Fluid Mechanics (CT-213)

- ***Course Instructor***

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Energy Consideration in Steady Flow

Lecture - 6

Introduction

- Up till now we have studied the motion of liquid particle without taking into consideration any force or energy causing the flow.
- This lecture deals with the motion of liquids and the forces causing the flow from a viewpoint of energy considerations.
- The first law of thermodynamics tells us that energy can neither be created nor destroyed. But it can of course, be changed from one form to other. It follows that all forms of energy are equivalent.

Energy

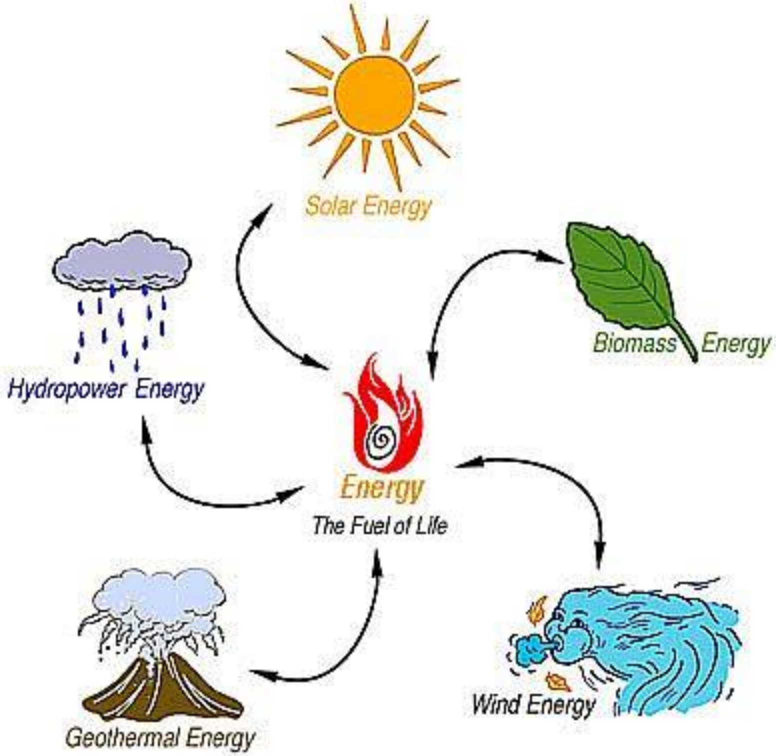
- The **energy**, in general, may be defined as:

  "The capacity to do work."

  Or

- "Quantity that is often understood as the ability of a physical system to do work on other physical systems."

- Since work is defined as a force acting through a distance (a length of space), *energy is always equivalent to the ability to exert pulls or pushes against the basic forces of nature, along a path of a certain length.*


Solar Energy
Biomass Energy
Hydropower Energy
Energy
The Fuel of Life
Geothermal Energy
Wind Energy

Energy

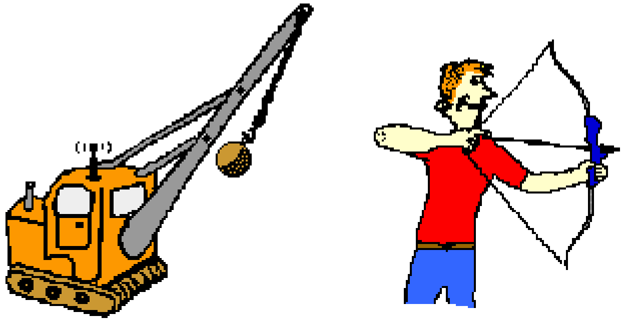
Energies of a flowing fluid

- Though the energy exists in many forms, yet the following are important from the subject point of view.

