Fluid Mechanics (CT-213)

- ***<u>Course Instructor</u>***

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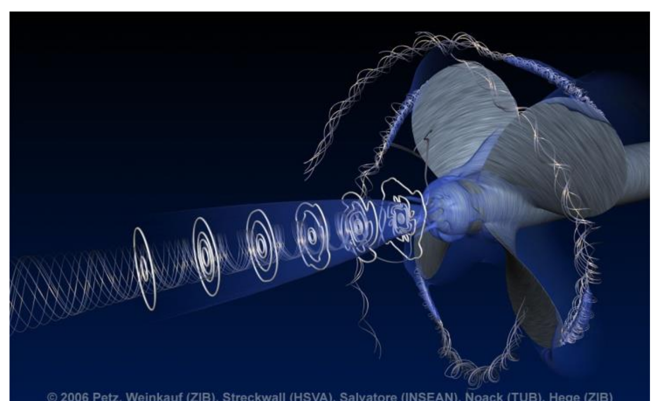
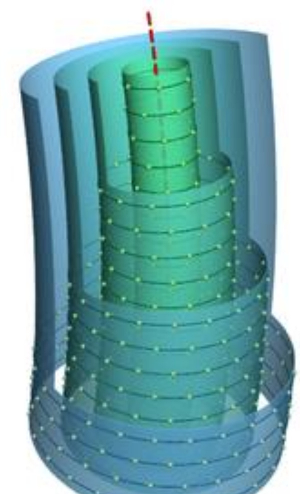
Vortex Flow + Flow Over Notches

Lecture - 8

Vortex

- *In fluid dynamics, a vortex is a region within a fluid where the flow is mostly a spinning motion about an imaginary axis, straight or curved. That motion pattern is called a vortical flow.*
- Some common examples are smoke rings, the whirlpools often seen around the boats and paddles, and the winds surrounding hurricanes, tornadoes and dust devils.





Vortex Flow:

- If we take a cylindrical vessel, containing some liquid, and start rotating it, about its vertical axis, we see that the liquid will also start revolving along with the vessel.
- After some time, we shall see that the liquid surface no longer remains level. But it has been depressed down at the axis of its rotation and has risen up near the wall of the vessel on all sides.
- This type of flow, in which a liquid flows continuously round a curved path about a fixed axis of rotation is called **vortex flow**.

Types:

1. Forced or Rotational Flow
2. Free or Ir-rotational Flow

1. Forced Vortex Flow:

- It is a type of vortex flow, in which the vessel, containing a liquid, is forced to rotate about the fixed vertical axis with the help of a torque.
- If the applied torque is removed the rotational motion will be slowly destroyed.
- Now consider a cylindrical vessel containing a liquid initially up to AA as shown in figure.

