Fluid Mechanics (CT-213)

- ***<u>Course Instructor</u>***

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Flow over Weirs

Lecture - 9

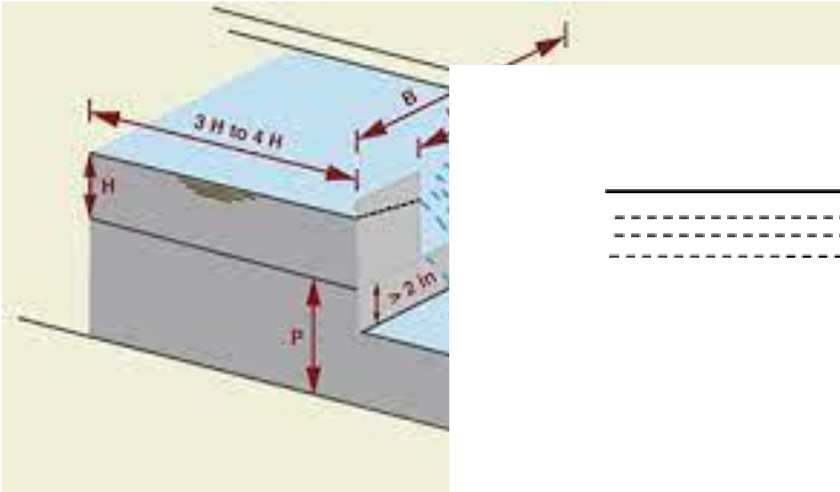
Introduction

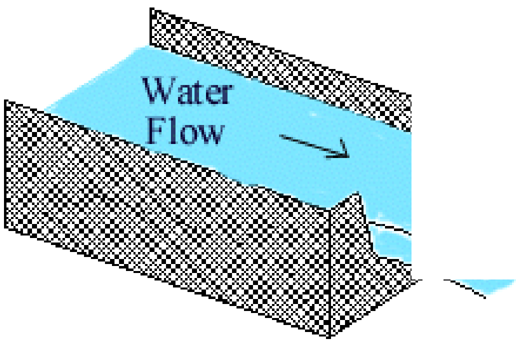
- **A structure, used to dam up a stream or river; over which the water flows, is called a weir.**
- The conditions of flow, in the case of a weir are practically the same, as those of a rectangular notch. That is why, a notch is sometimes called as a weir and vice versa.
- The only difference between a notch and a weir is, that the notch is of a small size; but the weir is of a bigger one. Moreover, a notch is usually made in a plate, whereas a weir is usually made of masonry or concrete.

Weirs

Types of Weirs

Shape	• Rectangular • Cippoletti (Trapezoidal)
Nature of Discharge	• Ordinary • Submerged or drowned
Width of Crest	• Narrow Crested • Broad Crested
Nature of Crest	• Sharp Crested • Ogee





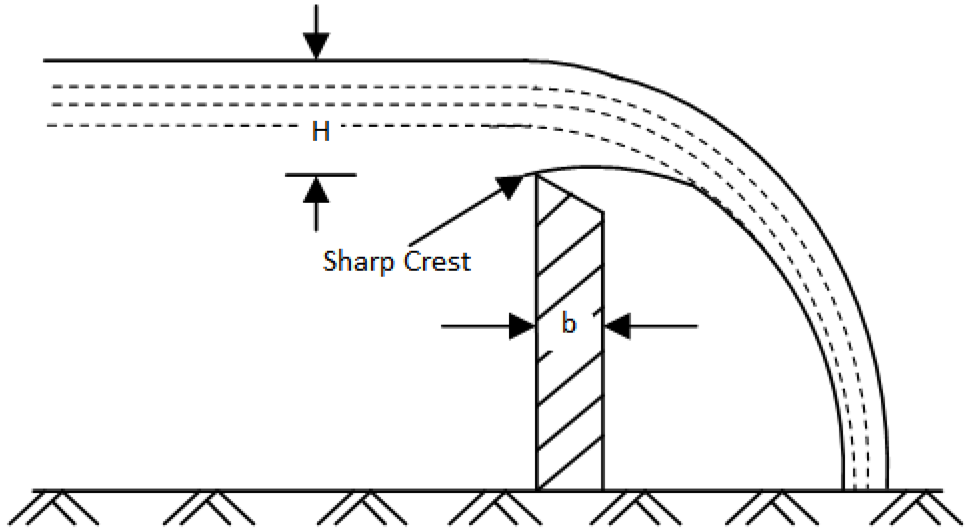
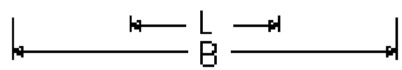


