Q:- Given $Q = 100 - 10P$ and $TC = 70 + 2Q$.

for an oligopolistic firm, determine mathematically the output at which the firm maximizes its (a) Total profit, and calculate P, TR & π. (b) TR and calculate P, TR & π (c) TR and calculate P, TR & π if the firm fails the total profit constraint of $85.

Solution:- (a) $Q = 100 - 10P$

$P = 100 - Q$

$P = \frac{100}{10} - \frac{1}{10}Q$

$P = 10 - 0.1Q$ ——— (1)

$TR = P \times Q = (10 - 0.1Q)Q$

$TR = 10Q - 0.1Q^2$ & $TC = 70 + 2Q$

$\pi = TR - TC = 10Q - 0.1Q^2 - (70 + 2Q)$

$= 10Q - 0.1Q^2 - 70 - 2Q$

$= 8Q - 0.1Q^2 - 70$

FOC / $\pi' = \frac{d\pi}{dQ} = 8 - 0.2Q$

set $\frac{d\pi}{dQ} = 0$ $\quad 8 - 0.2Q = 0$

$8 = 0.2Q$

$8/0.2 = Q$

$Q = \frac{80}{2} = 40$

$\boxed{Q = 40}$, put in (1) $\quad P = 10 - 0.1 \times 40$

$P = 10 - 4 = 6$

$\boxed{P = 6\$}$

SOC / $\pi'' = \frac{d^2\pi}{dQ^2} = -0.2 < 0$ at $Q = 40$,

π. f. maximized. For Finding the π value of TR & π, put $Q = 40$ in TR and π. functions: $\pi = 8(40) - 0.1(40)^2 - 70$

$= 320 - 70 - 160 = 90\$$

$TR = 10(40) - 0.1(40)^2 = 400 - 160 = 240\$$

Questions

(b) $TR = 10Q - 0.1Q^2$

FOC $TR' = 10 - 0.2Q$

Set $TR' = 0$

$10 - 0.2Q = 0$

$10 = 0.2Q$

$\frac{10}{0.2} = Q$

$\frac{100}{2} = Q$

$\boxed{Q = 50}$ put in (1)

$P = 10 - 0.1 \times 50$

$P = 10 - 5 = 5$

$\boxed{P = \$5}$

SOC

$TR = -0.2 < 0$

at $Q = 50$, TR maximized.

So $TR = 10(50) - 0.1(50)^2$

$= 500 - 250 = \$\ 250$

$\pi = -70 + 8(50) - 0.1(50)^2$

$= -70 + 400 - 250 = \$\ 80$

(c) To find Q, P & TR if the minimum profit constraint of the firm is 85 $.

ie $\pi = 85\$$

we proceed as follows:

we know that

$\pi = -70 + 8Q - 0.1Q^2$

so $\pi = 85$

$85 = -70 + 8Q - 0.1Q^2$

$0.1Q^2 - 8Q + 85 + 70 = 0$

$0.1Q - 8Q + 155 = 0$

$a = 0.1, \quad b = -8, \quad c = 155$

$$Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

By putting the values.

$$Q = \frac{8 \pm \sqrt{(-8)^2 - 4(0.1)(155)}}{2(0.1)}$$

$Q = 32.93$, $Q = 47.07$

Taking the largest of these outputs, we get $Q = 47.07$

$$P = 10 - 0.1(47.07)$$

$$P = \$5.29$$

$$TR = (47.01)(10) - 0.1(47.07)^2$$

$$= 470.70 - 221.56$$

$$TR = \$249.14$$

So $\pi = -70 + 8(47.07) - 0.1(47.07)^2$

$= -70 + 376.56 - 221.56$

$\pi = \$85$ (the minimum π required).

X

For Practice

Appendix Problems

Q. No. 5 on Page 455