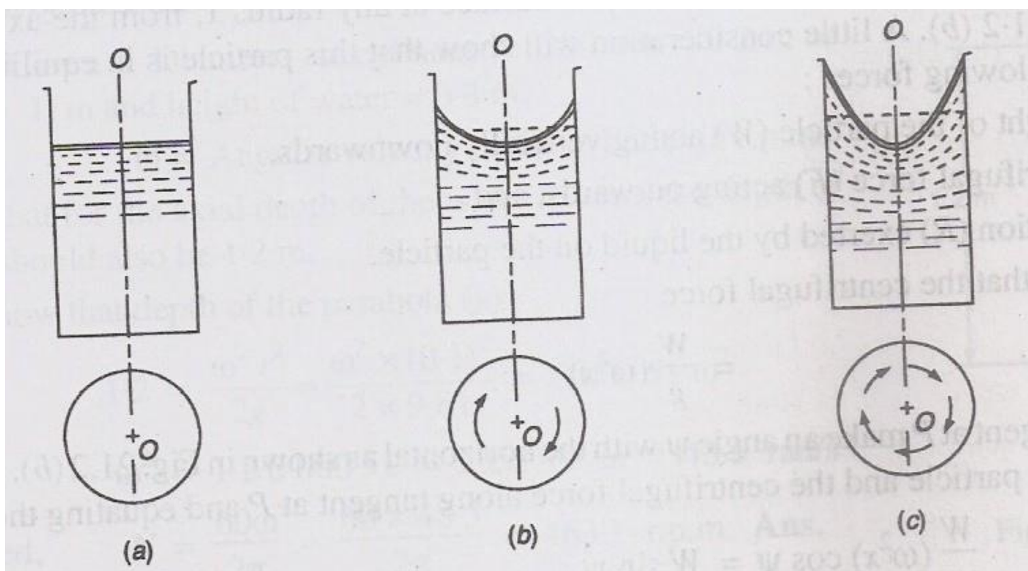
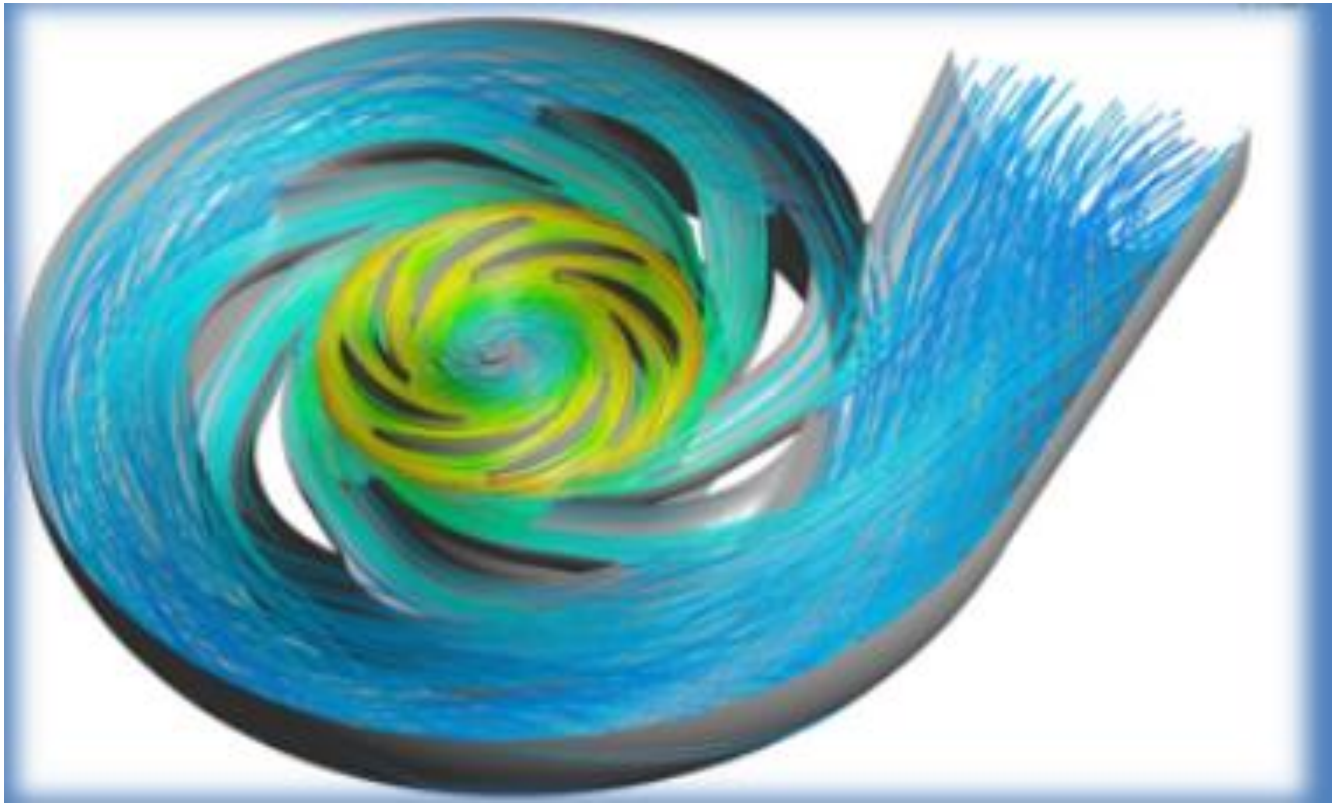


Fig. 21·1. Forced vortex flow.

1. Forced Vortex Flow:

- Let the vessel be rotated about its vertical axis O-O. It will be noticed that the liquid surface, in the vessel, no longer remains level. But it has depressed down at the axis of its rotation and has risen up near the wall of the vessel on all sides (Fig. b).
- If the vessel is revolved with the increased angular velocity, it will be noticed that the liquid has depressed down to greater extent at its axis of rotation, risen up to greater height near the walls of the vessel (Fig. c).
- If we further increase the velocity of rotation, the liquid will spill out of the vessel and ultimately the axial depth of liquid will become zero.



Flow through Impeller of Centrifugal Pump

2. Free Vortex:

- It is a type of flow, in which the liquid particles describe circular paths, about a fixed vertical axis, without any external force acting on the particles.
- The common example of a free vortex occurs when the water escapes, through the hole in the bottom of a wash basin.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Flow Over NOTCHES

Introduction

- A notch may be defined as:

  “An opening in one side of a tank or a reservoir, like a large orifice, with the upstream liquid level below the top edge of the opening”

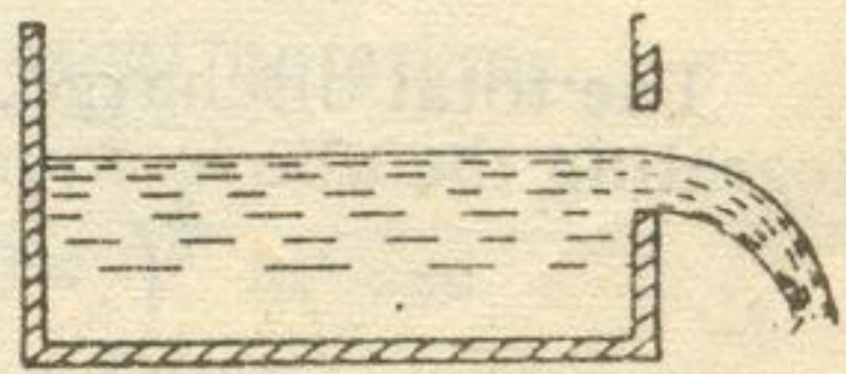

Fig. 11·1. A Notch.


