Ex # 7.4

Formula

$$\because \quad \frac{d}{dx}(a^x) = a^x \ln a \ (1)$$

$\boxed{Q_{12-22}}$

Q_{12} $y = 3^{-x}$

$$\frac{dy}{dx} = \frac{d}{dx}(3^{-x}) = 3^{-x}\ln 3 \frac{d}{dx}\cancel{(-x)}^{-1}$$

$$= -3^{-x}\ln 3 \quad \underline{\text{Ans}}$$

(20) $y = 3^{\tan\theta}\ln 3$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(3^{\tan\theta}\ln 3)$$

$$= \ln 3\left(\frac{d}{d\theta}3^{\tan\theta}\right)$$

$$= \ln 3\left(3^{\tan\theta}\ln 3 \cdot \frac{d}{d\theta}(\tan\theta)\right)$$

$$= (\ln 3)^2\, 3^{\tan\theta}(\sec^2\theta\,(1))$$

$$= (\ln 3)^2\, 3^{\tan\theta}\sec^2\theta \quad \underline{\text{Ans}}$$

$\boxed{13, 21, 22, 17, 18, 19}$

$$\because \int a^x dx = \frac{a^x}{\ln(a)} + c$$

(47)

$$\int 5^x dx = \frac{5^x}{\ln 5} + c \quad \underline{Ans}$$

(54)

$$\int_0^{\pi/4} \left(\frac{1}{3}\right)^{\tan t} \sec^2 t \, dt$$

$$= \left.\frac{\left(\frac{1}{3}\right)^{\tan t}}{\ln\left(\frac{1}{3}\right)}\right|_0^{\pi/4}$$

$$= \frac{1}{\ln\left(\frac{1}{3}\right)}\left[\left(\frac{1}{3}\right)^{\tan \pi/4 \to 1} - \left(\frac{1}{3}\right)^{\tan(0) \to 0}\right]$$

$$= \frac{1}{\ln\left(\frac{1}{3}\right)}\left[\left(\frac{1}{3}\right)^{1} - \left(\frac{1}{3}\right)^{0}\right]$$

$$= \frac{1}{\ln\left(\frac{1}{3}\right)}\left[\frac{1}{3} - 1\right]$$

$$= \frac{1}{\ln\left(\frac{1}{3}\right)}\left[-\frac{2}{3}\right] = \frac{1}{\cancel{\ln(1)}^{0} - \ln(3)}\left[-\frac{2}{3}\right]$$

$$\because \ln\left(\frac{a}{b}\right) = \ln a - \ln b \quad \Big| \quad = \frac{1}{-\ln 3}\left(-\frac{2}{3}\right) = \frac{2}{3}\left(\frac{1}{\ln 3}\right)$$

Ans

$\because \int a^x dx = \frac{a^x}{\ln(a)} + c$

(47)

$\int 5^x dx = \frac{5^x}{\ln 5} + c$ Ans

(54) $\int_0^{\pi/4} \left(\frac{1}{3}\right)^{\tan t} \sec^2 t \, dt$

$= \left. \frac{\left(\frac{1}{3}\right)^{\tan t}}{\ln\left(\frac{1}{3}\right)} \right|_0^{\pi/4}$

$= \frac{1}{\ln\left(\frac{1}{3}\right)} \left[\left(\frac{1}{3}\right)^{\tan \pi/4 \to 1} - \left(\frac{1}{3}\right)^{\tan(0) \to 0} \right]$

$= \frac{1}{\ln\left(\frac{1}{3}\right)} \left[\left(\frac{1}{3}\right)^{1} - \left(\frac{1}{3}\right)^{0} \right]$

$= \frac{1}{\ln\left(\frac{1}{3}\right)} \left[\frac{1}{3} - 1 \right]$

$= \frac{1}{\ln\left(\frac{1}{3}\right)} \left[-\frac{2}{3} \right] = \frac{1}{\ln(1)^{0} - \ln(3)} \left[-\frac{2}{3} \right]$

$\because \ln\left(\frac{a}{b}\right) = \ln a - \ln b$ | $= \frac{1}{-\ln 3} \left(-\frac{2}{3} \right) = \frac{2}{3} \left(\frac{1}{\ln 3} \right)$

Ans

Homework 51, 52, 53, 56,

Formulas for $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$

$\sec^{-1}x$, $\operatorname{cosec}^{-1}x$, $\cot^{-1}x$

Ex # 7.7

49 - 70

Find dy/dx

(49) $y = \cos^{-1}(x^2)$

$$\frac{dy}{dx} = \frac{d}{dx}\cos^{-1}(x^2)$$

$$\because \frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}} \quad (1)$$

$$= \frac{-1}{\sqrt{1-(x^2)^2}}\frac{d}{dx}(x^2)$$

$$= -\frac{1}{\sqrt{1-x^4}}(2x)(1)$$

$$= \frac{-2x}{\sqrt{1-x^4}} \quad \text{Ans}$$

(70) $y = \ln(x^2+4) - x\tan^{-1}\left(\frac{x}{2}\right)$ — ①

$\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$ (1)

$\because \frac{d}{dx}\ln x = \frac{1}{x}$ (1)

