Fluid Mechanics (CT-213)

- ***Course Instructor***

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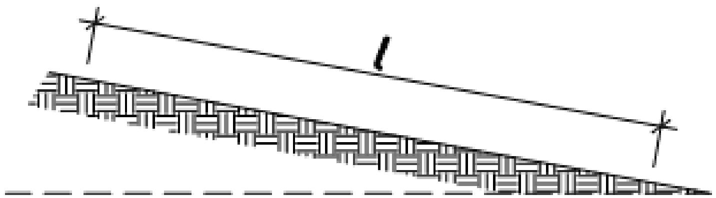
Steady Flow in Open Channel

Lecture - 11

Introduction

- An open channel is a passage through which the water flows under the force of gravity and atmospheric pressure.
- Or in other words, when the free surface of the flowing water is in contact with the atmosphere as in the case of canal, a sewer or an aqueduct, the flow is said to be through an open channel.
- A channel may be covered or open at the top.
- The flow of water in the channel is due the slope of the bed of channel instead of pressure as in the case of pipe flow.
- The velocity is different at different points in the channel, but calculations are based on the mean velocity of flow.
- Here we will assume the flow to be steady and uniform.

Chezy's Formula for Discharge through an Open Channel



Slopping Bed of Channel

- Consider an open channel of uniform cross section and bed slope as shown in figure.

  Let

  - l = Length of the channel
  - A = Area of flow
  - v = velocity of water
  - P = Wetted perimeter of the cross-section
  - f = Frictional resistance per unit area at unit velocity, and
  - i = Uniform slope in the bed.

It has been experimentally found that the total frictional resistance in the length l of the channel follows a law,

$$\text{Frictional Resistance} = f \times \text{Contact Area} \times (\text{Velocity})^{n}$$

$$= f \times Pl \times v^{n}$$

The value of n has been experimentally found to be nearly equal to 2. But, for all practical purposes, its value is taken to be 2.

Therefore frictional resistance

$$= f \times Pl \times v^{2}$$

Since the water moves through a distance v in one second, therefore workdone in overcoming the friction

$$= \text{Frictional resistance} \times \text{Distance}$$

$$= f \times Pl \times v^{2} \times v = f \times Pl \times v^{3}$$

We know that weight of water in the channel in a length of l metres.

$$= \gamma A l$$

Where γ is specific Weight of water.

This water will fall vertically down by the distance equal to (v. i) in one second. Therefore loss of potential energy

$$= \text{Weight of water x Height}$$

$$= \gamma A l \; x \; \text{v.i}$$

We also know that workdone in overcomong friction

$$= \text{Loss of potential energy}$$

$$\therefore \qquad \text{f x P}l \text{ x v}^3 = \gamma A l \; x \; \text{v.i}$$

$$\text{v}^2 = \frac{\gamma A \, \text{i}}{\text{f P}}$$

$$\text{v} = \sqrt{\frac{\gamma}{f}} \; \text{x} \; \sqrt{\frac{\text{A}}{\text{P}} x \; i} \quad = \text{C}\sqrt{\text{mi}}$$

Where $C = \sqrt{\frac{\gamma}{f}}$ (Known as Chezy's constant)

and $m = \frac{A}{P} =$ (Known as hydraulic mead depth or hydraulic radius)

$\therefore$ Discharge $Q = A \text{ x } v = A\, C\sqrt{mi}$

Problem-1

A rectangular channel is 1.5m deep and 6m wide. Find the discharge through channel, when it runs full. Take slope of the bed as 1 in 900 and Chezy's constant as 50.

Solution:

$Given:$

$d = 1.5\text{m}$

$b = 6\text{m}$

$i = 1/900 \quad \text{and} \quad C = 50$

We know that area of channel,

$A = b.d = 6 \text{ x } 1.5 = 9\text{m}^2$

And wetted perimeter $= D = b + 2d = 6 + 2(1.5) = 9\text{m}$

$\therefore$ Hydraulic mean depth $= m = \dfrac{A}{P} = \dfrac{9}{9} = 1m$

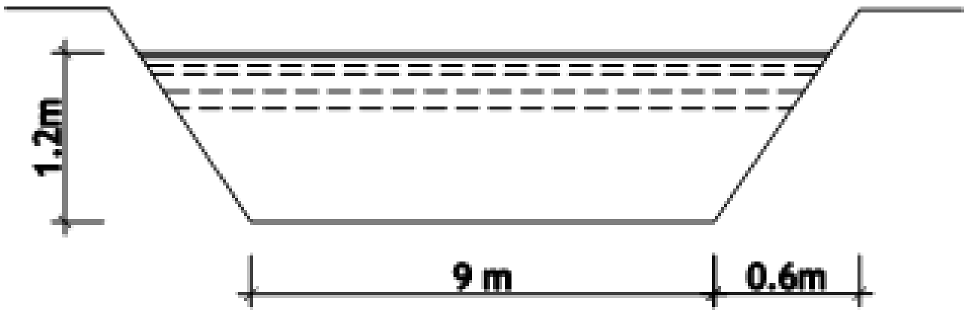
Discharge through channel,

$$Q = A.C\sqrt{mi} = 9x50\sqrt{1x\frac{1}{900}} = 15m^3/s$$

Problem-2

Water is flowing at the rate of 16.5 m³/s in an earthen trapezoidal channel with bed width 9m, water depth 1.2m and side slope 1:2. Calculate the bed slope, if the value of C is 49.5.

Solution:



Given :

$Q = 16.5 m^3 / s,$

$b = 9m,$

$d = 1.2m,$

Side Slope $= 1:2,$

$C = 49.5$

Let i = bed slope of trapezoidal channel.

We know that area of flow,

$A = 1/2 \text{ x } (9 + 10.2) \text{ x } 1.2m^2 = 11.52 m^2$

And wetted perimeter, $P = 9 + 2\sqrt{(1.2)^2 + (0.6)^2} = 11.68m$

$$\text{Hydraulic Mean Depth} = m = \frac{A}{P} = \frac{11.52}{11.68} = 0.968$$

Discharge through the pipe (Q),

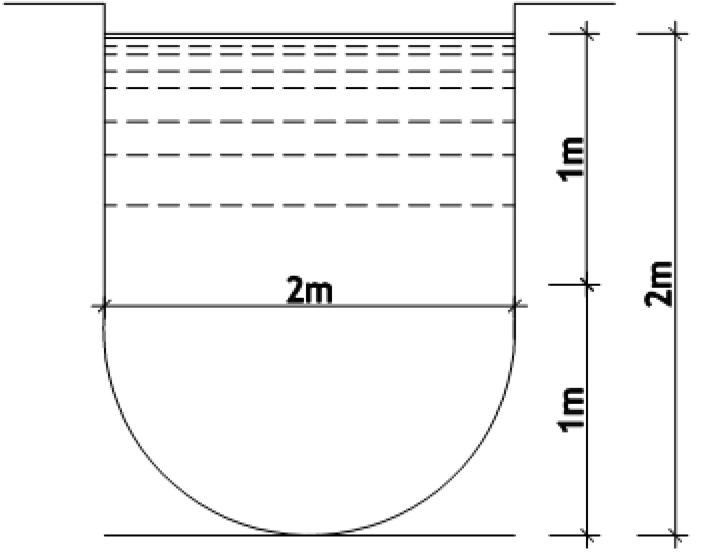
$$Q = A.C\sqrt{m.i}$$

$$16.5 = 11.52x49.5\sqrt{0.986 \ \ x \ \ i}$$

$$i = 8.47x10^{-4} = \frac{1}{1181}$$

Problem-3

A channel has two sides vertical and semi-circular bottom of 2 meters diameter. Calculate the discharge of water through the channel, when the depth of flow is 2m. Take C = 70 and slope of bed as 1 in 1000.



Solution:

Given :

Bottom Diameter = 2m, Depth of Water

C = 70 and i = 1/1000

We know that the area of flow,

$$A = (2x1) + \frac{\pi}{4}(2)^2$$

and Wetted Perimeter, $1 + (\pi \ x1) + 1 = 5.142\text{m}$

$$m = \frac{A}{P} = \frac{3.57}{5.14} = 0.695\text{m}$$

and Discharge of water through the channel,

$$Q = A.C\sqrt{mi} = 3.57 \text{ x } 70 \text{ x } \sqrt{0.695\text{x} \frac{1}{1000}} = 6.597\text{m}^3/\text{s}$$

Bazin's Formula for Discharge

- Bazin, after carrying out series of experiments, deducted the following relation for the value of C in the Chezy's formula for discharge,

$$C = \frac{157.6}{1.81 + \frac{K}{\sqrt{m}}}$$

- Where K is constant known as Bazin constant, whose value depends upon the roughness of the channel surface and m is the hydraulic mean depth.