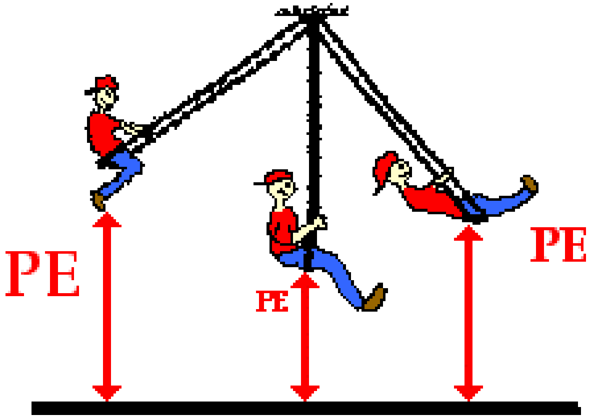
1. Potential Energy
2. Kinetic Energy
3. Pressure Energy

1. Potential Energy

- Energy of an object or a system due to the position of the body or the arrangement of the particles of the system.



The massive ball of a demolition machine and the stretched bow possesses stored energy of position - potential energy.



PE in terms of Fluid

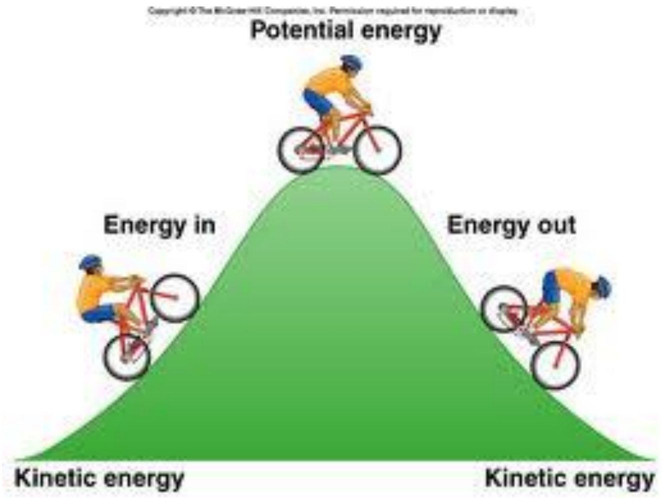
- **"It is the energy possessed by a liquid particle by virtue of its position."**
- A fluid particle of weight *W* situated a distance *z* above datum possesses a potential energy of *Wz*.
- Thus its potential energy per unit *weight* is *z*, measured in units ft.lb/lb=ft or N.m/N=m.
- The particle's potential energy per unit mass is *gz*, measured in units of ft^2/sec^2 or m^2/s^2;
- Its potential energy per unit volume is ***pgz***, measured in units of lb/ft^2 or N/m^2.

