CH #8 Techniques of Integration

Ex (8.2)

Integration by Parts

$$\int \underset{I}{u}\,\underset{II}{v}\,dx = u\int v\,dx - \int \int v\,dx \; \frac{d}{dx}(u)\; dx$$

Evaluate

Q1-24

(1) $\int x \sin\frac{x}{2}\,dx$

ILATE

Inverse Trignometric, logrithmetic, Algebric, Trignometric, Exponential

$$\int \underset{I}{x}\,\underset{II}{\sin\frac{x}{2}}\,dx = x\int \sin\frac{x}{2}\,dx - \int \int \sin\frac{x}{2}\,dx \; \frac{d}{dx}(x)\; dx$$

$$\because \int \sin x\,dx = \frac{-\cos x}{1} + C$$

$$= x\,\frac{-\cos\left(\frac{x}{2}\right)}{\frac{1}{2}} - \int \frac{-\cos\left(\frac{x}{2}\right)}{\left(\frac{1}{2}\right)}\,(1)\,dx$$

$$= -2x\cos\left(\frac{x}{2}\right) + 2\int \cos\left(\frac{x}{2}\right)dx$$

$$\because \int \cos x\,dx = \frac{\sin x}{1} + C$$

$$= -2x\cos\left(\frac{x}{2}\right) + 2\left[\frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{1}{2}\right)}\right] + C$$

$$= -2x\cos\left(\frac{x}{2}\right) + 4\sin\left(\frac{x}{2}\right) + C \quad \text{Ans}$$

(5) $\int_1^2 x \ln x \, dx$

ILATE

log Algebra

$\int_1^2 x \ln x \, dx = \int_1^2 \underset{I}{\ln x} \cdot \underset{II}{x} \, dx = ?$

$= \ln x \cdot \int_1^2 x \, dx - \int_1^2 \int_1^2 x \, dx \cdot \frac{d}{dx}(\ln x) \, dx$

$= \ln x \cdot \frac{x^2}{2} \Big|_1^2 - \int_1^2 \frac{x^{\cancel{2}}}{2} \cdot \frac{1}{\cancel{x}} \, dx$

$= \left[\frac{\ln 2 \, (2)^{\cancel{2}}}{\cancel{2}} - \frac{\cancel{\ln(1)}^{\,0} (1)^2}{2} \right] - \frac{1}{2} \int_1^2 x \, dx$

$= 2\ln 2 - \frac{1}{2} \left[\frac{x^2}{2} \Big|_1^2 \right] + C$

$= 2\ln 2 - \frac{1}{4} \left((2)^2 - (1)^2 \right) + C = 2\ln 2 - \frac{1}{4}(3) + C$ Ans

Homework

3, 7, 10, 11, 18, 19, 21, 24

Trignometric ← ILATE → Exponential

(23) $\int e^{2x}\cos 3x\,dx$

$$\int e^{2x}\cos 3x\,dx = \int \underset{I}{\cos 3x}\cdot \underset{II}{e^{2x}}\,dx = \cos 3x\int e^{2x}dx - \int\left(\int e^{2x}dx\ \frac{d}{dx}\cos(3x)\right)dx$$

let $I = \int e^{2x}\cos 3x\,dx$ $\boxed{\because \frac{d}{dx}(\cos x) = -\sin x(1)}$ $\boxed{\because \int e^{x}dx = \frac{e^{x}}{1}+C}$

$$I = \int e^{2x}\cos 3x\,dx = \cos 3x\cdot\frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}\cdot(-\sin 3x(3))\,dx$$

$$= \frac{e^{2x}\cos 3x}{2} + \frac{3}{2}\int \underset{I}{\sin 3x}\cdot \underset{II}{e^{2x}}\,dx$$

$$= \frac{e^{2x}\cos 3x}{2} + \frac{3}{2}\left[\sin 3x\int e^{2x}dx - \int\left(\int e^{2x}dx\ \frac{d}{dx}\sin(3x)\right)dx\right]$$

