Simple Harmonic Motion (SHM)

Overview

(Ref p368)

Terminology for Periodic Motion

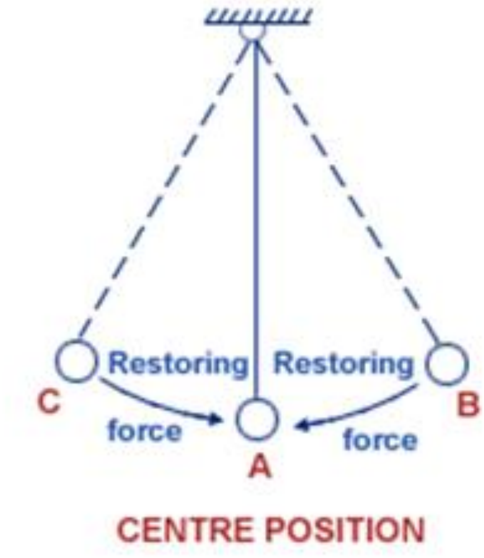
- Period (T)
  - The time, in seconds, it takes for a vibrating object to repeat its motion – seconds per vibration, oscillation or cycle
- Frequency (f)
  - The number of vibrations made per unit time – vibration, oscillation or cycles per second (Hz)
- $T = 1/f$
  - The relationship is reciprocal
- Amplitude (A or x)
  - The displacement from rest position

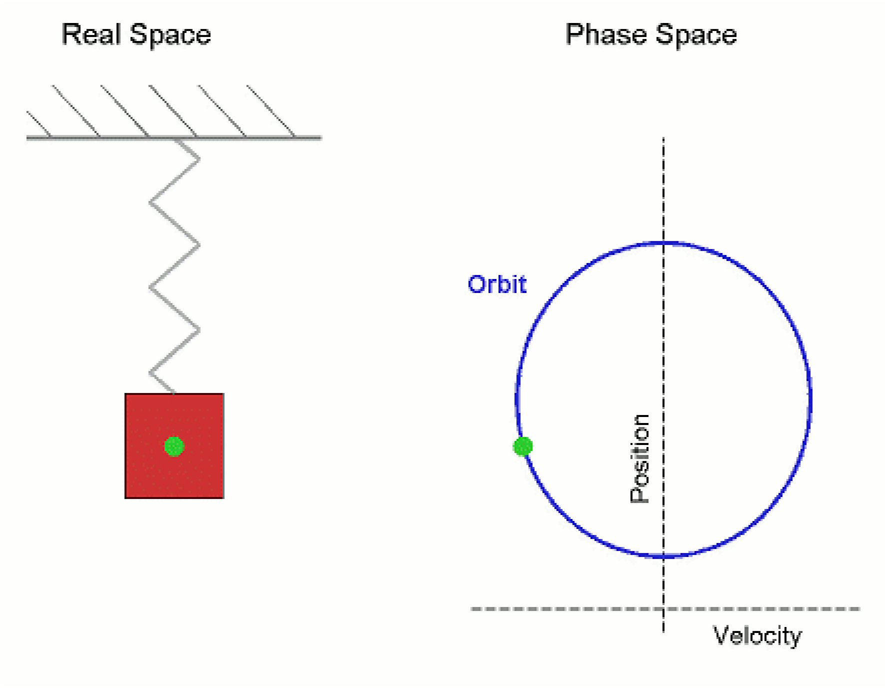
SHM - Description

- An object is said to be in simple harmonic motion if the following occurs:
  - It moves in a uniform path.
  - A variable force acts on it.
  - The magnitude of force is proportional to the displacement of the mass.
  - The force is always opposite in direction to the displacement direction.
  - The motion is repetitive and a round trip, back and forth, is always made in equal time periods.

