Important Limits

I. $\lim\limits_{x \to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$, where n is integer and $a > 0$.

II. $\lim\limits_{x \to 0} \dfrac{\sqrt{x+a} - \sqrt{a}}{x} = \dfrac{1}{2\sqrt{a}}$.

III. $\lim\limits_{n \to 0} \left(1 + \dfrac{1}{n}\right)^n = e$.

IV. $\lim\limits_{x \to \infty} (1 + x)^{\frac{1}{x}} = e$.

V. $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x} = \ln a$, where $a > 0$.

VI. $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x} = \ln e = 1$.

VII. If θ is measured in radian, then $\lim\limits_{\theta \to 0} \dfrac{\sin \theta}{\theta} = 1$.

Question # 1

(i) $\lim\limits_{x \to 3}(2x + 4) = \lim\limits_{x \to 3}(2x) + \lim\limits_{x \to 3}(4) = 2\lim\limits_{x \to 3}(x) + 4 = 2(3) + 4 = 10$.

(ii) $\lim\limits_{x \to 1}\left(3x^2 - 2x + 4\right) = 3(1)^2 - 2(1) + 4 = 3 - 2 + 4 = 5$.

(iii) $\lim\limits_{x \to 3}\sqrt{x^2 + x + 4} = \sqrt{(3)^2 + (3) + 4} = \sqrt{9 + 3 + 4} = \sqrt{16} = 4$.

(iv) $\lim\limits_{x \to 2} x\sqrt{x^2 - 4} = 2\sqrt{2^2 - 4} = 0$.

(v) $\lim\limits_{x \to 2}\left(\sqrt{x^3 + 1} - \sqrt{x^2 + 5}\right) = \lim\limits_{x \to 2}\left(\sqrt{x^3 + 1}\right) - \lim\limits_{x \to 2}\left(\sqrt{x^2 + 5}\right)$

$\qquad = \left(\sqrt{(2)^3 + 1}\right) - \left(\sqrt{(2)^2 + 5}\right)$

$\qquad = \sqrt{8 + 1} - \sqrt{4 + 5} = \sqrt{9} - \sqrt{9} = 0$.

(vi) $\lim\limits_{x \to -2} \dfrac{2x^3 + 5x}{3x - 2} = \dfrac{2(-2)^3 + 5(-2)}{3(-2) - 2} = \dfrac{-16 - 10}{-6 - 2} = \dfrac{-26}{-8} = \dfrac{13}{4}$.

Question # 2

(i) $\lim\limits_{x \to -1} \dfrac{x^3 - x}{x + 1} = \lim\limits_{x \to -1} \dfrac{x(x^2 - 1)}{x + 1} = \lim\limits_{x \to -1} \dfrac{x(x+1)(x-1)}{x + 1}$

$\qquad = \lim\limits_{x \to -1} x(x - 1) = (-1)(-1 - 1) = 2$

(ii) $\lim\limits_{x \to 0}\left(\dfrac{3x^3 + 4x}{x^2 + x}\right) = \lim\limits_{x \to 0} \dfrac{x(3x^2 + 4)}{x(x + 1)} = \lim\limits_{x \to 0} \dfrac{3x^2 + 4}{x + 1} = \dfrac{3(0) + 4}{0 + 1} = 4$.

(iii) $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2+x-6}$

$=\lim\limits_{x\to 2}\dfrac{x^3-(2)^3}{x^2+3x-2x-6}$ $=\lim\limits_{x\to 2}\dfrac{(x-2)(x^2+2x+4)}{x(x+3)-2(x+3)}$

$=\lim\limits_{x\to 2}\dfrac{(x-2)(x^2+2x+4)}{(x+3)(x-2)}$ $=\lim\limits_{x\to 2}\dfrac{(x^2+2x+4)}{(x+3)}$

$=\dfrac{(2)^2+2(2)+4}{(2+3)}$ $=\dfrac{12}{5}$

(iv) $\lim\limits_{x\to 1}\dfrac{x^3-3x^2+3x-1}{x^3-x}$

$=\lim\limits_{x\to 1}\dfrac{(x-1)^3}{x(x^2-1)}$ $=\lim\limits_{x\to 1}\dfrac{(x-1)^3}{x(x-1)(x+1)}$