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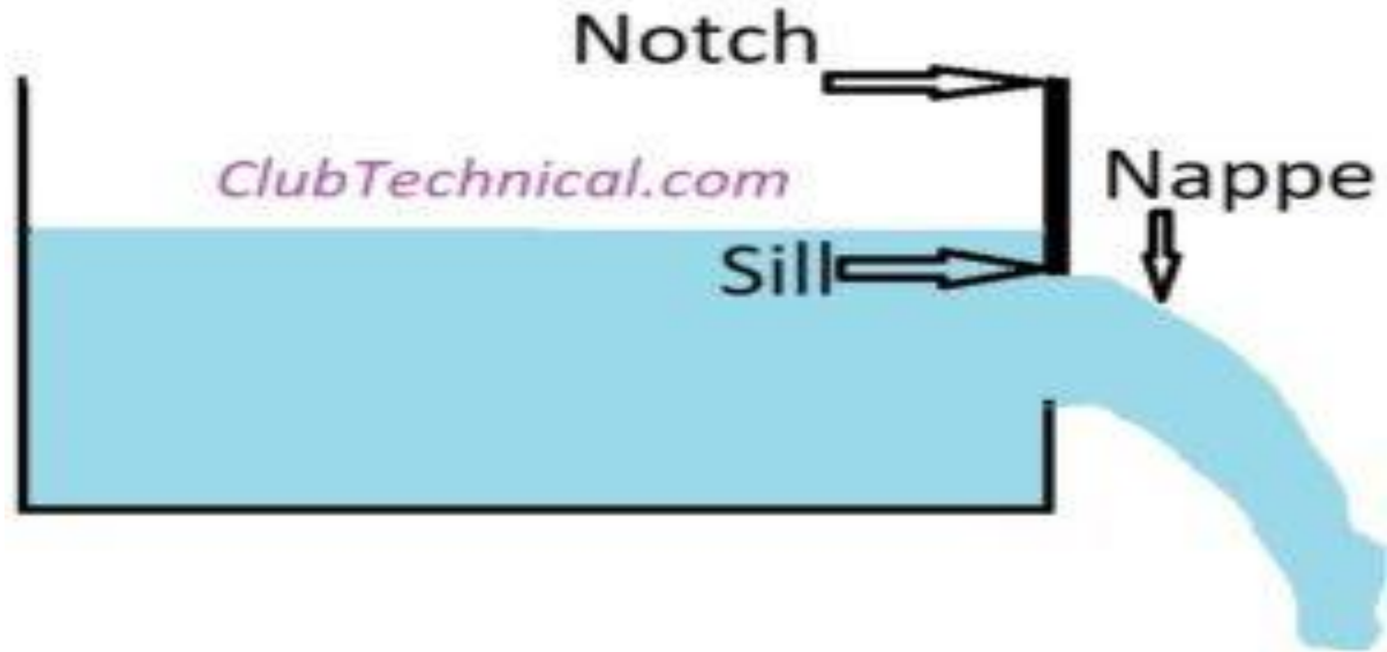
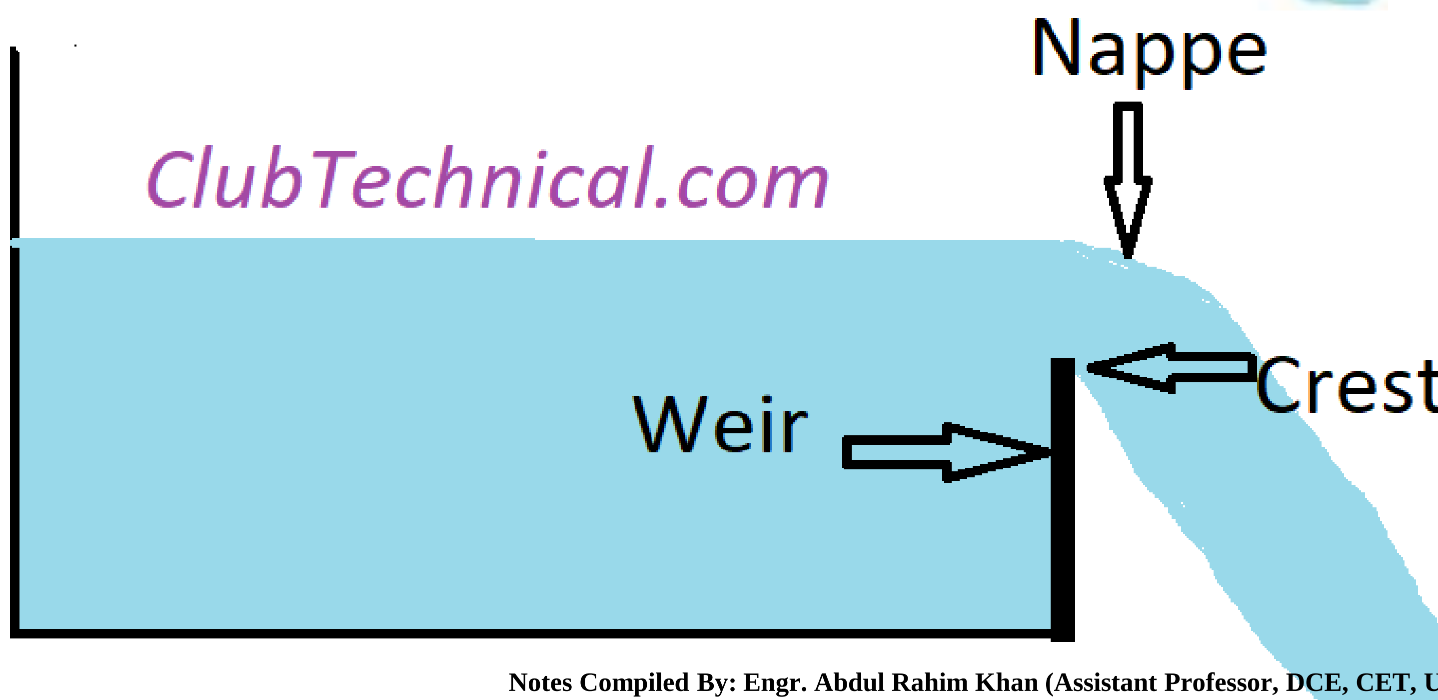
Introduction