Fig : Sharp-crested weir



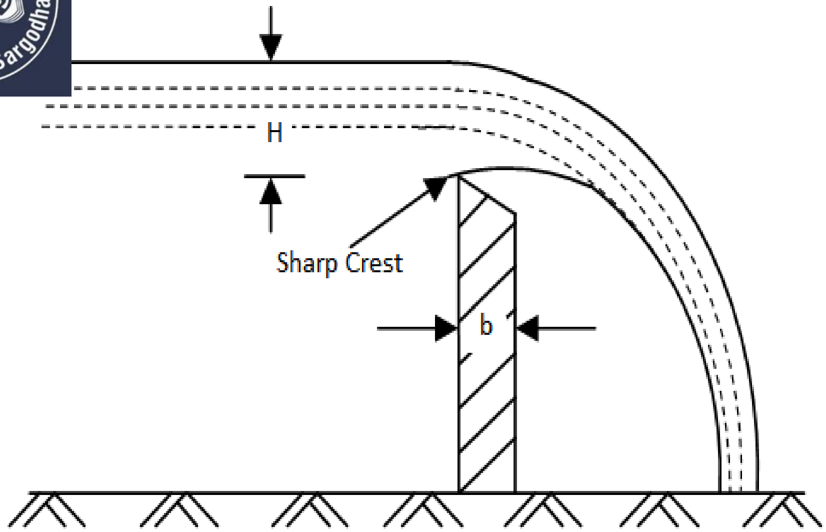
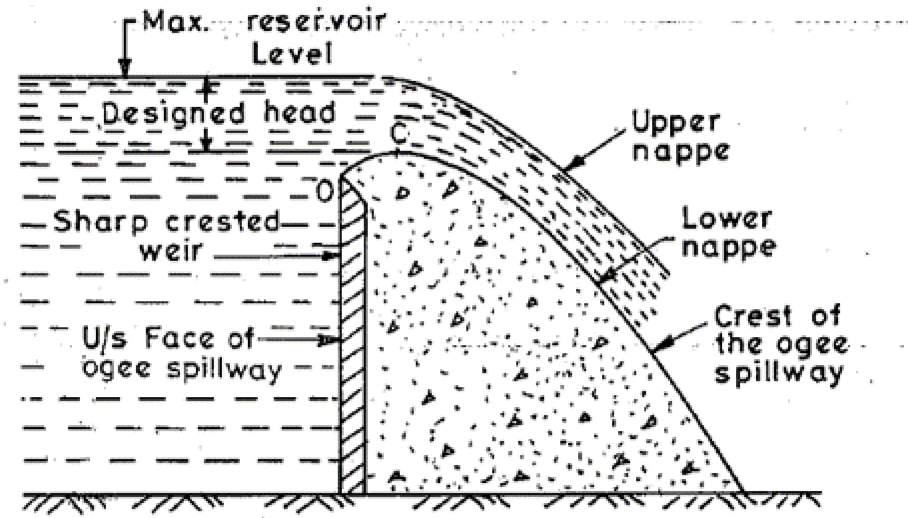
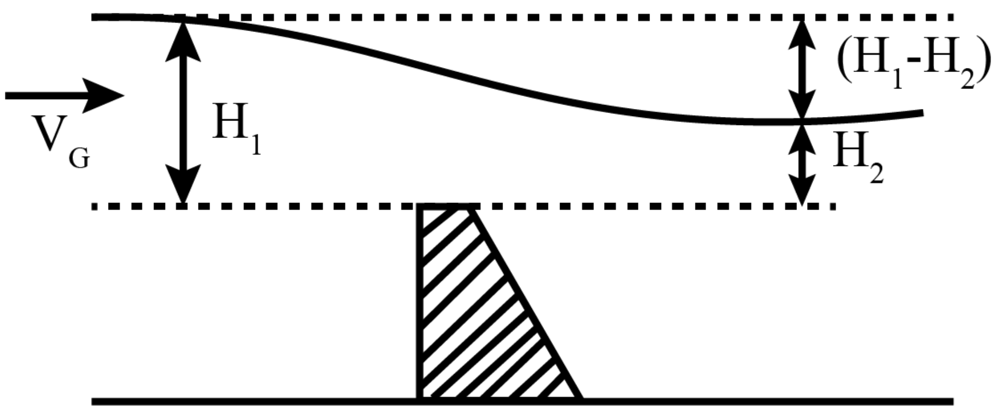