$=\lim\limits_{x\to 1}\dfrac{(x-1)^2}{x(x+1)}$ $=\lim\limits_{x\to 1}\dfrac{(1-1)^2}{(1)(1+1)}$ $=0$

(v) $\lim\limits_{x\to -1}\left(\dfrac{x^3+x^2}{x^2-1}\right)$ $=\lim\limits_{x\to -1}\dfrac{x^2(x+1)}{(x+1)(x-1)}$ $=\lim\limits_{x\to -1}\dfrac{x^2}{(x-1)}$

$=\dfrac{(-1)^2}{(-1-1)}=-\dfrac{1}{2}$

(vi) $\lim\limits_{x\to 4}\dfrac{2x^2-32}{x^3-4x^2}$ $=\lim\limits_{x\to 4}\dfrac{2(x^2-16)}{x^2(x-4)}$

$=\lim\limits_{x\to 4}\dfrac{2(x+4)(x-4)}{x^2(x-4)}$ $=\lim\limits_{x\to 4}\dfrac{2(x+4)}{x^2}$

$=\dfrac{2(4+4)}{4^2}$ $=\dfrac{16}{16}$ $=1.$

(vii) $\lim\limits_{x\to 2}\dfrac{\sqrt{x}-\sqrt{2}}{x-2}$ $=\lim\limits_{x\to 2}\dfrac{\sqrt{x}-\sqrt{2}}{x-2}\left(\dfrac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}\right)$

$=\lim\limits_{x\to 2}\dfrac{(\sqrt{x})^2-(\sqrt{2})^2}{(x-2)(\sqrt{x}+\sqrt{2})}$ $=\lim\limits_{x\to 2}\dfrac{x-2}{(x-2)(\sqrt{x}+\sqrt{2})}$

$=\lim\limits_{x\to 2}\dfrac{1}{\sqrt{x}+\sqrt{2}}$ $=\dfrac{1}{\sqrt{2}+\sqrt{2}}$ $=\dfrac{1}{2\sqrt{2}}$

(viii) $\lim\limits_{h\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$ $=\lim\limits_{h\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$=\lim\limits_{h\to 0}\dfrac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})}$ $=\lim\limits_{h\to 0}\dfrac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$

$$= \lim_{h \to 0} \frac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)} \qquad = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x+0} + \sqrt{x}} \qquad = \frac{1}{2\sqrt{x}}$$

(ix) $\displaystyle \lim_{x \to a} \frac{x^n - a^n}{x^m - a^m}$

$$= \lim_{x \to a} \frac{(x-a)\left(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \dots + a^{n-1}\right)}{(x-a)\left(x^{m-1} + x^{m-2}a + x^{m-3}a^2 + \dots + a^{m-1}\right)}$$

$$= \lim_{x \to a} \frac{\left(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + \dots + a^{n-1}\right)}{\left(x^{m-1} + x^{m-2}a + x^{m-3}a^2 + \dots + a^{m-1}\right)}$$

$$= \frac{a^{n-1} + a^{n-2}a + a^{n-3}a^2 + \dots + a^{n-1}}{a^{m-1} + a^{m-2}a + a^{m-3}a^2 + \dots + a^{m-1}}$$

$$= \frac{a^{n-1} + a^{n-1} + a^{n-1} + \dots + a^{n-1} \quad (n \text{ terms})}{a^{m-1} + a^{m-1} + a^{m-1} + \dots + a^{m-1} \quad (m \text{ terms})}$$

$$= \frac{n\,a^{n-1}}{m\,a^{m-1}} \qquad = \frac{n}{m} a^{n-1-m+1} \qquad = \frac{n}{m} a^{n-m}$$

Law of Sine

If θ is measured in radian, then $\displaystyle \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$

See proof on book at page 25

Question # 3

(i) $\displaystyle \lim_{x \to 0} \frac{\sin 7x}{x}$

Put $\quad t = 7x \quad \Rightarrow \frac{t}{7} = x$

When $\quad x \to 0$ then $t \to 0$, so

$$\lim_{x \to 0} \frac{\sin 7x}{x} = \lim_{t \to 0} \frac{\sin t}{t/7}$$

$$= 7 \lim_{t \to 0} \frac{\sin t}{t} \quad = 7(1) = 7 \qquad \text{By law of sine.}$$