- Since the top edge or the notch above the liquid level, serves no purpose, therefore a notch may have only the bottom edge and sides.
- The bottom edge, over which the liquid flows, is known as **sill or crest** of the notch and the sheet of liquid flowing over a notch (or a weir) is known as **nappe or vein**.
- A notch is usually made of a metallic plate and is used to measure the discharge of liquids.


Notch
ClubTechnical.com
Nappe
Sill


Nappe
ClubTechnical.com
Crest
Weir

Types:

- There are many types of notches, depending upon their shapes. But the following are important from the subject point of view.

1. Rectangular notch
2. Triangular notch
3. Trapezoidal notch
4. Stepped notch

Discharge over a Rectangular Notch

- Consider a rectangular notch. in one side of a tank over which the water is flowing as shown in Fig.

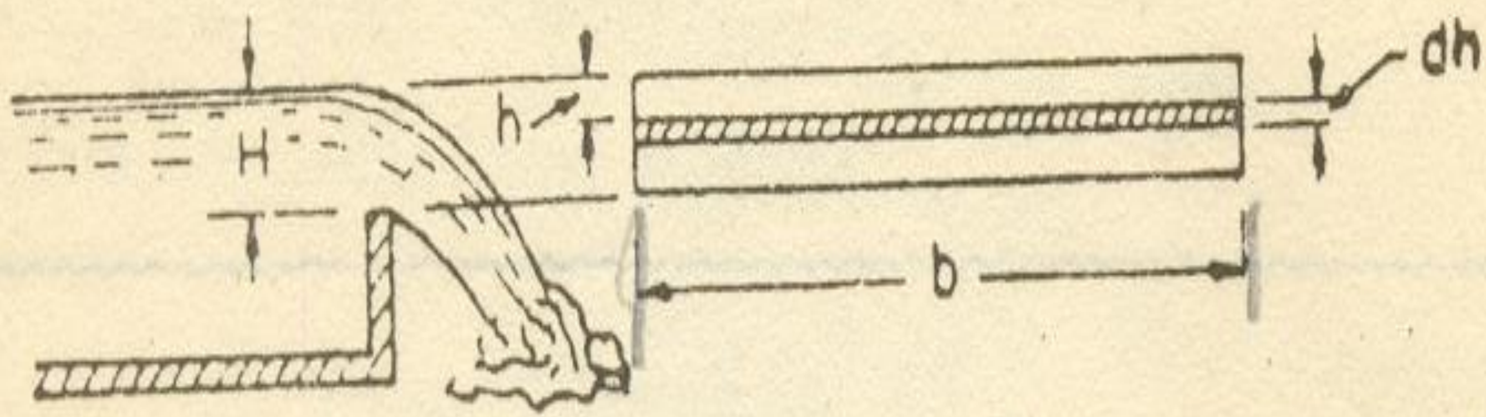
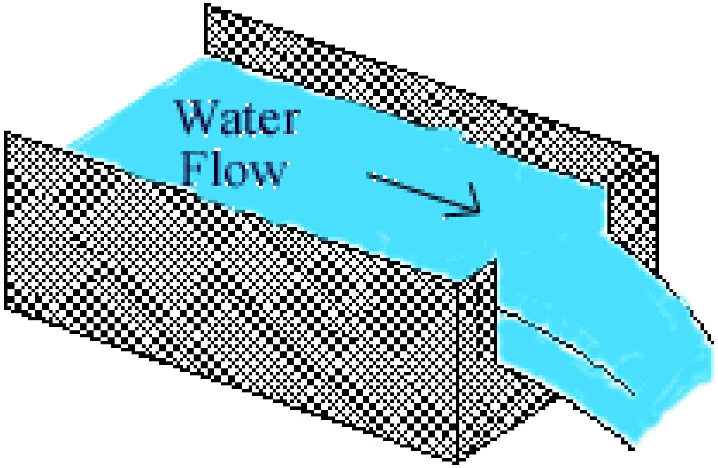
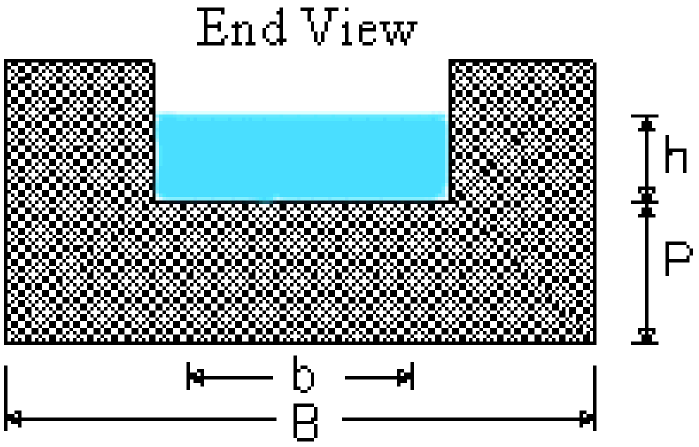