$\because \frac{d}{dx}\sin x = \cos x(1)$

$$= \frac{e^{2x}\cos 3x}{2} + \frac{3}{2}\left[\sin 3x\cdot\frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}\cos 3x(3)\,dx\right]$$

$$= \frac{e^{2x}\cos 3x}{2} + \frac{3e^{2x}}{4}\sin 3x - \frac{9}{4}\int e^{2x}\cos 3x\,dx$$

$$I = \frac{e^{2x}\cos 3x}{2} + \frac{3}{4}e^{2x}\sin 3x - \frac{9}{4}I$$

$$I + \frac{9}{4}I = \frac{e^{2x}\cos 3x}{2} + \frac{3}{4}e^{2x}\sin 3x + C$$

$$I\left(\frac{4+9}{4}\right) = \frac{e^{2x}\cos 3x}{2} + \frac{3}{4}e^{2x}\sin 3x + C$$

$$I\left(\frac{13}{4}\right) = \frac{e^{2x}\cos 3x}{2} + \frac{3}{4}e^{2x}\sin 3x + C$$

$$I = \frac{4}{13}\left[\frac{e^{2x}\cos 3x}{2} + \frac{3}{4}e^{2x}\sin 3x + C\right]$$ Ans

ILATE

I → Inverse Trign

A → Algebric

(20) $\int_0^{\frac{1}{\sqrt{2}}} 2x \sin^{-1}(x^2)dx$

$\int_0^{\frac{1}{\sqrt{2}}} 2x \sin^{-1}(x^2)dx = 2\int_0^{1/\sqrt{2}} \underset{I}{\sin^{-1}(x^2)} . \underset{II}{x}\, dx$

$= 2\left[\sin^{-1}(x^2)\int_0^{\frac{1}{\sqrt{2}}} x dx - \int_0^{1/\sqrt{2}} \left(\int_0^{\frac{1}{\sqrt{2}}} x dx\right)\left(\frac{d}{dx}\sin^{-1}(x^2)\right) dx\right]$

$\because \frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$ (1)

$= 2\left[\sin^{-1}(x)^2 \frac{x^2}{2}\Big|_0^{\frac{1}{\sqrt{2}}} - \int_0^{\frac{1}{\sqrt{2}}} \frac{x^2}{\cancel{2}}\left(\frac{1}{\sqrt{1-(x^2)^2}}(\cancel{2}x)\right)dx\right]$

$= 2\left[\left(\sin^{-1}(0)^{\nearrow 0}\cdot\frac{0^2}{2}\right) - \left(\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)^2 \cdot \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{2}\right) - \int_0^{\frac{1}{\sqrt{2}}} \frac{x^3}{\sqrt{1-x^4}}dx\right]$ put

$= 2\left[-\left(\sin^{-1}\left(\frac{1}{2}\right)\cdot\left(\frac{\frac{1}{2}}{2}\right)\right)\right] - \int_0^{\frac{1}{\sqrt{2}}} \frac{x^3}{\sqrt{1-x^4}} dx$ — 1

Put $u = 1-x^4$

$\Rightarrow \frac{du}{dx} = 0-4x^3$

$\Rightarrow du = -4x^3 dx$

$\Rightarrow \frac{du}{-4x^3} = dx$

when $\boxed{x = 0}$

$u = 1-(0)^4$

$\boxed{u = 1}$

when $\boxed{x = \frac{1}{\sqrt{2}}}$

$u = 1-\left(\frac{1}{\sqrt{2}}\right)^4$

$u = 1-\left(\left(\frac{1}{\sqrt{2}}\right)^2\right)^2$

$u = 1-\frac{1}{4} = \frac{4-1}{4}$

$\boxed{u = \frac{3}{4}}$

1 becomes.

$2\left\{-\frac{1}{\cancel{4}_2}(3\overset{15}{\cancel{0}}) - \int_1^{3/4} \frac{\cancel{x^3}}{\sqrt{u}}\cdot\left(\frac{du}{-4\cancel{x^3}}\right)\right.$

$2\left[\frac{-15}{2} + \frac{1}{4}\int_1^{3/4} u^{-1/2}du\right] = 2\left[\frac{-15}{2} - \frac{1}{4}\int_{3/4}^{1} u^{-1/2}du\right]$

$= 2\left[\frac{-15}{2} - \frac{1}{4}\left(\frac{u^{-1/2+1}}{-1/2+1}\Big|_{3/4}^{1}\right)\right] = 2\left[\frac{-15}{2} - \frac{1}{4}\left(\frac{u^{1/2}}{1/2}\Big|_{3/4}^{1}\right)\right]$

$= -15 - \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{1}\left((1)^{1/2} - (3/4)^{1/2}\right)$ Ans

$-15-(1-(3/4)^{1/2})$