Fig : Sharp-crested weir

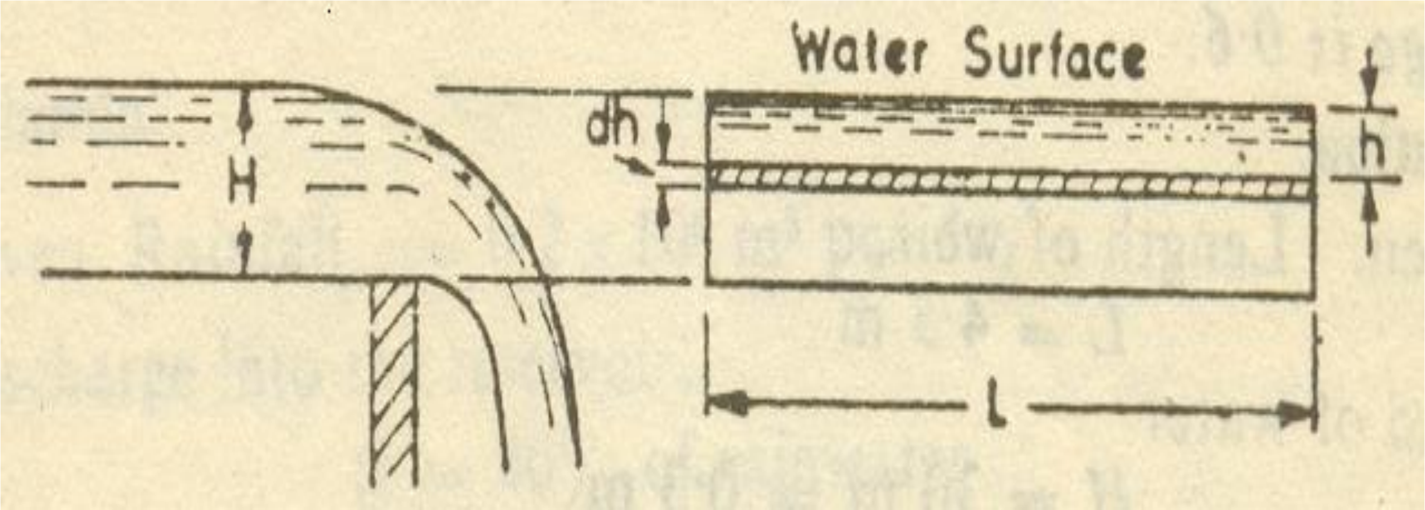




Submerged weir

Discharge over a Rectangular Weir

- Consider a rectangular weir, over which the water is flowing.



- Let

  H = Height of the water, above the crest of the weir,

  L = Length of the weir, and

  C_d = Coefficient of discharge.