Fig. 11·2 Rectangular notch.


Water
Flow


End View
h
P
b
B

Discharge over a Rectangular Notch

- Let

  H = Height of water above sill of the notch,

  b = Width or length of the notch, and

  C_d = Coefficient of discharge.

  Let us consider a horizontal strip of water of thickness dh at a depth of h from the water level as shown in Fig.

- Area of the strip = $b.dh$ *(i)*
- We know that theoretical velocity of water through the strip

  $= \sqrt{2gh}$ *(ii)*

Discharge over a Rectangular Notch

- Discharge through the strip,

$$dq = C_d \text{ x Area of strip x Theoretical velocity}$$

$$= C_d \; b.dh \; \sqrt{2gh}$$

- The total discharge, over the whole notch, may be found out by integrating the above equation within the limits 0 and *H*.

$$Q = \int_0^H C_d .bdh\sqrt{2gh}$$

$$= C_d .b\sqrt{2g} \int_0^H h^{1/2} dh$$

Discharge over a Rectangular Notch

$$= C_d .b\sqrt{2g}\left[\frac{h^{3/2}}{\frac{3}{2}}\right]_0^H \qquad = \frac{2}{3}C_d .b\sqrt{2g}\left[h^{\frac{3}{2}}\right]_0^H$$

$$= \frac{2}{3}C_d .b\sqrt{2g}\,(H)^{\frac{3}{2}}$$

- **Note:** Sometimes the limits of integration, in the above equation, are from H_1 to H_2 (i.e. the liquid level is at a height of H_1 above the top of the notch and H_2 above the bottom of the notch, instead of 0 to H; then the discharge over such a notch will be given by the equation.

$$Q = \frac{2}{3}C_d .b\sqrt{2g}\left(H_2^{3/2} - H_1^{3/2}\right)$$

Problem-1

A rectangular notch 0.5 meters wide has a constant head of 400 mm. Find the discharge over the notch, in litres per second, if coefficient of discharge for the notch is 0.62.

Solution:

Given:

$\mathrm{b} = 0.5\mathrm{m}$

$\mathrm{H} = 400\mathrm{mm} = 0.4\mathrm{m}$

$\mathrm{C_d} = 0.62$

Discharge over the rectangular notch,

$$Q = \frac{2}{3} C_d . b \sqrt{2g} (H)^{\frac{3}{2}}$$

Putting values,

$$Q = \frac{2}{3} x 0.62 x 0.5 \sqrt{2 x 9.81} (0.4)^{3/2} m^3 / \sec$$

$$Q = 0.915 x 0.253 = 0.231 m^3 / s = 231 \ \ litres / s$$

Problem-2

A rectangular notch has a discharge of 21.5 cubic meters per minute, when the head of water is half the length of the notch. Find the length of the notch. Assume C_d = 0.6.

Solution:

$$Given:$$

$$Q = 21.5m^3/\min = 21.5/60 = 0.358m^3/s$$

$$\mathrm{H} = b/2 = 0.5\mathrm{b}$$

$$\mathrm{C_d} = 0.6$$

Discharge over the rectangular notch,

$$\mathrm{Q} = \frac{2}{3} C_d . b\sqrt{2g}(H)^{\frac{3}{2}}$$

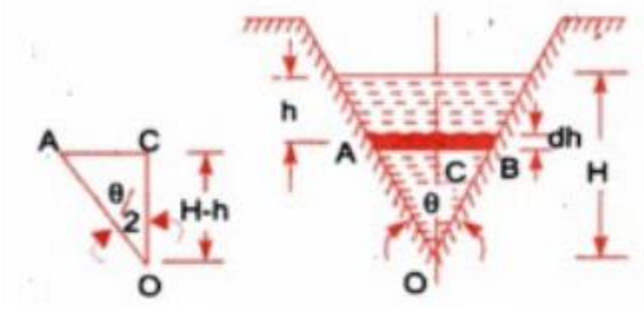
Putting values,

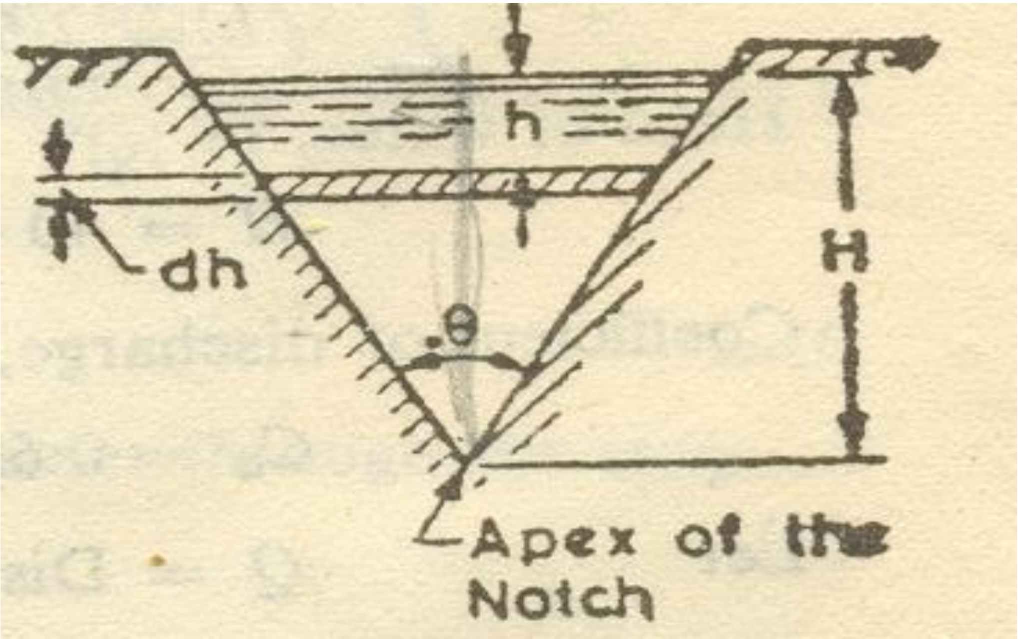
$$0.358 = \frac{2}{3} x0.6xb\sqrt{2x9.81}\left(\frac{b}{2}\right)^{\frac{3}{2}} = 0.626b^{\frac{5}{2}}$$

$$b^{\frac{5}{2}} = \frac{0.358}{0.626} = 0.572 \qquad \mathrm{b} = 0.8\mathrm{m}$$

Discharge over a Triangular Notch

- A triangular notch it also called a **V-notch**. Consider a triangular notch, in one side of the tank, over which the water is flowing.





Discharge over a Triangular Notch

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$$AC = (H-h)\tan\frac{\theta}{2}$$

$$\text{Width of strip } = AB = 2AC = 2\,(H-h)\tan\frac{\theta}{2}$$

$$\text{Area of strip } = 2\,(H-h)\tan\frac{\theta}{2}\times dh$$

- We know that theoretical velocity of water through the strip

$$= \sqrt{2gh}$$

- and discharge over the notch,

$$dq = C_d \text{ x Area of strip x Theoretical velocity}$$

$$= C_d .2(H-h)\tan\frac{\theta}{2}\,dh\sqrt{2gh}$$

Discharge over a Triangular Notch

- The total discharge, over the whole notch, may be found out by integrating the above equation, within the limits 0 and *H*.

$$Q = \int_0^H C_d .2(H-h)\tan\frac{\theta}{2}\,dh.\sqrt{2gh}$$

$$= 2C_d\sqrt{2g}\,\tan\frac{\theta}{2}\int_0^H (H-h)\sqrt{h}.dh$$

$$= C_d\sqrt{2g}\,\tan\frac{\theta}{2}\int_0^H \left[Hh^{1/2} - h^{3/2}\right]dh$$

$$= 2C_d\sqrt{2g}\,\tan\frac{\theta}{2}\left[\frac{H.h^{\frac{3}{2}}}{\frac{3}{2}} - \frac{h^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^H$$

$$= \frac{8}{15}C_d\sqrt{2g}\,\tan\frac{\theta}{2}\,xH^{\frac{5}{2}}$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{2}{3}H.H^{3/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{4}{15}H^{5/2}\right]$$

$$= \frac{8}{15}C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\times H^{5/2}$$

If $\theta = 90^o$,
$C_d = 0.6$ and
$g = 9.81 m/s^2$
then $Q = 1.417\,H^{3/2}$

Advantages of a Triangular Notch over a Rectangular Notch

1. Only one reading i.e., head (H) is required to be taken for the measurement of discharge, in a given triangular notch.
2. If, in a triangular notch the angle of the notch i.e. $\theta = 90°$, the formula becomes very simple (i.e., $Q = 1.417H^{5/3}$) to remember.
3. A triangular notch gives more accurate results for low discharges than a rectangular notch.
4. The same triangular notch can measure a wide range of flows accurately.