(ii) $\displaystyle \lim_{x \to 0} \frac{\sin x^\circ}{x}$

Since $\quad 180^\circ = \pi$ rad $\qquad \Rightarrow 1^\circ = \frac{\pi}{180}$ rad $\qquad \Rightarrow x^\circ = \frac{x\pi}{180}$ rad

So $\quad \displaystyle \lim_{x \to 0} \frac{\sin x^\circ}{x} = \lim_{x \to 0} \frac{\sin \pi x/180}{x}$

Now put $\quad \dfrac{\pi x}{180} = t \quad$ i.e. $\quad x = \dfrac{180t}{\pi}$

When $\;x \to 0\;$ then $\;t \to 0$, so

$$\lim_{x \to 0} \dfrac{\sin \pi x /180}{x} \;=\; \lim_{x \to 0} \dfrac{\sin t}{180t/\pi}$$

$$= \; \dfrac{\pi}{180} \lim_{x \to 0} \dfrac{\sin t}{t} \;=\; \dfrac{\pi}{180}\,(1) \;=\; \dfrac{\pi}{180} \qquad \text{by law of sine}$$

(iii) $\quad \displaystyle\lim_{\theta \to 0} \dfrac{1 - \cos \theta}{\sin \theta} \;=\; \lim_{\theta \to 0} \dfrac{1 - \cos \theta}{\sin \theta} \cdot \dfrac{1 + \cos \theta}{1 + \cos \theta}$

$$= \lim_{\theta \to 0} \dfrac{1 - \cos^2 \theta}{\sin \theta (1 + \cos \theta)} = \lim_{\theta \to 0} \dfrac{\sin^2 \theta}{\sin \theta (1 + \cos \theta)}$$

$$= \lim_{\theta \to 0} \dfrac{\sin \theta}{(1 + \cos \theta)} \;=\; \dfrac{\sin(0)}{1 + \cos(0)} \;=\; \dfrac{0}{1 + 1} \;=\; 0$$

(iv) $\quad \displaystyle\lim_{x \to \pi} \dfrac{\sin x}{\pi - x}$

Put $\quad t = \pi - x \quad \Rightarrow \quad x = \pi - t$

When $\;x \to \pi\;$ then $\;t \to 0$, so

$$\lim_{x \to \pi} \dfrac{\sin x}{\pi - x} \;=\; \lim_{t \to 0} \dfrac{\sin(\pi - t)}{t}$$

$$= \lim_{t \to 0} \dfrac{\sin t}{t} \qquad\qquad \because \; \sin(\pi - t) = \sin\left(2 \cdot \dfrac{\pi}{2} - t\right) = \sin t$$

$$= 1 \qquad\qquad\qquad \text{By law of sine.}$$

(v) $\quad \displaystyle\lim_{x \to 0} \dfrac{\sin ax}{\sin bx} \;=\; \lim_{x \to 0} \sin ax \cdot \dfrac{1}{\sin bx}$

$$= \lim_{x \to 0} \sin ax \cdot \left(\dfrac{ax}{ax}\right) \dfrac{1}{\sin bx \cdot \left(\dfrac{bx}{bx}\right)} \;=\; \lim_{x \to 0} \dfrac{\sin ax}{ax} \cdot ax \, \dfrac{1}{\dfrac{\sin bx}{bx} \cdot bx}$$

$$= \dfrac{a}{b} \lim_{x \to 0} \dfrac{\sin ax}{ax} \cdot \dfrac{1}{\displaystyle\lim_{x \to 0} \dfrac{\sin bx}{bx}} \;=\; \dfrac{a}{b} \cdot (1) \cdot \dfrac{1}{(1)} \;=\; \dfrac{a}{b} \qquad \text{by law of sine}$$

(vi) $\quad \displaystyle\lim_{x \to 0} \dfrac{x}{\tan x} = \lim_{x \to 0} \dfrac{x}{\dfrac{\sin x}{\cos x}} = \lim_{x \to 0} \dfrac{x}{\sin x} \cdot \cos x$

$$= \lim_{x \to 0} \dfrac{1}{\dfrac{\sin x}{x}} \cdot \cos x \;=\; \dfrac{1}{\displaystyle\lim_{x \to 0} \dfrac{\sin x}{x}} \cdot \lim_{x \to 0} \cos x \;=\; \dfrac{1}{1} \cdot 1 \;=\; 1$$