Discharge over a Rectangular Weir

- Let us consider a horizontal strip of water of thickness *dh* at a depth *h* from the water surface as shown in.

  Area of strip = $L.dh$

- We know that theoretical velocity of water through the strip is,

$$= \sqrt{2gh}$$

- dq = C_d x Area of strip x Theoretical velocity

$$= C_d .L.dh\sqrt{2gh}$$

- The total discharge, over the weir, may be found out by integrating the above equation within the limits 0 and *H*.

$$Q = \int_0^H C_d L . dh \sqrt{2gh}$$

$$= C_d . L \sqrt{2g} \int_0^H h^{1/2} dh$$

$$= C_d . L \sqrt{2g} \left[\frac{h^{3/2}}{\frac{3}{2}} \right]_0^H$$

$$Q = \frac{2}{3} C_d . L \sqrt{2g} \left[h^{3/2} \right]_0^H$$

$$= \frac{2}{3} C_d . L \sqrt{2g} \, x H^{3/2}$$

Discharge over a Rectangular Weir

Note:

Sometimes, the limit of integration to the above equation, are from H_1 to H_2. (i.e,. the liquid level is at a height of H_1 above the top of the weir and H_2 above the bottom of the weir) instead of 0 and H; then the discharge over such a weir will be given be the equation.

$$Q = \frac{2}{3} C_d \; L \; \sqrt{2g} \left(H_2^{3/2} - H_1^{3/2} \right)$$

Problem-1

A rectangular weir 4.5 meters long has a head of water 300mm, Determine the discharge over the weir, if coefficient of discharge is 0.60.

Solution:

$$Given:$$

$$\mathrm{L} = 4.5\mathrm{m}$$

$$\mathrm{H} = 300\mathrm{mm} = 0.3\mathrm{m}$$

$$\mathrm{C_d} = 0.60$$

Discharge over the rectangular weir,

$$Q = \frac{2}{3} C_d . L \sqrt{2g} \left(H\right)^{\frac{3}{2}}$$

Putting values,

$$Q = \frac{2}{3} x 0.6 x 4.5 \sqrt{2 x 9.81} (0.3)^{3/2} m^3 / \sec$$

$$Q = 7.972 x 0.164 = 1.31 \ m^3 / s = 1310 \ \ litres / s$$

Problem-2

A weir 8 m long is to be built across a rectangular channel to discharge a flow of 9 m^3/sec. If the maximum depth of water on the upstream side of the weir is to be 2 m. what should be the height of the weir? Adopt $C_d = 0.62$

Solution:

Given :

$$\mathrm{L} = 8\,\mathrm{m} \qquad \mathrm{Q} = 9\,\mathrm{m}^3 / s$$

$$\text{Depth of water} = 2\,\mathrm{m} \quad \& \quad \mathrm{C_d} = 0.62$$

Let H = Height of water above the sill of water

$$\mathrm{Q} = \frac{2}{3} C_d . L \sqrt{2g} (H)^{\frac{3}{2}}$$

Putting values,

$$9 = \frac{2}{3} x 0.62 x 8 \sqrt{2 x 9.81} x H^{\frac{3}{2}} \qquad H^{\frac{3}{2}} = \frac{9}{14.645} = 0.614$$

$$H = 0.72\, m$$

Notes Compiled By: Engr. Abdul Rahim Khan (Assistant Professor, DCE, CET, UOS)

Therefore the height of weir should be $2 - 0.72 = 1.28\mathrm{m}$

Problem-3

The daily record of rainfall over a catchment area is 0.2 million cubic meters. It has been found that 80% of the rainwater reaches the storage reservoir and then passes over a rectangular weir. What should be the length of the weir, if the water is not to rise more than 400mm above the crest?

Assume suitable value of coefficient of discharge for the weir as 0.61.