Problem-3

A right-angled V-notch was used to measure the discharge of a centrifugal pump. If the depth of water at V-notch is 200 mm, calculate the discharge over the notch in litres per minute. Assume coefficient of discharge as 0.62.

Solution:

Given :

$\theta = 90^{o}$, $\mathrm{H} = 200\mathrm{mm} = 0.2\mathrm{m}$

$\mathrm{C_d} = 0.62$

Discharge over the triangula r notch,

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} x H^{\frac{5}{2}}$$

Putting values,

$$Q = \frac{8}{15} x 0.62 x \sqrt{2 x 9.81} \tan 45^{0} x (0.2)^{\frac{5}{2}} m^3 / \sec$$

$$Q = 1.465 x 0.018 = 0.026 m^3 / \sec$$

$$Q = 26 Lit / s = 1560 Lit / \min$$

Problem-4

During an experiment in a laboratory, 280 litres of water flowing over a right-angled notch was collected in one minute. If the head of the sill is 100mm, calculate the coefficient of discharge of the notch.

Solution:

Given :

$$Q = 280 lit/\min = 0.0047 m^3/s$$

$$\mathrm{H} = 100\mathrm{mm} = 0.1\mathrm{m}$$

Discharge over the triangula r notch,

$$\mathrm{Q} = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} x H^{\frac{5}{2}}$$

Putting values,

$$0.0047 = \frac{8}{15} C_d \sqrt{2x981} \tan 45^0 x (0.1)^{\frac{5}{2}} = 0.0075 C_d$$

$$C_d = 0.0047/0.0075 = 0.627$$

Problem-5

A Rectangular channel 1.5 meter wide is used to carry 0.2 cubic meter of water. The rate of flow is measured by placing a V-notch weir. If the maximum depth of water is not to exceed 1.2 meter, find the position of the apex of the notch from the bed of the channel. Assume C_d = 0.6.

Solution:

Given :

Width of rectangular notch (b) = 1.5m

$Q = 0.2 m^3/s, \qquad \theta = 90^o, \qquad C_d = 0.6$

Let H = Height of water above the apex of notch.

We know that Discharge over the triangular notch (Q),

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} x H^{\frac{5}{2}}$$

Putting values,

$$0.2 = \frac{8}{15} x 0.6 \sqrt{2x9.81} \tan 45^0 x (H)^{\frac{5}{2}} = 1.417 H^{5/2}$$

$H = 0.46m$

Problem-6

Water flows over a rectangular notch of 1 meter length over a depth, of 150 mm. Then the same quantity of water passes through a triangular right-angled notch. Find the depth of water through the notch.

Take the coefficients of discharges, for the rectangular and triangular notch, as 0.62 and 0.59 respectively.

Solution:

$$\text{Given :}$$

$$\text{For rectangular notch } b = 1.5\text{m}$$

$$H_1 = 150mm = 0.15m$$

$$C_d = 0.62$$

$$\text{For triangular notch}$$

$$\theta = 90^o \quad \text{and} \quad C_d = 0.59$$

First of all, consider the flow of water over rectangular notch.

$$Q = \frac{2}{3} C_d \sqrt{2g} \tan\frac{\theta}{2} x H^{\frac{5}{2}}$$

Putting values,

$$Q = \frac{2}{3} x 0.62 x 1 x \sqrt{2x9.81} \tan 45^0 x (0.15)^{\frac{5}{2}}$$

$$Q = 1.831 x 0.058 = 0.106 m^3 / s$$

Now consider the flow of water over the triangular notch.

We know that discharge over the triangular notch,

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} x H^{\frac{5}{2}}$$

$$0.106 = \frac{8}{15} x 0.59 \sqrt{2x9.81} \tan 45^0 x H_2^{\frac{5}{2}}$$

$$H_2 = 0.357 m$$

Discharge over a Trapezoidal Notch

- A trapezoidal notch is a combination of a rectangular notch and two triangular notches as shown in Fig.
- It is thus obvious, that the discharge over such notch will be the sum of the discharges over the rectangular and triangular notches.