Taking derivative of ① both sides w.r.t 'x'

$$\frac{dy}{dx} = \frac{d}{dx}\left[\ln(x^2+4) - x\tan^{-1}\left(\frac{x}{2}\right)\right]$$

$$\frac{dy}{dx} = \frac{d}{dx}\ln(x^2+4) - \frac{d}{dx}\left[x\tan^{-1}\left(\frac{x}{2}\right)\right]$$

$$= \frac{1}{x^2+4}\frac{d}{dx}(x^2+4) - \left(x\frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{2}\right)\right) + \tan^{-1}\left(\frac{x}{2}\right)\frac{d}{dx}(x)\right)$$

$$= \frac{1}{x^2+4}(2x+0) - \left(x\frac{1}{1+\left(\frac{x}{2}\right)^2}\frac{d}{dx}\left(\frac{x}{2}\right) + \tan^{-1}\left(\frac{x}{2}\right)(1)\right)$$

$$= \frac{2x}{x^2+4} - \left(x\frac{1}{1+\frac{x^2}{4}}\left(\frac{1}{2}\right)\frac{d}{dx}(x) + \tan^{-1}\left(\frac{x}{2}\right)\right)$$

$$= \frac{2x}{x^2+4} - \left(x\frac{1}{\frac{4+x^2}{4}}\left(\frac{1}{2}\right)(1) + \tan^{-1}\left(\frac{x}{2}\right)\right)$$

$$= \frac{2x}{x^2+4} - \left(\frac{\cancel{4}^{2}x}{4+x^2}\left(\frac{1}{\cancel{2}_{1}}\right) + \tan^{-1}\left(\frac{x}{2}\right)\right)$$

$$= \cancel{\frac{2x}{x^2+4}} - \cancel{\frac{2x}{x^2+4}} - \tan^{-1}\left(\frac{x}{2}\right) = -\tan^{-1}\left(\frac{x}{2}\right)$$

<u>Ans</u>

55, 65, 67, 68

71-94 (72) Evaluate the integrals

$$\int \frac{dx}{\sqrt{1-4x^2}}$$

$$\because \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{1-4x^2}} = \int \frac{dx}{\sqrt{(1)^2-(2x)^2}}$$

$$= \sin^{-1}\left(\frac{2x}{1}\right) + C$$

$$= \sin^{-1}2x + C \quad \text{Ans}$$

(74) $$\int \frac{dx}{9+3x^2} \qquad \because \int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$$

$$= \int \frac{dx}{(3)^2+(\sqrt{3}x)^2}$$

$$= \frac{1}{3}\tan^{-1}\frac{\sqrt{3}x}{3} + C \quad \text{Ans}$$

(90) $$\int_{\pi/6}^{\pi/4} \frac{\operatorname{cosec}^2 x\,dx}{1+(\cot x)^2} \quad \text{— (A)}$$

Put $u = \cot x$ — (1)

$$\Rightarrow \frac{du}{dx} = -\operatorname{cosec}^2 x$$

$$-du = \operatorname{cosec}^2 x\,dx \quad \text{(2)}$$

1.732

$u = \cot x$

$\boxed{x = \frac{\pi}{6}}$ | $\boxed{x = \frac{\pi}{4}}$

$u = \cot\frac{\pi}{6}$ | $u = \cot\frac{\pi}{4}$

$\boxed{u = \sqrt{3}}$ | $\boxed{u = 1}$

using ①, ② and required limits in Ⓐ

$$\int_{\pi/6}^{\pi/4} \frac{\operatorname{cosec}^2 x\,dx}{1+(\cot x)^2} = \int_{\sqrt{3}}^{1} \frac{-du}{1+(u)^2} = -\int_{1}^{\sqrt{3}} -\frac{du}{1+u^2}$$

$$= \int_{1}^{\sqrt{3}} \frac{du}{1+u^2} = \int_{1}^{\sqrt{3}} \frac{du}{(1)^2+(u)^2}$$

$$\because \int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

$$= \frac{1}{1}\tan^{-1}\frac{u}{1}\Big|_{1}^{\sqrt{3}}$$

$$= \tan^{-1}\sqrt{3} - \tan^{-1}(1)$$

$$= \frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi - 3\pi}{12}$$

$$= \frac{\pi}{12}$$ Ans

(87) $\int \frac{dx}{(2x-1)\sqrt{(2x-1)^2-4}}$

$\because \int \frac{1}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$

$$\int \frac{dx}{(2x-1)\sqrt{(2x-1)^2-4}} = \int \frac{dx}{(2x-1)\sqrt{(2x-1)^2-(2)^2}}$$

$$= \frac{1}{2}\sec^{-1}\left|\frac{2x-1}{2}\right| + C$$

<u>Ans</u>

$\boxed{81, \ 83, \ 89, \ 93, \ 94}$

$\sinh x$, $\cosh x$, $\tanh x$, $\coth x$, $\operatorname{sech} x$
$\operatorname{cosech} x$
and $\sinh^{-1} x$, $\cosh^{-1} x$, $\tanh^{-1} x$, $\operatorname{sech}^{-1} x$
$\coth^{-1} x$, $\operatorname{cosech}^{-1} x$

(Formulas of derivatives)