Solution:

$$Given:$$

$$\text{Rainfall} = 0.2\text{x}10^6 m^3/day$$

$$\text{Discharge in the reservoir} = 80\% \text{ of rain water}$$

$$\text{H} = 400\text{mm} = 0.4\text{m}$$

$$\text{C}_\text{d} = 0.61$$

We know that the volume of water which reaches the reservoir from the catchment area,

$$Q = 80\% \text{ of rain water } = 0.8 \text{ x } (0.2 \text{ x } 10^6) m^3 / day$$

$$= 0.16 \text{ x } 10^6 \; m^3 / day = \frac{0.16 \text{ x } 10^6}{24 \text{ x } 60 \text{ x } 60} = 1.85 m^3 / s$$

Discharge over the rectangular weir,

$$Q = \frac{2}{3} C_d . L \sqrt{2g} \left(H \right)^{\frac{3}{2}}$$

Putting values,

$$1.85 = \frac{2}{3} x 0.61 x L \sqrt{2 x 9.81} x (0.4)^{\frac{3}{2}} = 0.456 L$$

$$L = 1.85 / 0.456 = 4.06 m$$

Bazin's formula for discharge over a Rectangular Weir

- Bazin, after carrying out series of experiments, proposed an empirical formula for the discharge over rectangular weir.
- He found that the value of coefficient of discharge varies with the height of water over the sill of the weir. Thus, he proposed an amendment in formula for rectangular weir.
- We know that discharge over rectangular weir,

$$Q = \frac{2}{3} C_d \, L \sqrt{2g} \;\; x \;\; H^{3/2}$$

- Bazin proposed that the discharge over a weir,

$$Q = m \, L \sqrt{2g} \;\; x \;\; H^{3/2}$$

Where m = 2/3 C_d

Bazin's formula for discharge over a Rectangular Weir

- He found that the value of (m) varies with the head of water, whose value may be obtained from the relation:

$$m = 0.405 + \frac{0.003}{H}$$

$$\text{Where H} = \text{Height of water in meters}$$

- He found the above relation by experiments in which he avoided the effect of end contractions.

Problem-6

Find the discharge over a rectangular weir 4.5m long under head of 600mm by using Bazin's formula.

Solution:

Given :

$$L = 4.5m$$

$$H = 600mm = 0.6m$$

From Bazin's relation :

$$m = 0.405 + \frac{0.003}{H} = 0.405 + \frac{0.003}{0.6} = 0.41$$

and the discharge over the weir (By Bazin's formula)

$$Q = m\,L\,\sqrt{2g}\;\;x\;\;H^{3/2}$$

$$= 0.41 \text{ x } 4.5 \text{ x } \sqrt{2 \text{ x } 9.81}\; x\, 0.6^{3/2}$$

$$= 3.8 \text{ m}^3 / s$$

Problem-7

A rectangular weir 6m long is discharging water under a head of 300mm. Calculate the discharge over the weir by using

i) Bazin's formula

Solution:

i) Discharge over the weir by using Bazin's formula,

From Bazin's relation:

$$m = 0.405 + \frac{0.003}{H} = 0.405 + \frac{0.003}{0.3} = 0.415$$

and the discharge over the weir (By Bazin's formula)

$$Q = m\ L\ \sqrt{2g}\ \ x\ \ H^{3/2}$$

$$= 0.41\ \text{x}\ 4.5\ \text{x}\ \sqrt{2\ \text{x}\ 9.81}\ x\ 0.6^{3/2}$$

$$=\ 1.18\ \text{m}^3 / s$$

Velocity of Approach:

It is defined as:

- "The velocity with which water approaches or reaches a weir before it flows over it is called velocity of approach."
- If V_a is the velocity of approach then additional head h_a equals to $V^2/2g$ due to velocity of approach is acting over the water flowing over the weir.
- The initial height of water becomes $H + h_a$ and the final height becomes h_a.
- It is determined by finding discharge over the weir neglecting velocity of approach. Then dividing discharge by the cross sectional area of the channel on the upstream side of the weir, the velocity of approach is obtained.