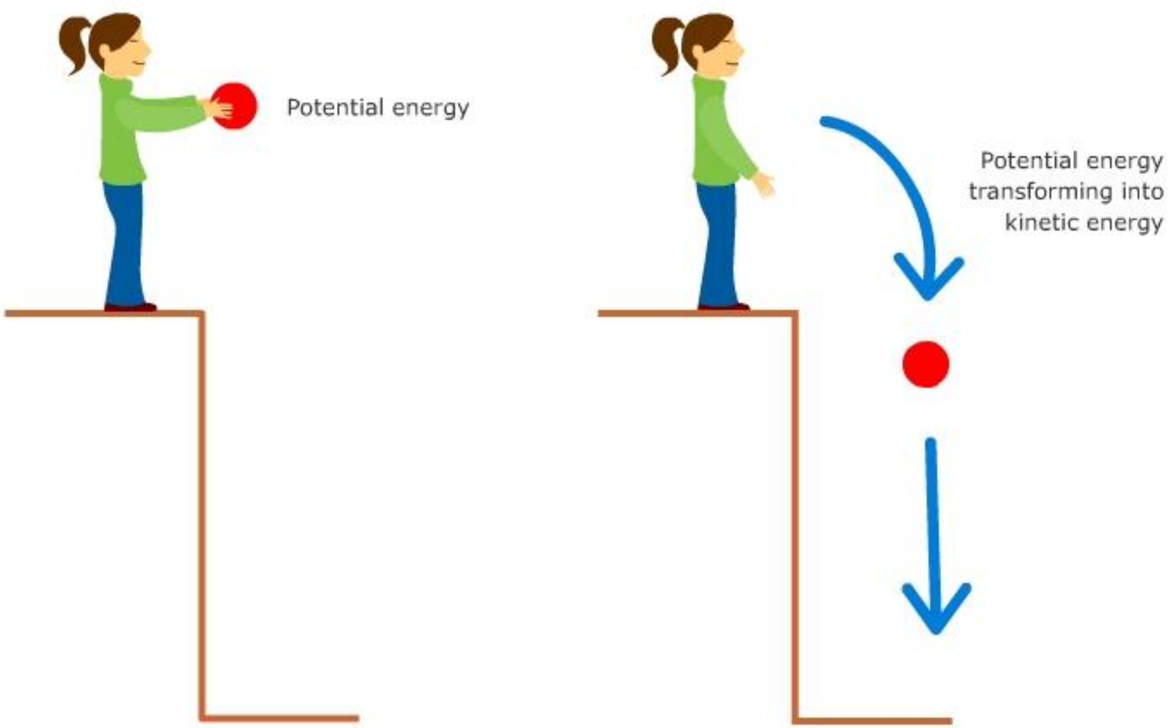
2. Kinetic Energy

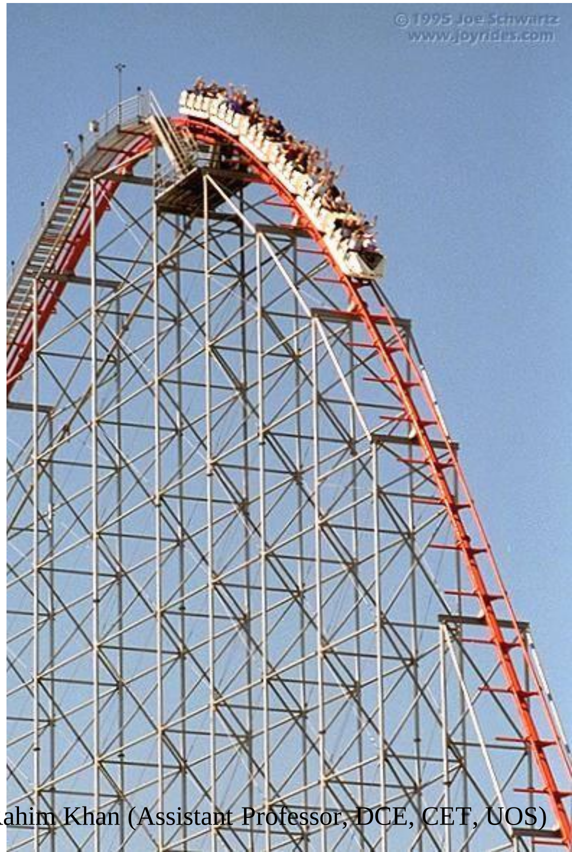
- *Anything that is moving contains kinetic energy.*
- The **kinetic energy** of an object is the energy which it possesses due to its motion.
- It is defined as the work needed to accelerate a body of a given mass from rest to its stated velocity.
- E.g. Wind, waves,

  falling rocks

Energy Conversion







KE in terms of Fluid

- A body of mass *m* when moving at a velocity *V* possesses a kinetic energy, **KE = ½ *m V*^{2}.**
- Thus if a fluid were flowing with all particles moving at the same velocity, its kinetic energy would also be ½ *m V*^{2};
- for unit *weight* of the fluid we can write this as:

$$\frac{KE}{Weight} = \frac{\frac{1}{2}mV^2}{\gamma\forall} = \frac{\frac{1}{2}(p\forall)V^2}{pg\forall} = \frac{V^2}{2g} \qquad (5.1a)$$

- where $\forall$ represents the volume of the fluid mass. In BG units we express $V^2/2g$ in ft-lb/lb = ft and in SI units as N .m/N = m.

KE in terms of Fluid

- Similarly,

$$\frac{KE}{Mass} = \frac{\frac{1}{2}mV^2}{m} = \frac{V^2}{2} \qquad (5.1b)$$

$$\frac{KE}{Volume} = \frac{\frac{1}{2}mV^2}{\forall} = \frac{\frac{1}{2}(p\forall)V^2}{\forall} = \frac{pV^2}{2} \qquad (5.1c)$$

- The units of $V^2/2$ of course are ft^2/sec^2 in BG (British Gravitational) units or m^2/s^2 in SI units. The units of $pV^2/2$ are lb/ft^2 or N/m^2, which are units of pressure.