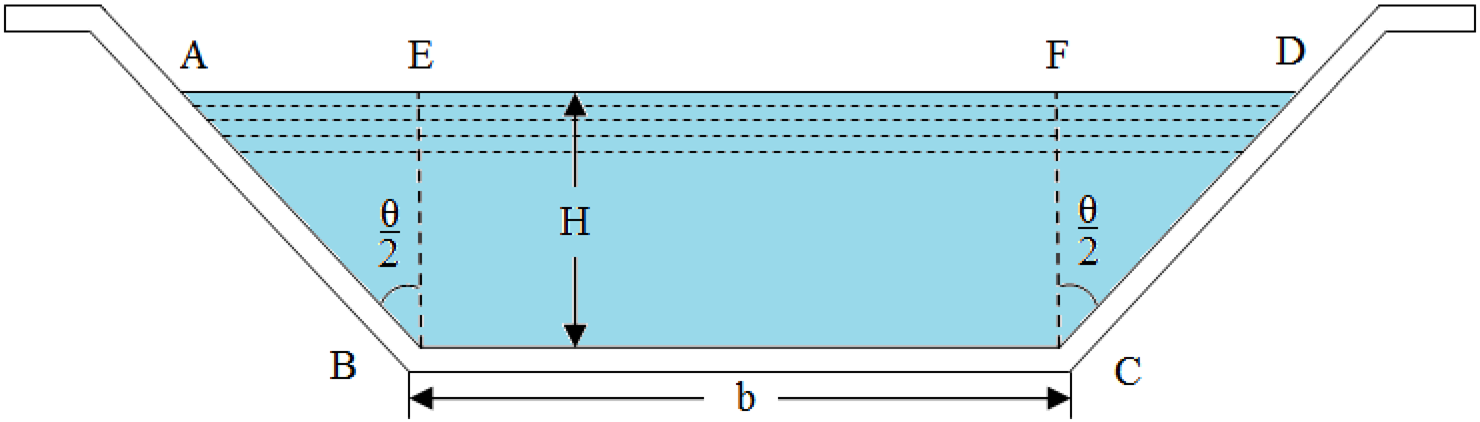


Fig : Trapezoidal Notch

Discharge over a Trapezoidal Notch

- Consider a trapezoidal notch *ABCD* as shown in Fig. For analysis is purpose split up the notch into a rectangular notch *BCFE* and two triangular notches *ABE* and *DCF*. The discharge over these two triangular notches is equivalent to the discharge over a single triangular notch of angle θ.
- Let

H = Height of the liquid above the sill of the notch.

Cd_1 = Coefficient of discharge for the rectangular portion

Cd_2 = Coefficient of discharge for the triangular portions

b = Breadth of the rectangular portion of the notch, and

θ =Angle, which the sides make, with the vertical.

Discharge over a Trapezoidal Notch

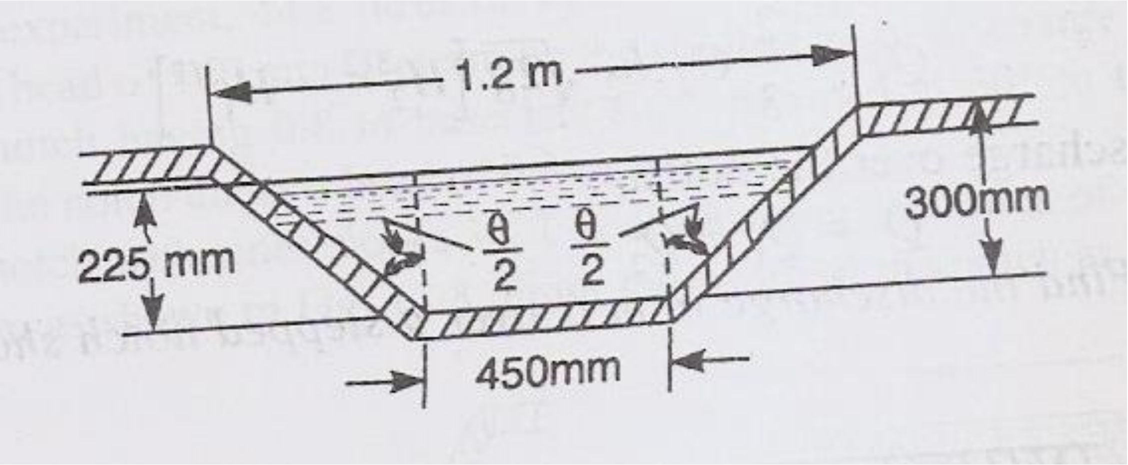
- Discharge over the trapezoidal notch,

$Q =$ Discharge over the Rectangular notch+ Discharge over the Triangular notch.

$$= \frac{2}{3} C_{d1} . b \sqrt{2g} H^{\frac{3}{2}} + \frac{8}{15} C_{d2} \sqrt{2g} \tan \frac{\theta}{2} H^{\frac{5}{2}}$$

Problem-7

A trapezoidal notch of 1.2m wide at the top and 450mm at the bottom is 300mm high. Find the discharge through the notch, if the head of water is 225mm. Take coefficient of discharge as 0.6.



Solution:

Given :

Width of notch $= 1.2\text{m}$

$\text{b} = 450\text{mm} = 0.45\text{m}$

Height of notch $= 300\text{mm} = 0.3\text{m}$

$\text{H} = 225\text{mm} = 0.225\text{m}$ and $\text{C}_\text{d} = 0.6$

From the geometry of notch, we find that,

$$\tan\frac{\theta}{2} = \frac{1200-450}{2} x \frac{1}{300} = \frac{750}{600} = 1.25$$

and the discharge over the trapezoidal notch,

$$Q = \frac{2}{3} C_d . b\sqrt{2g} H^{\frac{3}{2}} + \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H^{\frac{5}{2}}$$

$$Q = \frac{2}{3} x0.6x0.45\sqrt{2x9.81}x(0.225)^{\frac{3}{2}} + \frac{8}{15} x0.6x\sqrt{2x9.81}x1.25x(0.225)^{\frac{5}{2}}$$

$$Q = 0.085 + 0.043 = 0.0128 m^3 / s = 128 lit / s$$