Derivation of Energy of an Electron in an Orbit

The energy of an electron in an orbit is the sum of its potential and kinetic energy

$$E_T = K.E + P.E$$

$$E_T = \frac{1}{2}mv^2 + \left(-\frac{Ze^2}{4\pi\varepsilon_o r}\right).....(7)$$

$$E_T = \frac{1}{2}mv^2 - \frac{Ze^2}{4\pi\varepsilon_o r}$$

From eq(1) $$mv^2 = \frac{Ze^2}{4\pi\varepsilon^\circ r}$$

Putting value in eq (7)

$$E_T = \frac{Ze^2}{2.4\pi\varepsilon^\circ r} - \frac{Ze^2}{4\pi\varepsilon^\circ r}$$

$$E_n = \frac{Ze^2}{8\pi\varepsilon_\circ r} - \frac{Ze^2}{4\pi\varepsilon_\circ r}$$

$$E_n = \frac{Ze^2}{4\pi\varepsilon_\circ r}\left(\frac{1}{2} - 1\right)$$

$$E_n = \frac{-Ze^2}{8\pi\varepsilon_\circ r} \ldots\ldots\ldots\ldots\ldots (8)$$

Now putting the value of *r* from eq(5) into eq(8),

$$E_n = \frac{-Ze^2}{8\pi\varepsilon_o} \times \frac{Ze^2\pi n}{\varepsilon_o n^2 h^2}$$

$$E_n = \frac{-Z^2 e^4 m}{8\varepsilon_o^2 n^2 h^2} \ldots\ldots\ldots\ldots(9)$$

For Hydrogen atom; Z=1

$$E_n = \frac{-me^4}{8\varepsilon_o^2 n^2 h^2}$$

$$E_n = -\frac{me^4}{8\varepsilon_o^2 h^2}\left[\frac{1}{n^2}\right]$$

But $\frac{me^4}{8\varepsilon_o^2 h^2} = -2.178\times10^{-18}\,\mathrm{J}$

$$E_n = -2.178\times10^{-18}\left(\frac{1}{n^2}\right)\mathrm{J}\ldots\ldots\ldots(10)$$

$$E_n = -\frac{k}{n^2}$$

$$\text{where } k = 2.178\times10^{-18}$$

The negative sign indicated Decrease in energy of the electron.

For 1 mol of electron, multiply by Avogadro's No.

$$E_n = -\left(\frac{k}{n^2}\right) \times 6.02 \times 10^{23}\ \text{J/mol}$$

$$E_n = -\left(\frac{k}{n^2}\right) \times \frac{6.02 \times 10^{23}}{1000}\ \text{KJ/mol}$$

$$E_n = 1313.315\left(\frac{1}{n^2}\right)\text{KJ/mol}$$

This energy is associated with 1.008 gram-atoms of hydrogen.

If n=1, 2, 3,…. then;

$$E_1 = -1313.315\left(\frac{1^2}{1^2}\right) = -1313.315\ kJmol^{-1}$$

$$E_2 = -1313.315\left(\frac{1^2}{2^2}\right) = -328.32\ kJmol^{-1}$$

$$E_3 = -1313.315\left(\frac{1^2}{3^2}\right) = -145.92\ kJmol^{-1}$$

$$E_4 = -1313.315\left(\frac{1^2}{4^2}\right) = -82.08\ kJmol^{-1}$$

$$E_5 = -1313.315\left(\frac{1^2}{5^2}\right) = -52.53\ kJmol^{-1}$$

The first energy level when n=1 is called Ground state of H atom. All others are called Excited states.

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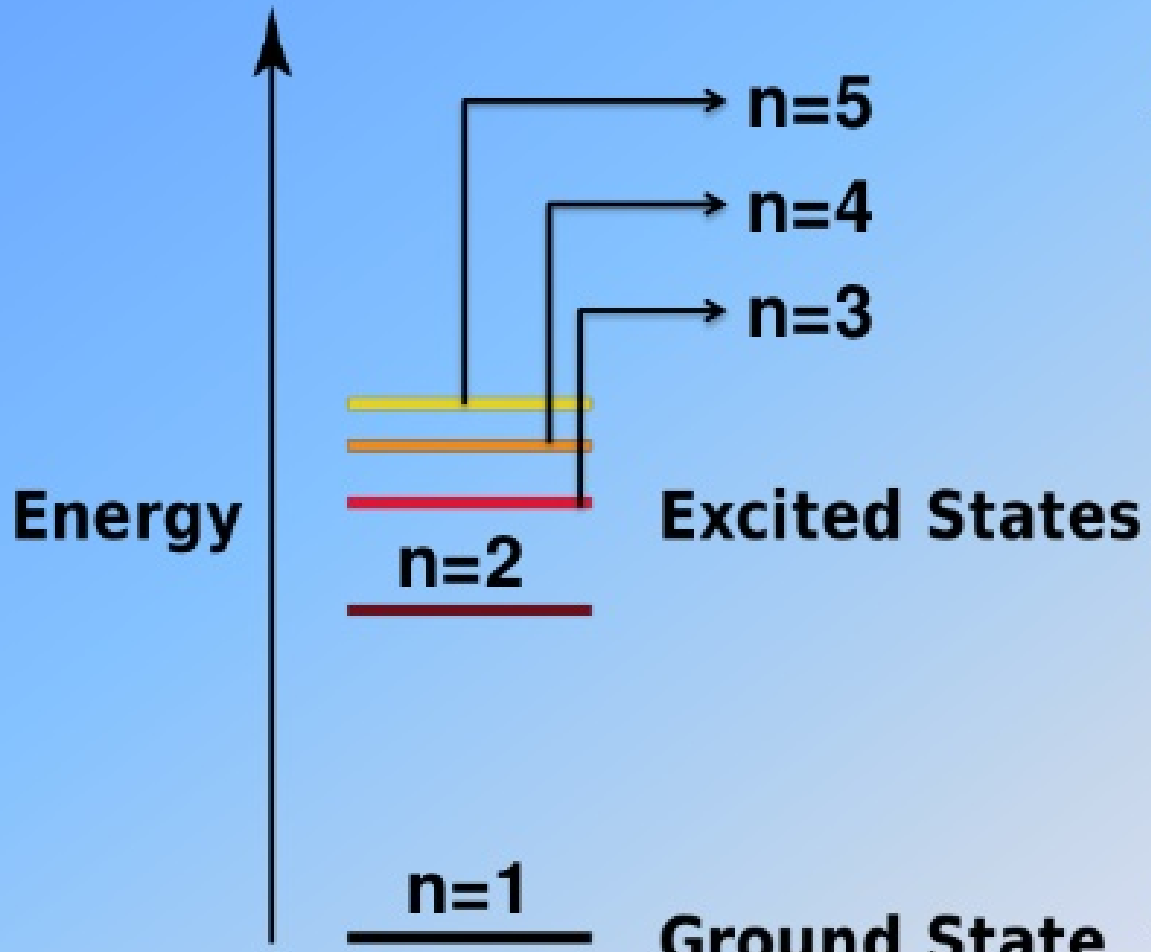

$E_5 = -52.53\ kJmol^{-1}$

$E_4 = -82.08\ kJmol^{-1}$

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