True KE

- In most situations the velocities of the different fluid particles crossing a section are not the same, so it is necessary to integrate all portions of the stream to obtain the true value of the kinetic energy.
- It is convenient to express the true value in terms of the mean velocity *V* and a factor *a* (alpha), known as the *kinetic-energy correction factor or coriolis coefficient*. Then,

$$\frac{True\ KE}{Weight} = \alpha \frac{V^2}{2g} \qquad (5.2)$$

Correction Factor

- As the average of cubes is always greater than the cube of the average, the value of α will always be more than 1. The greater the variation in velocity across the section, the larger will be the value of α. For laminar flow in a circular pipe, $\alpha=2$ (Problem 5.1), for turbulent flow in pipes, ranges from 1.01 to 1.15, but it is usually between 1.03 and 1.06.

Correction Factor

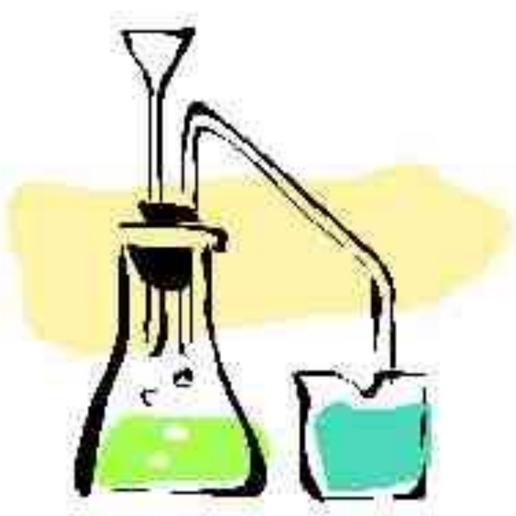
- In some instances it is very desirable to use the proper value of α, but in most cases the error made in neglecting its divergence from 1.0 is negligible.
- As precise values of α are seldom known, it is customary in the case of turbulent flow to assume that = 1, i.e., that the kinetic energy is $V^2/2g$ per unit weight of fluid, measured in units of ft.lb/lb=ft or N.m/N=m.
- In laminar flow the velocity is usually so small that the kinetic energy per unit weight of fluid is negligible.

3. Pressure Energy

- It is the energy, possessed by a liquid particle, by virtue of its existing pressure.
- A particle of fluid has energy due to its pressure above datum, most usually its pressure above atmospheric, although we normally do not refer to this as pressure energy.
- Pressure is $p = \gamma h$, and so the depth of liquid that would produce this pressure, or the "pressure head", is $h = p/\gamma$.

4. Internal Energy

- Internal energy is stored energy that is associated with the molecular, or internal state of matter; it may be stored in many forms, including thermal, nuclear, chemical, and electrostatic.



EQUATION FOR STEADY MOTION OF AN IDEAL FLUID ALONG A STREAMLINE, AND BERNOULLI'S THEOREM

- **BERNOULLI'S THEOREM**
- The total mechanical energy of the moving fluid comprising the gravitational potential energy of elevation, the energy associated with the fluid pressure and the kinetic energy of the fluid motion, remains constant.
- Bernoulli’s principle can be derived from the principle of conservation of energy.
- Bernoulli’s Principle Formula
- Bernoulli’s equation formula is a relation between pressure, kinetic energy, and gravitational potential energy of a fluid in a container.