(vii) $\quad \displaystyle\lim_{x \to 0} \dfrac{1 - \cos 2x}{x^2}$

$$= \lim_{x \to 0} \dfrac{2 \sin^2 x}{x^2} \qquad\qquad \because \; \sin^2 x = \dfrac{1 - \cos 2x}{2}$$

$$\qquad\qquad\qquad\qquad\qquad \therefore \; 2 \sin^2 x = 1 - \cos 2x$$

$$= 2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 = 2\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2 = 2(1)^2 = 2$$

(vii)   *Do yourself by rationalizing*

(viii) $\lim_{\theta\to 0}\dfrac{\sin^2\theta}{\theta} = \lim_{\theta\to 0}\dfrac{\sin\theta}{\theta}\cdot\sin\theta$

$$= \lim_{\theta\to 0}\frac{\sin\theta}{\theta}\cdot\lim_{\theta\to 0}\sin\theta = (1)\cdot(0) = 0$$

(x) $\lim_{x\to 0}\dfrac{\sec x - \cos x}{x}$

$$= \lim_{x\to 0}\frac{\dfrac{1}{\cos x}-\cos x}{x} = \lim_{x\to 0}\frac{\dfrac{1-\cos^2 x}{\cos x}}{x}$$

$$= \lim_{x\to 0}\frac{1-\cos^2 x}{x\cos x} = \lim_{x\to 0}\frac{\sin^2 x}{x\cos x} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin x}{\cos x}$$

$$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\sin x}{\cos x} = (1)\frac{\sin(0)}{\cos(0)} = (1)\cdot\frac{0}{1} = 0$$

(xi) $\lim_{x\to 0}\dfrac{1-\cos p\theta}{1-\cos q\theta}$

$$= \lim_{x\to 0}\frac{2\sin^2\dfrac{p\theta}{2}}{2\sin^2\dfrac{q\theta}{2}} \qquad\qquad \because\ \sin^2\frac{x}{2}=\frac{1-\cos x}{2}$$

$$= \lim_{x\to 0}\ \sin^2\frac{p\theta}{2}\cdot\frac{1}{\sin^2\dfrac{q\theta}{2}} = \lim_{x\to 0}\ \sin^2\frac{p\theta}{2}\cdot\frac{\left(\dfrac{p\theta}{2}\right)^2}{\left(\dfrac{p\theta}{2}\right)^2}\cdot\frac{1}{\sin^2\dfrac{q\theta}{2}\cdot\dfrac{\left(\dfrac{q\theta}{2}\right)^2}{\left(\dfrac{q\theta}{2}\right)^2}}$$

$$= \lim_{x\to 0}\ \frac{\sin^2\dfrac{p\theta}{2}}{\left(\dfrac{p\theta}{2}\right)^2}\cdot\left(\frac{p\theta}{2}\right)^2\cdot\frac{1}{\dfrac{\sin^2\dfrac{q\theta}{2}}{\left(\dfrac{q\theta}{2}\right)^2}\cdot\left(\dfrac{q\theta}{2}\right)^2} = \lim_{x\to 0}\ \left(\frac{\sin\dfrac{p\theta}{2}}{\dfrac{p\theta}{2}}\right)^2\cdot\frac{1}{\left(\dfrac{\sin\dfrac{q\theta}{2}}{\dfrac{q\theta}{2}}\right)^2}\cdot\frac{p^2\theta^2\!\!\big/4}{q^2\theta^2\!\!\big/4}$$

$$= \frac{p^2}{q^2}\left(\lim_{x\to 0}\frac{\sin\dfrac{p\theta}{2}}{\dfrac{p\theta}{2}}\right)^2\cdot\frac{1}{\left(\lim_{x\to 0}\dfrac{\sin\dfrac{q\theta}{2}}{\dfrac{q\theta}{2}}\right)^2} = \frac{p^2}{q^2}(1)^2\cdot\frac{1}{(1)^2} = \frac{p^2}{q^2}$$

