M in these equations, the coordinates of the center of mass can be found.

9.2 CG / CM & CENTROID OF A BODY



A rigid body can be considered as made up of an infinite number of particles. Hence, using the same principles as in the previous slide, we get the coordinates of G by simply replacing the discrete summation sign ($\sum$) by the continuous summation sign ($\int$) and W by dW.

$$\bar{x} = \frac{\int \tilde{x}\, dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\, dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\, dW}{\int dW}$$

Similarly, the coordinates of the center of mass and the centroid of volume, area, or length can be obtained by replacing W by m, V, A, or L, respectively.

STEPS FOR DETERMING AREA CENTROID

1. Choose an appropriate differential element dA at a general point (x,y). Hint: Generally, if y is easily expressed in terms of x (e.g., $y = x^2 + 1$), use a vertical rectangular element. If the converse is true, then use a horizontal rectangular element.

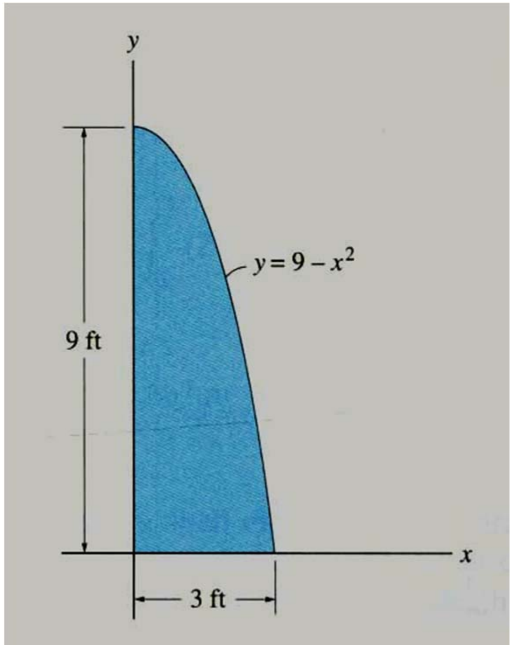
2. Express dA in terms of the differentiating element dx (or dy).

3. Determine coordinates ($\tilde{x}$, $\tilde{y}$) of the centroid of the rectangular element in terms of the general point (x,y).

4. Express all the variables and integral limits in the formula using either x or y depending on whether the differential element is in terms of dx or dy, respectively, and integrate.

Note: Similar steps are used for determining CG, CM, etc.. These steps will become clearer by doing a few examples.

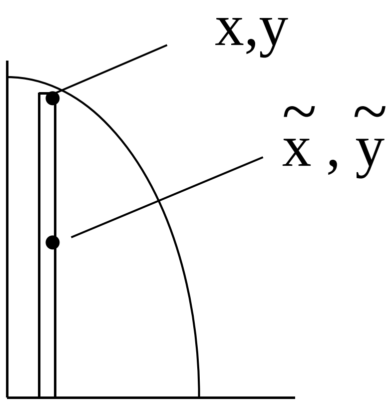
EXAMPLE



Given: The area as shown.

Find: The centroid location $(\bar{x}\ ,\ \bar{y})$

Plan: Follow the steps.



Solution

1. Since y is given in terms of x, choose dA as a vertical rectangular strip.

2. $dA \;=\; y\,dx \;=\; (9 - x^2)\,dx$

3. $\tilde{x} \;=\; x$ and $\tilde{y} \;=\; y / 2$

EXAMPLE (continued)

4. $\bar{x} = \left(\int_A \tilde{x} \, dA \right) / \left(\int_A dA \right)$

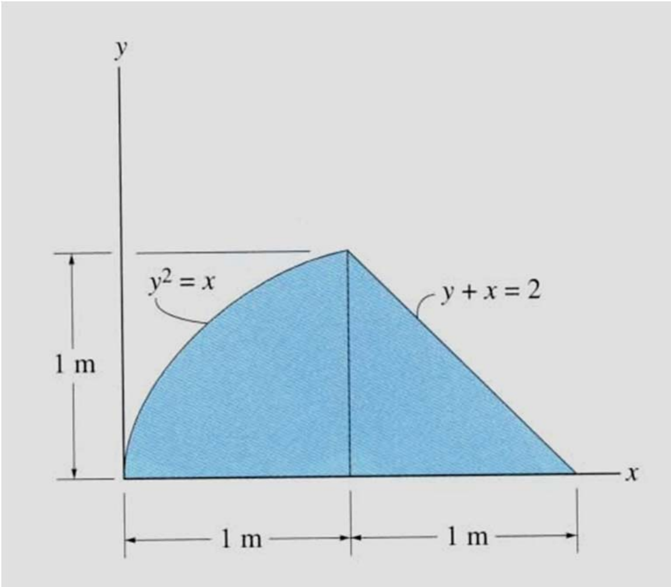
$$= \frac{\int_0^3 x\,(9 - x^2)\,dx}{\int_0^3 (9 - x^2)\,dx} = \frac{\left[\,9\,(x^2)/2 - (x^4)/4\right]_0^3}{\left[\,9x - (x^3)/3\,\right]_0^3}$$

$$= (\,9\,(\,9\,)\,/\,2 - 81\,/\,4\,)\,/\,(\,9\,(\,3\,) - (\,27\,/\,3\,)\,)$$

$$= 1.13 \text{ ft}$$

$$\bar{\mathbf{y}} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{½\,\int_0^3 (9 - x^2)\,(9 - x^2)\,dx}{\int_0^3 (9 - x^2)\,dx} = 3.60 \text{ ft}$$

GROUP PROBLEM SOLVING

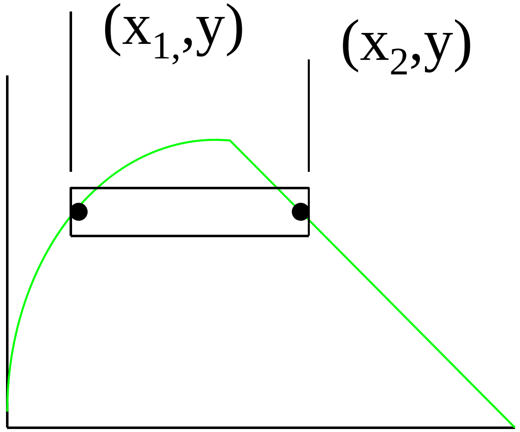


Given: The area as shown.

Find: The $\bar{x}$ of the centroid.

Plan: Follow the steps.

Solution



1. Choose dA as a horizontal rectangular strip.
2. $dA = (x_2 - x_1)\ dy$

   $= ((2 - y) - y^2)\ dy$
3. $x = (x_1 + x_2) / 2$

   $= 0.5 ((2 - y) + y^2)$

GROUP PROBLEM SOLVING (continued)

4. $\bar{x} = (\int_A \tilde{x} \, dA) / (\int_A dA)$

$$\int_A dA = \int_0^1 (2 - y - y^2) \, dy$$

$$[2 y - y^2 / 2 - y^3 / 3]_0^1 = 1.167 \text{ m}^2$$

$$\int_A \tilde{x} \, dA = \int_0^1 0.5 (2 - y + y^2) (2 - y - y^2) \, dy$$

$$= 0.5 \int_0^1 (4 - 4 y + y^2 - y^4) \, dy$$

$$= 0.5 [4 y - 4 y^2 / 2 + y^3 / 3 - y^5 / 5]_0^1$$

$$= 1.067 \text{ m}^3$$

$$\bar{x} = 1.067 / 1.167 = 0.914 \text{ m}$$