Velocity of Approach:

- Mathematically, $$V_a = \frac{Q}{\text{Area of Channel}}$$

- This velocity of approach is used to find an additional head:

$$h_a = \frac{{V_a}^2}{2\text{g}}$$

- Again the discharge is calculated and above process is repeated for more accurate discharge.

- Discharge over the rectangular weir with velocity of approach is:

$$Q = \frac{2}{3} C_d \; L \; \sqrt{2g} \;\; x \;\; [(H + h_a)^{3/2} - {h_a}^{3/2}]$$

Problem-8

A weir 2.0m long has 0.6m head of water over the crest. Using Francis' formula, determine the new discharge, considering the velocity of approach. The weir is 6m wide and 1.2m deep. Discharge is 1.95 m^3/sec

Also **Solution:**

Given:

$L = 2.0m$

$H = 0.6m$

Width of channel $=$ 6m and depth $=$ 1.2m

ii) Discharge over the weir considering the velocity of approach

Cross sectional area of water flowing in the channel,

$$A = 6 \text{ x } 0.6 = 3.6\text{m}^3$$

$$\therefore \text{ Velocity of approach, v } = \frac{\text{Q}}{\text{A}} = \frac{1.95}{3.6} = 0.54 m/s$$

and head due to velocity of approach,

$$\text{h}_\text{a} = \frac{\text{v}^2}{2g} = \frac{0.54^2}{2x9.81} = 0.015m$$

Putting values,

Total Head $\text{H}_1 = \text{H} + \text{h}_\text{a} = 0.6 + 0.015 = 0.615m$

$$Q = Q = \frac{2}{3} C_d \; L \; \sqrt{2g} \;\; x \;\; [(H + h_a)^{3/2} - h_a{}^{3/2}]$$

$$Q = ??? \;\; m^3 / s$$

Flow through Mouthpieces

- We know that discharge through an orifice depends upon its coefficient of discharge. It was felt by the engineers that the discharge through an orifice is too less (due to low value of coefficient of discharge).
- It was found after conducting series of experiments by engineers that if a short pipe be fitted to an orifice, it will increase the value of coefficient of discharge and of course discharge too.
- Such a pipe whose length is generally more than 2 times the diameter of the orifice and is fitted (externally or internally) to the orifice is known as mouthpiece.