SHM Visually

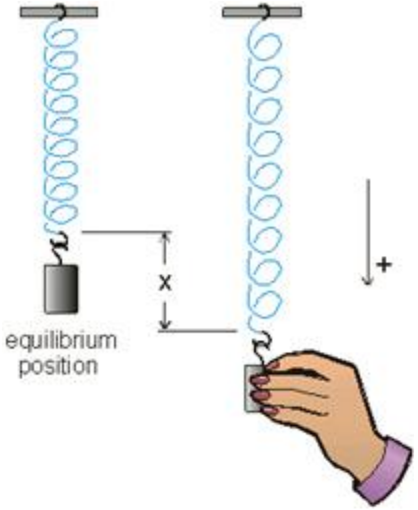
- Examples
  - Spring
  - Pendulum





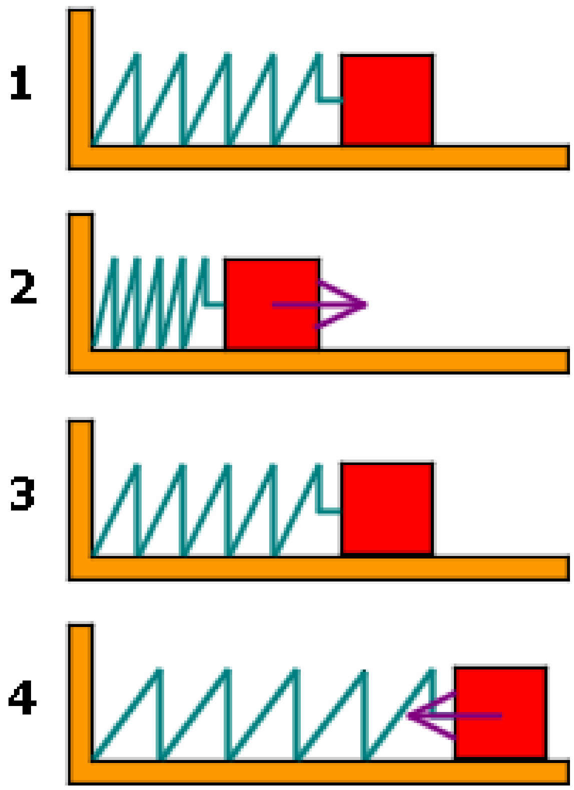
SHM – Hooke's Law

- SHM describes any periodic motion that results from a restoring force (F) that is proportional to the displacement (x) of an object from its equilibrium position.
- $F_{rest} = -kx$, where k = spring constant
- **Note:**
  - **Elastic limit** – if exceeded, the spring does **not** return to its original shape
- Law applies equally to horizontal and vertical models



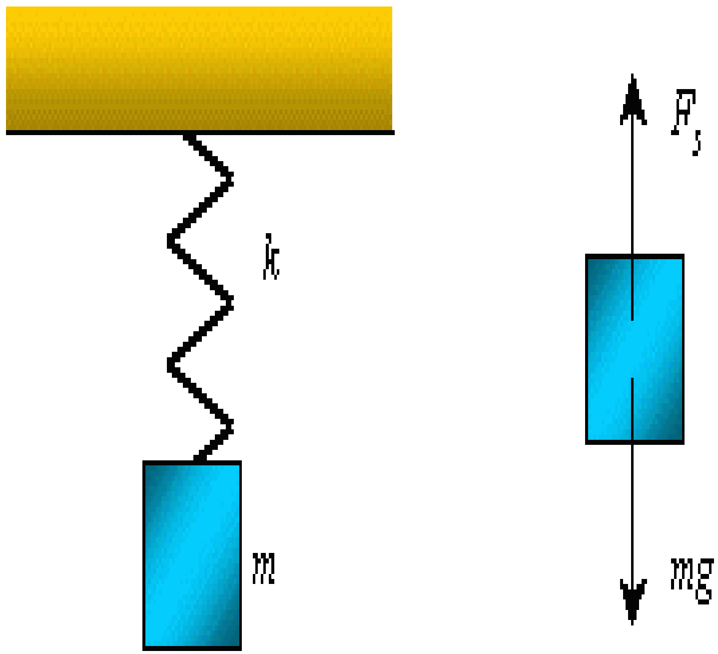
Hooke's Law – Horizontal Springs

- At max displacement (2 & 4), spring force and acceleration reach a maximum and velocity (thus KE) is zero.
- At zero displacement (1 & 3) PE is zero, thus KE and velocity are maximum
- The larger the k value the stiffer the spring
- Negative sign indicates the restoring force is opposite the displacement