(xii) $\lim\limits_{\theta \to 0} \dfrac{\tan\theta - \sin\theta}{\sin^3\theta}$

$$= \lim\limits_{\theta \to 0} \dfrac{\dfrac{\sin\theta}{\cos\theta} - \sin\theta}{\sin^3\theta} \quad = \lim\limits_{\theta \to 0} \dfrac{\dfrac{\sin\theta - \sin\theta\cos\theta}{\cos\theta}}{\sin^3\theta}$$

$$= \lim\limits_{\theta \to 0} \dfrac{\sin\theta - \sin\theta\cos\theta}{\sin^3\theta\cos\theta} \quad = \lim\limits_{\theta \to 0} \dfrac{\sin\theta(1 - \cos\theta)}{\sin^3\theta\cos\theta}$$

$$= \lim\limits_{\theta \to 0} \dfrac{1 - \cos\theta}{\sin^2\theta\cos\theta} \quad = \lim\limits_{\theta \to 0} \dfrac{1 - \cos\theta}{\sin^2\theta\cos\theta} \cdot \dfrac{1 + \cos\theta}{1 + \cos\theta}$$

$$= \lim\limits_{\theta \to 0} \dfrac{1 - \cos^2\theta}{\sin^2\theta\cos\theta(1 + \cos\theta)} \quad = \lim\limits_{\theta \to 0} \dfrac{\sin^2\theta}{\sin^2\theta\cos\theta(1 + \cos\theta)}$$

$$= \lim\limits_{\theta \to 0} \dfrac{1}{\cos\theta(1 + \cos\theta)} \quad = \lim\limits_{x \to 0} \dfrac{1}{\cos\theta(1 + \cos\theta)}$$

$$= \dfrac{1}{\cos(1)(1 + \cos(1))} \quad = \dfrac{1}{1\cdot(1+1)} \quad = \dfrac{1}{2}$$

Note:

a) $\lim\limits_{n \to \infty}\left(1 + \dfrac{1}{n}\right)^n = e$

b) $\lim\limits_{x \to 0}(1 + x)^{\frac{1}{x}} = e$ where $e = 2.718281...$

            See proof of (a) and (b) on book at page 23

c) $\lim\limits_{x \to 0}\dfrac{a^x - 1}{x} = \log_e a$ or $\ln a$

Proof:

Put $y = a^x - 1$ .... (i)

When $x \to 0$ then $y \to 0$

Also from (i) $1 + y = a^x$

Taking log on both sides

      $\ln(1 + y) = \ln a^x \implies \ln(1 + y) = x\ln a$ $\because \ln x^m = m\ln x$

    $\implies x = \dfrac{\ln(1 + y)}{\ln a}$

Now $\lim\limits_{x \to 0}\dfrac{a^x - 1}{x} = \lim\limits_{y \to 0}\dfrac{y}{\dfrac{\ln(1 + y)}{\ln a}}$

             $= \lim\limits_{y \to 0}\dfrac{y\ln a}{\ln(1 + y)} \quad = \lim\limits_{y \to 0}\dfrac{\ln a}{\dfrac{1}{y}\ln(1 + y)}$

$$= \lim_{y \to 0} \frac{\ln a}{\ln(1+y)^{\frac{1}{y}}} \quad = \quad \frac{\ln a}{\lim_{y \to 0} \ln(1+y)^{\frac{1}{y}}} \qquad \because \ln x^m = m \ln x$$

$$= \frac{\ln a}{\ln\left(\lim_{y \to 0}(1+y)^{\frac{1}{y}}\right)} \quad = \quad \frac{\ln a}{\ln(e)} \qquad \because \lim_{x \to 0}(1+x)^{\frac{1}{x}} = e$$

$$= \frac{\ln a}{1} \quad = \quad \ln a \qquad \because \ln e = 1$$

Question # 4

(i) $\quad \lim_{n \to +\infty}\left(1+\dfrac{1}{n}\right)^{2n} \quad = \left[\lim_{n \to +\infty}\left(1+\dfrac{1}{n}\right)^{n}\right]^{2} \quad = \quad e^2$

(ii) $\quad \lim_{n \to +\infty}\left(1+\dfrac{1}{n}\right)^{\frac{n}{2}} \quad = \left[\lim_{n \to +\infty}\left(1+\dfrac{1}{n}\right)^{n}\right]^{\frac{1}{2}} \quad = \quad e^{\frac{1}{2}} \quad = \quad \sqrt{e}$

