Ex # 5.5

(7) $\int \frac{9r^2\,dr}{\sqrt{1-r^3}}$ — (1) Put $u = 1-r^3$ — (2)

(2) $u = 1-r^3$

Taking derivative both sides w.r.t 'r'

$$\frac{du}{dr} = 0 - 3r^2$$

$$du = -3r^2 \cdot dr$$

$$\frac{du}{-3r^2} = dr \Rightarrow dr = \frac{du}{-3r^2} \quad \text{— (3)}$$

using (2) and (3) in (1)

$$\int \frac{9r^2}{\sqrt{1-r^3}}\,dr = \int \frac{9\cancel{r^2}}{\sqrt{u}} \cdot \frac{du}{-3\cancel{r^2}} = \int \frac{\overset{3}{\cancel{9}}\,du}{\sqrt{u} \cdot -\cancel{3}_1}$$

$$= \int -\frac{3\,du}{\sqrt{u}} = -3\int \frac{1}{\sqrt{u}}\,du$$

$$\int \frac{9r^2}{\sqrt{1-r^3}}\,dr = -3\int u^{-1/2}\,du$$

$$\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$= -3\,\frac{u^{-1/2+1}}{-1/2+1} + C \qquad \frac{-1+2}{2} = \frac{1}{2}$$

$$= -3\,\frac{u^{1/2}}{1/2} + C = -3 \times \frac{2}{1}\,u^{1/2} + C$$

$$= -6u^{1/2} + C \qquad \text{Put } u = 1-r^3$$

$$\int \frac{9r^2}{\sqrt{1-r^3}}\,dr = -6(1-r^3)^{1/2} + C \quad \underline{\underline{\text{Ans}}}$$

Ex # 5.5

Integration by substitution

$\because \int 1\,dx = x \quad \because \int \cos x\,dx = \frac{\sin x}{1} + C$

$= \frac{1}{3}\left[\int 1\,du - \int \cos 2u\,du\right]$

$= \frac{1}{3}\left[u - \frac{\sin 2u}{2}\right] + C$

put $u = x^{3/2} - 1$

$= \frac{1}{3}\left[(x^{3/2} - 1) - \frac{\sin 2(x^{3/2} - 1)}{2}\right] + C$

Ans

$\because \boxed{\cos^2 x = \frac{1 + \cos 2x}{2}}$

(11) $\int \operatorname{cosec}^2 2\theta \cot 2\theta \, d\theta \quad -(1)$

(a) $u = \cot 2\theta \quad -(2)$

Diff (2) both sides w.r.t θ

$\frac{du}{d\theta} = \frac{d}{d\theta}(\cot 2\theta) \qquad \because \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x \ (1)$

$\frac{du}{} = -\operatorname{cosec}^2 2\theta \cdot \frac{d}{}(2\theta) = -\operatorname{cosec}^2 2\theta \cdot 2(1)$

$= \frac{1}{3}\left[\int 1du - \int \cos 2u\, du\right]$ $\because \int 1dx = x$ $\because \int \cos x\, dx = \frac{\sin x}{1} + C$

$= \frac{1}{3}\left[u - \frac{\sin 2u}{2}\right] + C$

Put $u = x^{3/2} - 1$

$= \frac{1}{3}\left[(x^{3/2} - 1) - \frac{\sin 2(x^{3/2} - 1)}{2}\right] + C$

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Diff (2) both sides w.r.t θ

$\frac{du}{d\theta} = \frac{d}{d\theta}(\cot 2\theta)$ $\because \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x\ (1)$

$\frac{du}{d\theta} = -\operatorname{cosec}^2 2\theta \cdot \frac{d}{d\theta}(2\theta) = -\operatorname{cosec}^2 2\theta \cdot 2(1)$

$\frac{du}{d\theta} = -2\operatorname{cosec}^2 2\theta$

$du = -2\operatorname{cosec}^2 2\theta\, d\theta$

$\frac{du}{-2\operatorname{cosec}^2 2\theta} = d\theta \Rightarrow d\theta = \frac{-du}{2\operatorname{cosec}^2 2\theta}$ — (3)

using (2) & (3) in (1)

$\int \operatorname{cosec}^2(2\theta)\cot(2\theta)\, d\theta = \int \cancel{\operatorname{cosec}^2 2\theta} \cdot u \times \frac{-du}{2\cancel{\operatorname{cosec}^2 2\theta}}$

$= \int \frac{-u\,du}{2} = -\frac{1}{2}\int u\,du = -\frac{1}{2}\left[\frac{u^2}{2}\right] + C$

Ex # 5.5

Evaluate

(25) $\int \sec^2(3x+2)\,dx$ — (1)

Put $u = 3x+2$ — (2)