Hooke's Law – Vertical Springs

- Hooke's Law applies equally to a vertical model of spring motion, in which the weight of the mass provides a force.
- @ Equilibrium position with no motion:
  - Spring force↑ = weight↓



Practice

- A load of 50 N stretches a vertical spring by 0.15 m. What is the spring constant?
- Solve F = -kx for k
  - 50 = -k*0.15
  - k = - 50/0.15 = 333.3 N/m (drop the – sign)

Mass-Spring System - Period

- The period of a mass-spring can be calculated as follows:

$$T = 2\pi\sqrt{\frac{mass}{spring\ const}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Practice

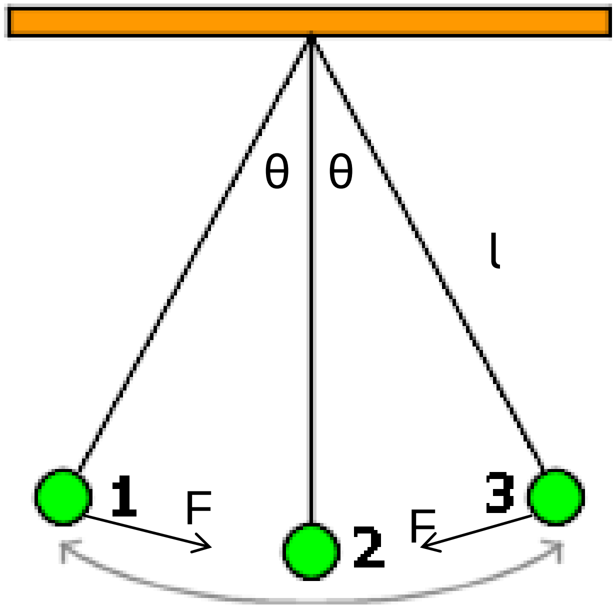
- What is the spring constant of a mass-spring system that has a mass of 0.40 kg and oscillates with a period of 0.2 secs?
- Solve $T = 2\pi\sqrt{\frac{m}{k}}$ for k
  - 0.2 = 2π*√(0.4/k)
  - k = 394.8 N/m

Practice

- If a mass of 0.55 kg stretches a vertical spring 2 cm from its rest position, what is the spring constant (k)?
- Solve F = -kx for k (or ΔF = -k*Δx)
  - k = F/x, where F = weight (mg) of the mass
  - k = mg/x = 0.55 x 9.8/0.02
  - k = 269.5 N/m

SHM - Simple Pendulum

- If a pendulum of length l is disturbed through an angle θ (1 or 3), the restoring force (F) component drives the bob back (and through) the rest (2) position



$$Period(T) = 2\pi\sqrt{\frac{length}{grav\,acc}} = 2\pi\sqrt{\frac{l}{g}}$$

Practice - pendulum

- What period would you expect from a pendulum of length 0.5 m on the moon where g = 1.6 m/s^2?
- Solve T = $2\pi\sqrt{\frac{l}{g}}$
  - T = 2π √(0.5/1.6)
  - T = 3.51 seconds