EQUATION FOR STEADY MOTION OF AN IDEAL FLUID ALONG A STREAMLINE, AND BERNOULLI'S THEOREM

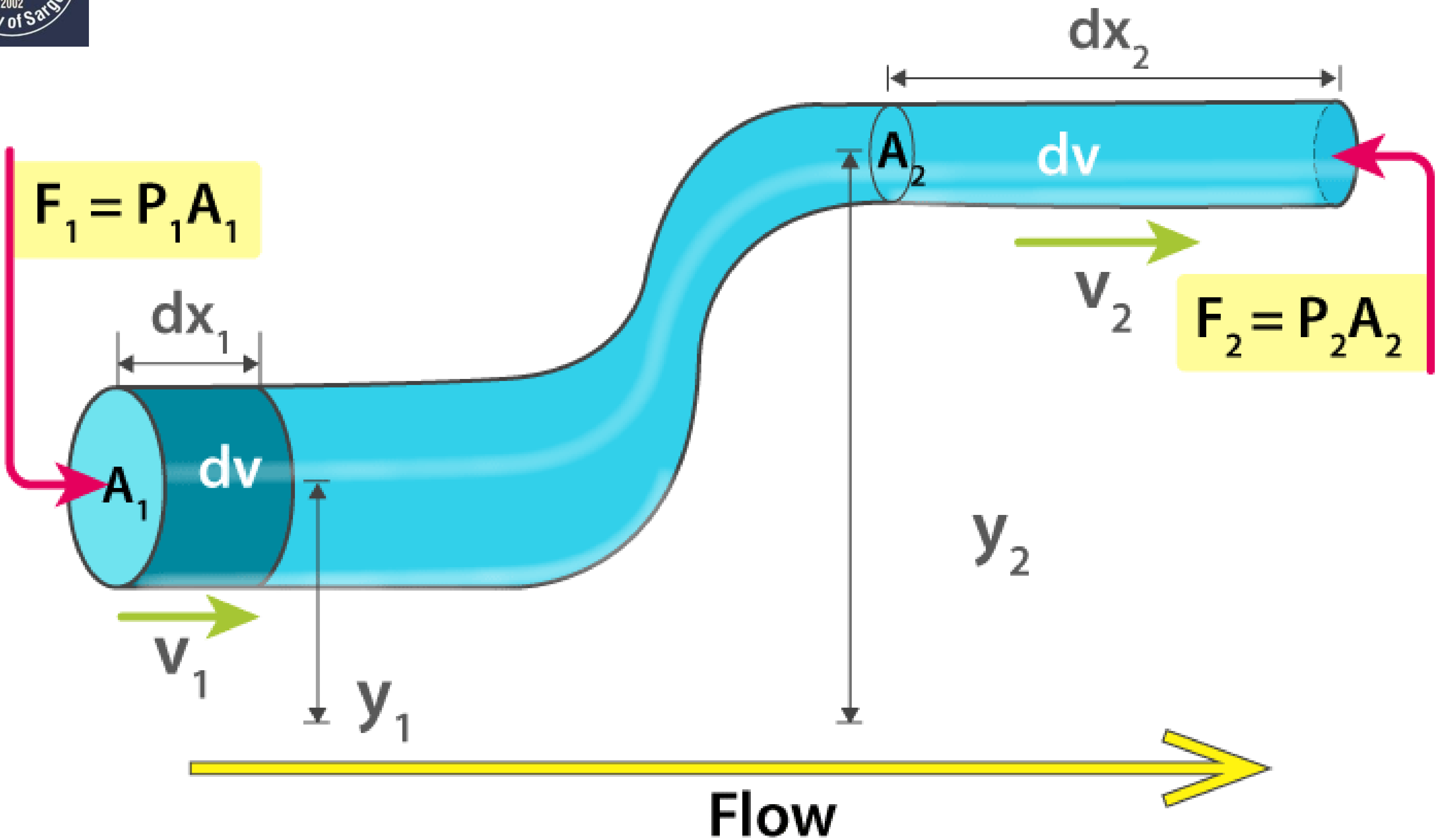
- The formula for Bernoulli’s principle is given as follows:

$$p+1/2\rho v^2+\rho gh=\text{constant}$$

- Where (p) is the pressure exerted by the fluid, (v) is the velocity of the fluid, (ρ) is the density of the fluid and h is the height of the container.
- Bernoulli’s equation gives great insight into the balance between pressure, velocity and elevation.