Flow through Mouthpieces

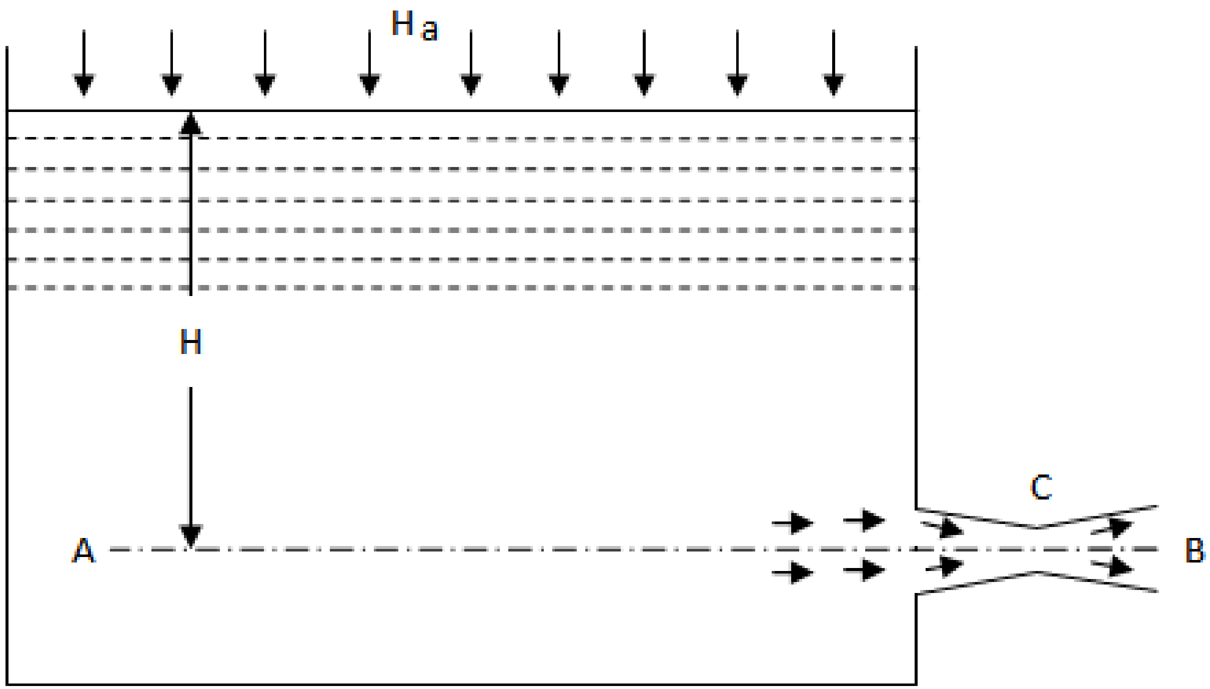
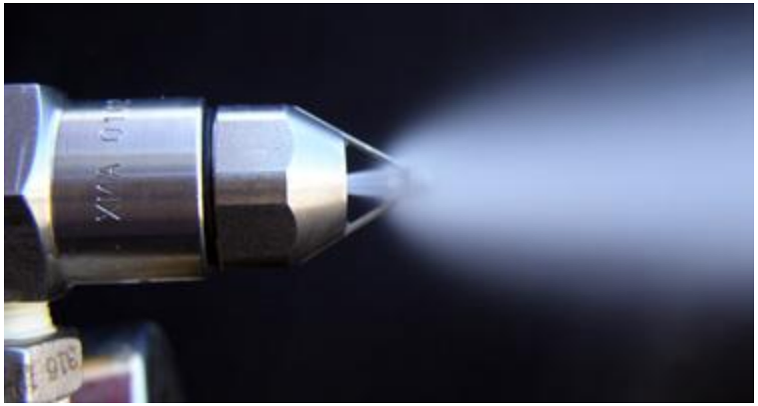


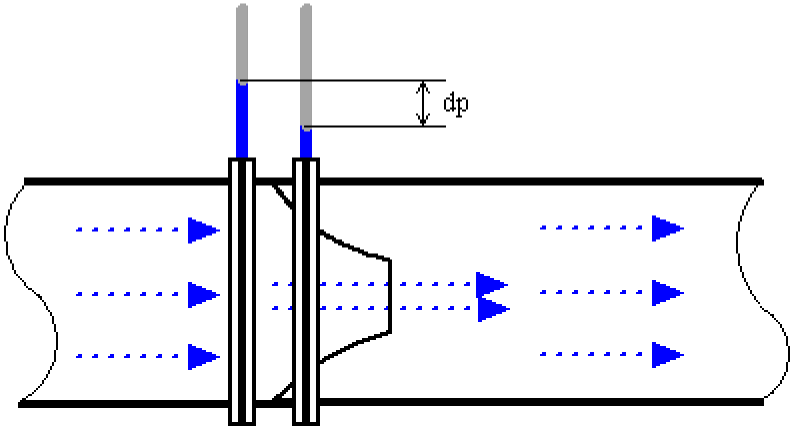
Fig-2 : Pressure in convergent-divergent mouthpiece


SM3M

Flow through Nozzles

- A nozzle is a tapering mouthpiece, which is fitted to the outlet end of a pipe.
- A nozzle is generally, used to have a high velocity of water, as it converts pressure head into kinetic head at its outlet.
- A high velocity of water is required in fire fighting, service station, mining power development etc.





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