Value of K:

S. No.	Type of inside surface of Channel	Value of K
1	Smooth Cement plaster or planed wood	0.11
2	Brickwork, stone or unplaned wood	0.21
3	Poor brickwork or rubble stone	0.83
4	Earth of very good surface	1.54
5	Earth of ordinary surface	2.35
6	Earth of rough surface	3.17

Problem-4

A rectangular channel 1.2m wide and 1m deep has longitudinal slope of 1 in 3000. Using Bazin's formula, find the discharge through the channel.

Solution:

Given :

$$b = 1.2m,\ d = 1m,\ i = 1/3000 \ \text{ and } K = 1.54$$

We know that the area of flow,

$$A = b.d = 1.2 \text{ x } 1 = 1.2m^2$$

and Wetted Perimeter $= P = 1 + 1.2 + 1 = 3.2m$

$$\text{Hydraulic mean depth} = m = \frac{A}{P} = \frac{1.2}{3.2} = 0.375m$$

We know that Chezy's Constant with Bazin's formula,

$$C = \frac{157.6}{1.81 + \frac{K}{\sqrt{m}}} = \frac{157.6}{1.81 + \frac{1.54}{\sqrt{0.375}}} = 36.4$$

and Discharge of water through the channel,

$$Q = A.C\sqrt{mi} = 1.2 \text{ x } 36.4 \text{ x } \sqrt{0.375\text{x } \frac{1}{3000}} = 0.489m^3/s$$

Value of N:

S. No.	Type of inside surface of Channel	Value of N
1	Smooth Cement plaster or planed wood	0.010
2	Brickwork, stone or unplaned wood	0.012
3	Poor brickwork or rubble stone	0.017
4	Earth of very good surface	0.020
5	Earth of ordinary surface	0.025
6	Earth of rough surface	0.030

Manning Formula for Discharge

- Manning, after carrying out series of experiments, deducted the following relation for the value or C in Chezy's formula for discharge:

$$C = \frac{1}{N} \times m^{1/6}$$

- Where N is constant and has same value as previous table.
- Now we see that the velocity,

$$v = C\sqrt{mi} = \frac{1}{N} \times m^{1/6}\sqrt{mi} \quad = \frac{1}{N} \times m^{2/3} \times i^{1/2}$$

$$= M \times m^{2/3} \times i^{1/2}$$

- M = 1/N and is known as Manning's constant.
- Now the Discharge, Q = Area x Velocity

$$= A \text{ x } M \text{ x } m^{2/3} \text{ x } i^{1/2}$$

Problem-6

A cement lined rectangular channel 6m wide carries water at the rate of 30m³/s. Find the value of Manning's constant, if the slope required to maintain a depth of 1.5m is 1/625.

Solution:

$Given:$

$\mathrm{b = 6m,\ Q = 30m^3/s,\ d = 1.5m\ and\ i = 1/625}$

Let $\quad \mathrm{N}$ = Value of Manning's constant

We know that the area of flow,

$\mathrm{A = b.d = 6\ x\ 1.5 = 9m^2}$

and Wetted Perimeter $= \mathrm{P} = 1.5 + 6 + 1.5 = 9m$

Hydraulic mean depth $= \mathrm{m} = \dfrac{\mathrm{A}}{\mathrm{P}} = \dfrac{9}{9} = 1\mathrm{m}$

We know that discharge through the channel (Q),

$$Q = A\ x\ \frac{1}{N} \times m^{2/3} \times i^{1/2}$$

$$30 = 9 \times \frac{1}{N} \times (1)^{2/3} \times \left(\frac{1}{625}\right)^{1/2} \quad \Rightarrow \quad N = 0.012$$

Channels of Most Economical Cross- Sections

- **A channel which gives maximum discharge for a given cross sectional area and bed slope is called a channel of most economical cross-section.**

  It can also be defined as:

- It gives maximum discharge for a given cross sectional area and bed slope
- It has minimum wetted perimeter
- It required lesser excavation for design amount of the discharge

Channels of Most Economical Cross- Sections

- The most economical section of a rectangular channel is one which has hydraulic radius equal to half the depth of flow.
- The most economical section of a trapezoidal channel is one which has hydraulic mean depth equal to half the depth of flow.
- The most economical section of a triangular channel is one which has its sloping sides at an angle of 45 degree with the vertical.
- The discharge through a channel of rectangular section is maximum when its breadth is twice the depth.

Channels of Most Economical Cross- Sections

- The discharge through a channel of trapezoidal section is maximum when the sloping side is equal to half the width at the top.
- The discharge through a channel of circular section is maximum when the depth of water is equal to 0.95 times the diameter of the circular channel.
- The velocity through a channel of circular section is maximum when the depth of water is equal to 0.81 times the diameter of circular channel.
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