Electrodynamics-II

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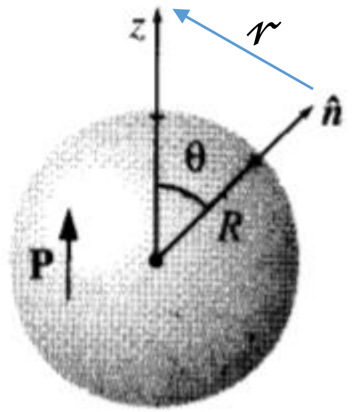
Example 4.2

Find the electric field produced by a uniformly polarized sphere of radius R.

Sphere is polarized means it has net dipole moment per unit volume.



We take the polarization along z axis.

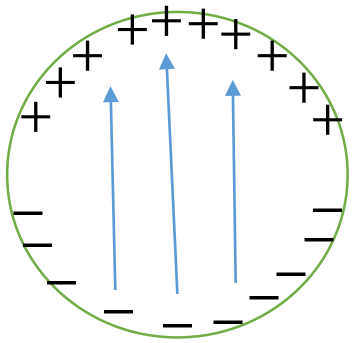
The volume bound charge density

ρ_b

Because $\vec{P}$ is uniform

So

$$\rho_b = -\vec{\nabla}.\vec{P} = 0$$



$$\sigma_b = \vec{P}.\hat{n} = P\ cos\theta$$

Where θ is usual spherical coordinate.

Example 4.2

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \oint \frac{\sigma_b}{\mathscr{r}} da'$$

Using Law of Cosines

$$\mathscr{r} = \sqrt{R^2 + z^2 - 2Rz\cos\theta}$$

$$\mathscr{r}^2 = R^2 + z^2 - 2Rz\cos\theta$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \oint \frac{\sigma_b R^2 \sin\theta d\theta d\varphi}{\sqrt{R^2 + z^2 - 2Rz\cos\theta}}$$

Example 4.2

$$V(\boldsymbol{r}) = \frac{PR^2}{4\pi\epsilon_0}\oint \frac{cos\theta\ sin\theta d\theta d\varphi}{\sqrt{R^2 + z^2 - 2Rzcos\theta}}$$

$$V(r,\theta) = \begin{cases} \dfrac{P}{3\epsilon_0} rcos\theta & \text{For } r \leq R \\ \dfrac{P}{3\epsilon_0}\dfrac{R^3}{r^2} cos\theta & \text{For } r \geq R \end{cases}$$

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$$r cos\theta = z$$

$$V(r,\theta) = \frac{P}{3\epsilon_0} r cos\theta = \frac{Pz}{3\epsilon_0}$$ For $r \leq R$

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\right) = -\frac{\partial}{\partial z}\left(\frac{Pz}{3\epsilon_0}\right)\hat{z} = \frac{-P}{3\epsilon_0}\hat{z} = \frac{-\vec{P}}{3\epsilon_0}$$

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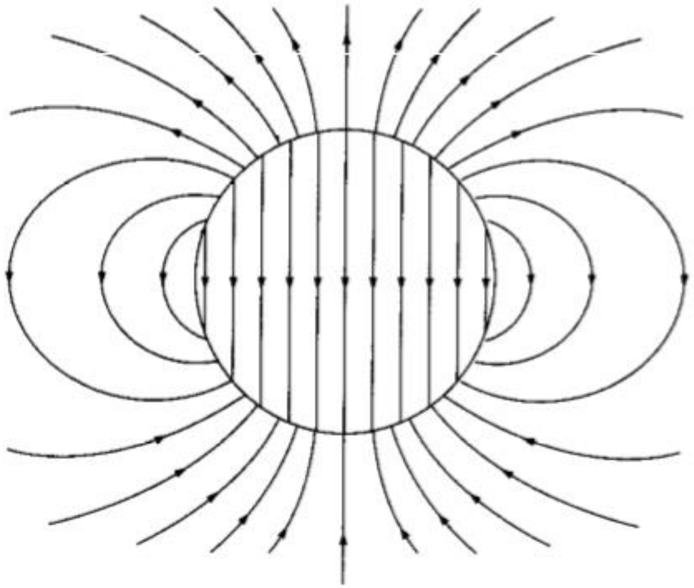
Now For $r \geq R$

$$\frac{\vec{p}}{\frac{4}{3}\pi R^3} = \vec{P}$$

$$V(r,\theta) = \frac{P}{3\epsilon_0}\frac{R^3}{r^2}cos\theta = \frac{\vec{P}.\hat{r}}{3\epsilon_0}\frac{R^3}{r^2} = \frac{\vec{p}}{\frac{4}{3}\pi R^3}.\frac{\hat{r}}{3\epsilon_0}\frac{R^3}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}.\hat{r}}{r^2}$$

Which is the potential for dipole

Example 4.2



Graph represents the electric field of dipole.