BERNOULLI'S THEOREM DERIVATION

- Consider a pipe with varying diameter and height through which an incompressible fluid is flowing.
- The relationship between the areas of cross-sections (A), the flow speed (v), height from the ground y, and pressure (p) at two different points 1 and 2 are given in the figure below.
- **Assumptions:**
- The density of the incompressible fluid remains constant at both points.
- The energy of the fluid is conserved as there are no viscous forces in the fluid.



dx2
A2
dv
v2
F2 = P2A2
F1 = P1A1
dx1
A1
dv
v1
y1
y2
Flow

$dW = F_1dx_1 - F_2dx_2$

$dW = p_1A_1dx_1 - p_2A_2dx_2$

$dW = p_1dv - p_2dv = (p_1 - p_2)dv$

We know that the work done on the fluid was due to the conservation of change in gravitational potential energy and change in kinetic energy. The change in kinetic energy of the fluid is given as:

$$dK = \frac{1}{2}m_2v_2^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}\rho dv(v_2^2 - v_1^2)$$

The change in potential energy is given as:

$dU = m_2gy_2 - m_1gy_1 = \rho dvg(y_2 - y_1)$

Therefore, the energy equation is given as:

$dW = dK + dU$

$$(p_1 - p_2)dv = \frac{1}{2}\rho dv(v_2^2 - v_1^2) + \rho dvg(y_2 - y_1)$$

$$p_1 - p_2) = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$

Rearranging the above equation, we get

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho gy_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho gy_2$$

This is Bernoulli's equation.

Assumptions:

1. It assumes viscous (friction) effects are negligible

2. It assumes the flow is steady

3. The equation applies along a streamline

4. It assumes the fluid to be incompressible

5. It assumes no energy is added to or removed from the fluid along the streamline

BERNOULLI'S THEOREM APPLICATION

- Bernoulli's principle is used for studying the unsteady potential flow which is used in the theory of ocean surface waves and acoustics. It is also used for approximation of parameters like pressure and speed of the fluid.
- The other applications of Bernoulli's principle are:
- **Venturi meter:** It is a device that is based on Bernoulli's theorem and is used for measuring the rate of flow of liquid through the pipes. Using Bernoulli's theorem, Venturi meter formula is given as: $V = A_1 A_2 \sqrt{\frac{2hg}{A_1^2 - A_2^2}}$

BERNOULLI'S THEOREM APPLICATION

- **Working of an aeroplane:** The shape of the wings is such that the air passes at a higher speed over the upper surface than the lower surface. The difference in airspeed is calculated using Bernoulli's principle to create a pressure difference.
- When we are standing at a railway station and a train comes we tend to fall towards the train. This can be explained using Bernoulli's principle as the train goes past, the velocity of air between the train and us increases. Hence, from the equation, we can say that the pressure decreases. So the pressure from behind pushes us towards the train. This is based on Bernoulli's effect.

Problem:

- **Calculate the pressure in the hose whose absolute pressure is 1.01×10^5 N.m^{-2} if the speed of the water in the hose increases from 1.96 m.s^{-1} to 25.5 m.s^{-1}. Assume that the flow is frictionless and density 10^3 kg.m^{-3}**

Ans: Given,

Pressure at point 2, $p_2 = 1.01 \times 10^5$ N.m^{-2}

Density of the fluid, $\rho = 10^3$ kg.m^{-3}

Velocity of the fluid at point 1, $v_1 = 1.96$ m.s^{-1}

Velocity of the fluid at point 2, $v_2 = 25.5$ m.s^{-1}

From Bernoulli's principle for p_1,

$$p_1 = p_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho(v_2^2 - v_1^2)$$

Substituting the values in the above equation, we get

$$p_1 = (1.01 \times 10^5) + \frac{1}{2}(10^3)[(25.5)^2 - (1.96)^2]$$

$p_1 = 4.24 \times 10^5$ N.m^{-2}

Problem:

- A horizontal pipe of non-uniform cross-section allows water to flow through it with a velocity 1 ms−1 when pressure is 50 kPa at a point. If the velocity of flow has to be 2 ms−1 at some other point, what will the pressure at that point?

$$\Rightarrow P_1 + \frac{1}{2}\rho_1 V_1^2 = P_2 + \frac{1}{2}\rho_2 V_2^2$$

$$\Rightarrow 50 \times 10^3 + \frac{1}{2} \times 10^3 (1)^2 = P_2 + \frac{1}{2} \times 10^3 \times 4$$

$$\Rightarrow 5 \times 10^4 + 500 - 2000 = p_2$$

$$\Rightarrow P_2 = 48.5 \text{ kPa}$$