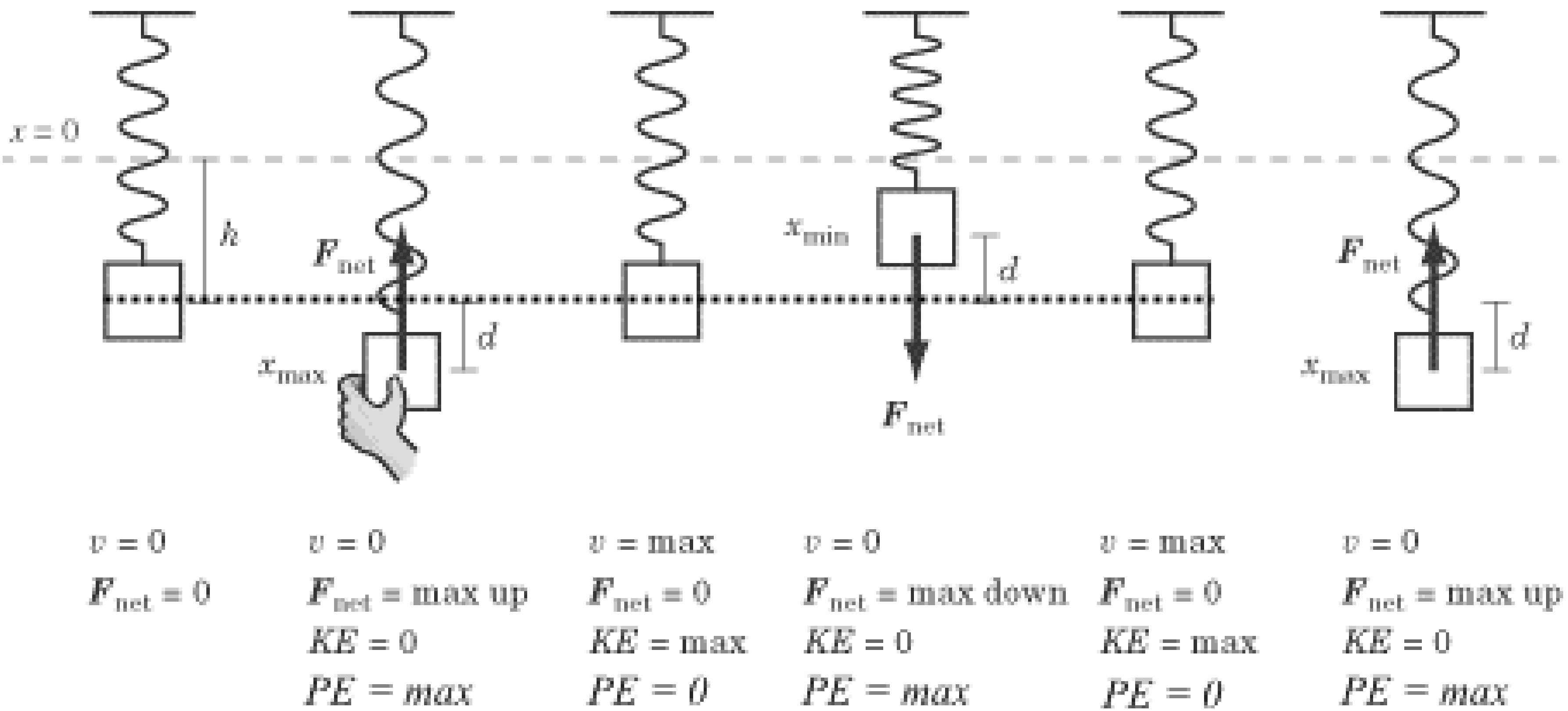
Practice - pendulum

- What is the frequency (f) of a 3 m (l) swing at the North Pole, where g = 9.83 m/s^2?
- Solve $T = 2\pi\sqrt{(l/g)}$
  - $T = 2\pi\sqrt{(3/9.83)}$
  - T = 3.47 sec, therefore…
  - f = 1/T = 0.29 Hz