(iii) $\quad \lim_{n \to +\infty}\left(1-\dfrac{1}{n}\right)^{n} \quad = \left[\lim_{n \to +\infty}\left(1-\dfrac{1}{n}\right)^{-n}\right]^{-1} \quad = \quad e^{-1} \quad = \quad \dfrac{1}{e}$

(iv) $\quad \lim_{n \to +\infty}\left(1+\dfrac{1}{3n}\right)^{n}$

$$= \lim_{n \to +\infty}\left(1+\dfrac{1}{3n}\right)^{\frac{3n}{3}} = \left[\lim_{n \to +\infty}\left(1+\dfrac{1}{3n}\right)^{3n}\right]^{\frac{1}{3}} = e^{\frac{1}{3}}$$

(v) $\quad \lim_{n \to +\infty}\left(1+\dfrac{4}{n}\right)^{n} = \lim_{n \to +\infty}\left(1+\dfrac{4}{n}\right)^{\frac{4n}{4}} = \left[\lim_{n \to +\infty}\left(1+\dfrac{4}{n}\right)^{\frac{n}{4}}\right]^{4} = e^4.$

(vi) $\quad \lim_{x \to 0}(1+3x)^{\frac{2}{x}}$

$$= \lim_{x \to 0}(1+3x)^{\frac{6}{3x}} = \left[\lim_{x \to 0}(1+3x)^{\frac{1}{3x}}\right]^{6} = e^6$$

(vii) $\quad \lim_{x \to 0}\left(1+2x^2\right)^{\frac{1}{x^2}} = \lim_{x \to 0}\left(1+2x^2\right)^{\frac{2}{2x^2}} = \left[\lim_{x \to 0}\left(1+2x^2\right)^{\frac{1}{2x^2}}\right]^{2} = e^2$

(viii) $\quad \lim_{h \to 0}(1-2h)^{\frac{1}{h}} = \lim_{h \to 0}(1-2h)^{\frac{-2}{-2h}} = \left[\lim_{h \to 0}(1-2h)^{\frac{1}{-2h}}\right]^{-2} = e^{-2} = \dfrac{1}{e^2}$

(ix) $\quad \lim_{x \to \infty}\left(\dfrac{x}{1+x}\right)^{x}$

$$= \lim_{x \to \infty}\left(\dfrac{1+x}{x}\right)^{-x} = \lim_{x \to \infty}\left(\dfrac{1}{x}+\dfrac{x}{x}\right)^{-x} = \lim_{x \to \infty}\left(\dfrac{1}{x}+1\right)^{-x}$$

$$= \left[\lim_{x \to \infty} \left(1 + \frac{1}{x} \right)^x \right]^{-1} = e^{-1} = \frac{1}{e}$$

(x) $\displaystyle\lim_{x \to 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$; $x < 0$

Put $x = -t$ where $t > 0$

When $x \to 0$ then $t \to 0$, so

$$\lim_{x \to 0} \frac{e^{1/x} - 1}{e^{1/x} + 1} = \lim_{t \to 0} \frac{e^{-1/t} - 1}{e^{-1/t} + 1} = \frac{e^{-1/0} - 1}{e^{-1/0} + 1}$$

$$= \frac{e^{-\infty} - 1}{e^{-\infty} + 1} = \frac{0 - 1}{0 + 1} \qquad\qquad \because \; e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$$= -1$$

(xi) $\displaystyle\lim_{x \to 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$; $x > 0$

$$= \lim_{x \to 0} \frac{e^{1/x} \left(1 - \dfrac{1}{e^{1/x}} \right)}{e^{1/x} \left(1 + \dfrac{1}{e^{1/x}} \right)} = \lim_{x \to 0} \frac{\left(1 - \dfrac{1}{e^{1/x}} \right)}{\left(1 + \dfrac{1}{e^{1/x}} \right)}$$

$$= \frac{1 - \dfrac{1}{e^{1/0}}}{1 + \dfrac{1}{e^{1/0}}} = \frac{1 - \dfrac{1}{e^{\infty}}}{1 + \dfrac{1}{e^{\infty}}} = \frac{1 - \dfrac{1}{\infty}}{1 + \dfrac{1}{\infty}} = \frac{1 - 0}{1 + 0} = 1$$

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