Diff (2) w.r.t 'x'

$$\frac{du}{dx} = \frac{d}{dx}(3x+2) = 3\frac{d}{dx}x + \cancel{\frac{d}{dx}(2)}^{\,0}$$

$$\frac{du}{dx} = 3$$

$$du = 3dx \Rightarrow dx = \frac{du}{3} \quad \text{— (3)}$$

using (2) & (3) in (1)

$$\int \sec^2(3x+2)\,dx = \int \sec^2 u \,\frac{du}{3} = \frac{1}{3}\int \sec^2 u\,du$$

$$\because \int \sec^2 x\,dx = \operatorname{Tan} x + c$$

$$= \frac{1}{3}\operatorname{Tan} u + c$$

Put $u = 3x+2$

$$\int \sec^2(3x+2)\,dx = \frac{1}{3}\operatorname{Tan}(3x+2) + c$$ Ans

Homework

23, 31, 34, 36, 40

(28) $\int \tan^7 \frac{x}{2} \sec^2 \frac{x}{2} \, dx$ — (1)

Put $u = \tan \frac{x}{2}$ — (2)

Diff eq (2) w.r.t 'x'

$\because \frac{d}{dx}(\tan x) = \sec^2 x (1)$

$$\frac{du}{dx} = \sec^2 \frac{x}{2} \frac{d}{dx} \frac{x}{2}$$

$$\frac{du}{dx} = \sec^2 \frac{x}{2} \left(\frac{1}{2}\right)$$

$$\Rightarrow du = \frac{1}{2} \sec^2 \frac{x}{2} \cdot dx$$

$$\Rightarrow \frac{2du}{\sec^2 \frac{x}{2}} = dx \Rightarrow dx = \frac{2du}{\sec^2 \frac{x}{2}} \quad \text{— (3)}$$

Using (2) & (3) in (1)

$$\int \tan^7 \frac{x}{2} \sec^2 \frac{x}{2} \, dx = \int u^7 \cdot \cancel{\sec^2 \frac{x}{2}} \cdot \frac{2du}{\cancel{\sec^2 \frac{x}{2}}}$$

$$= 2 \int u^7 du$$

$$\because \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$= 1\cancel{2} \frac{u^8}{\cancel{8}4} + C = \frac{1}{4} u^8 + C$$

Put $u = \tan \frac{x}{2}$

$$\int \tan^7 \frac{x}{2} \sec^2 \frac{x}{2} \, dx = \frac{1}{4} \left(\tan \frac{x}{2}\right)^8 + C = \frac{1}{4} \tan^8 \frac{x}{2} + C$$

Ans

$(33) \int \sec(v+\frac{\pi}{2})\tan(v+\frac{\pi}{2})\,dv$ — (1)

Put $u = v+\frac{\pi}{2}$ — (2)

Diff eq (2) w.r.t 'v'

$\frac{du}{dv} = \frac{d}{dv}(v+\frac{\pi}{2}) = 1+0 = 1$

$du = dv$ — (3)

using (2) & (3) in (1)

$\int \sec u \tan u\, du$

$\because \int \sec x \tan x\, dx = \sec x + c$

$= \sec u + c$

put $u = v+\pi/2$

$\int \sec(v+\pi/2)\tan(v+\pi/2)\,dv = \sec(v+\pi/2) + c$ Ans

$(35) \int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt$ — (1)

Put, $u = \cos(2t+1)$ — (2)

Diff (2) w.r.t 't' both sides

$\frac{du}{dt} = -\sin(2t+1)\frac{d}{dt}(2t+1)$ $\quad \because \frac{d}{dx}\cos x = -\sin x(1)$

$\frac{du}{dt} = -\sin(2t+1)(2(1)+0) = -2\sin(2t+1)$

$$\frac{du}{dt} = -2\sin(2t+1)$$

$$\Rightarrow du = -2\sin(2t+1)\,dt \Rightarrow dt = \frac{du}{-2\sin(2t+1)} \quad -3$$

using 2 & 3 in 1

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt = \int \frac{\cancel{\sin(2t+1)}}{u^2} \times \frac{du}{-2\cancel{\sin(2t+1)}} = \int \frac{du}{-2u^2}$$

$$= -\frac{1}{2}\int \frac{1}{u^2}\,du = -\frac{1}{2}\int u^{-2}\,du \qquad \because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$= -\frac{1}{2}\,\frac{u^{-2+1}}{-2+1} + c$$

$$= -\frac{1}{2}\,\frac{u^{-1}}{-1} + c = \frac{1}{2}\,\frac{1}{u} + c$$

Put $u = \cos(2t+1)$

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt = \frac{1}{2(\cos(2t+1))} + c \qquad \underline{\underline{\text{Ans}}}$$

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(31) $\int \sqrt{\cot y}\,\operatorname{cosec}^2 y\,dy \quad -1$

Put $u = \cot y \quad -2$

Taking derivative of 2 w.r.t 'y'

$$\frac{du}{dy} = \frac{d}{dy}\cot y \qquad \because \frac{d}{dx}\cot x = -\operatorname{cosec}^2 x\,(1)$$

$$\frac{du}{dy} = -\operatorname{cosec}^2 y \Rightarrow du = -\operatorname{cosec}^2 y\,dy$$

$$\Rightarrow \frac{du}{-\operatorname{cosec}^2 y} = dy \quad -3$$

using 2 & 3 in 1

$$\int \sqrt{\cot y}\,\operatorname{cosec}^2 y\,dy = \int \sqrt{u}\cdot\cancel{\operatorname{cosec}^2 y}\,\frac{du}{-\cancel{\operatorname{cosec}^2 y}} = -\int u^{1/2}\,du$$

<u>Do yourself</u>