Energy Considerations for SHM

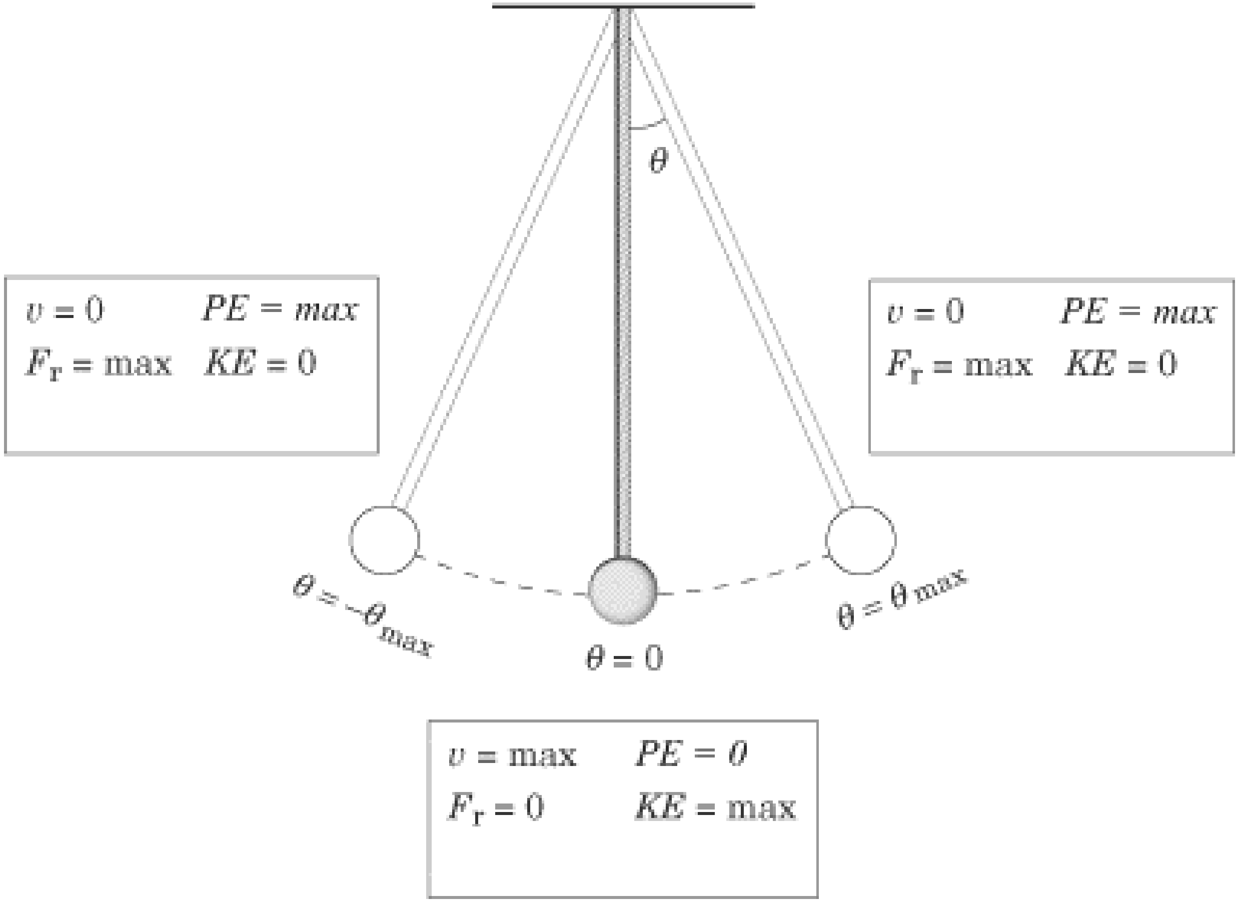
- Springs
  - PE max at maximum displacement
  - KE max while passing through the rest position

Spring Energy Summary



- <u>Pendulums</u>
  - PE max at maximum disturbance angle
  - KE max at bottom of arc

Pendulum Energy summary



Summary formulas

- Period (T) = 1/frequency (f)
  - $T = 1/f$
- Hooke's Law
  - Force = spring const x displacement
  - $F = kx$ (*drop negative sign)*
- Spring period
  - $T = 2\pi\sqrt{(m/k)}$
- Pendulum
  - $T = 2\pi\sqrt{(l/g)}$