Fluid Mechanics (CT-213)

- ***Course Instructor***

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Fluid Statics

Lecture -4

Notes Compiled By Engr. Abdul
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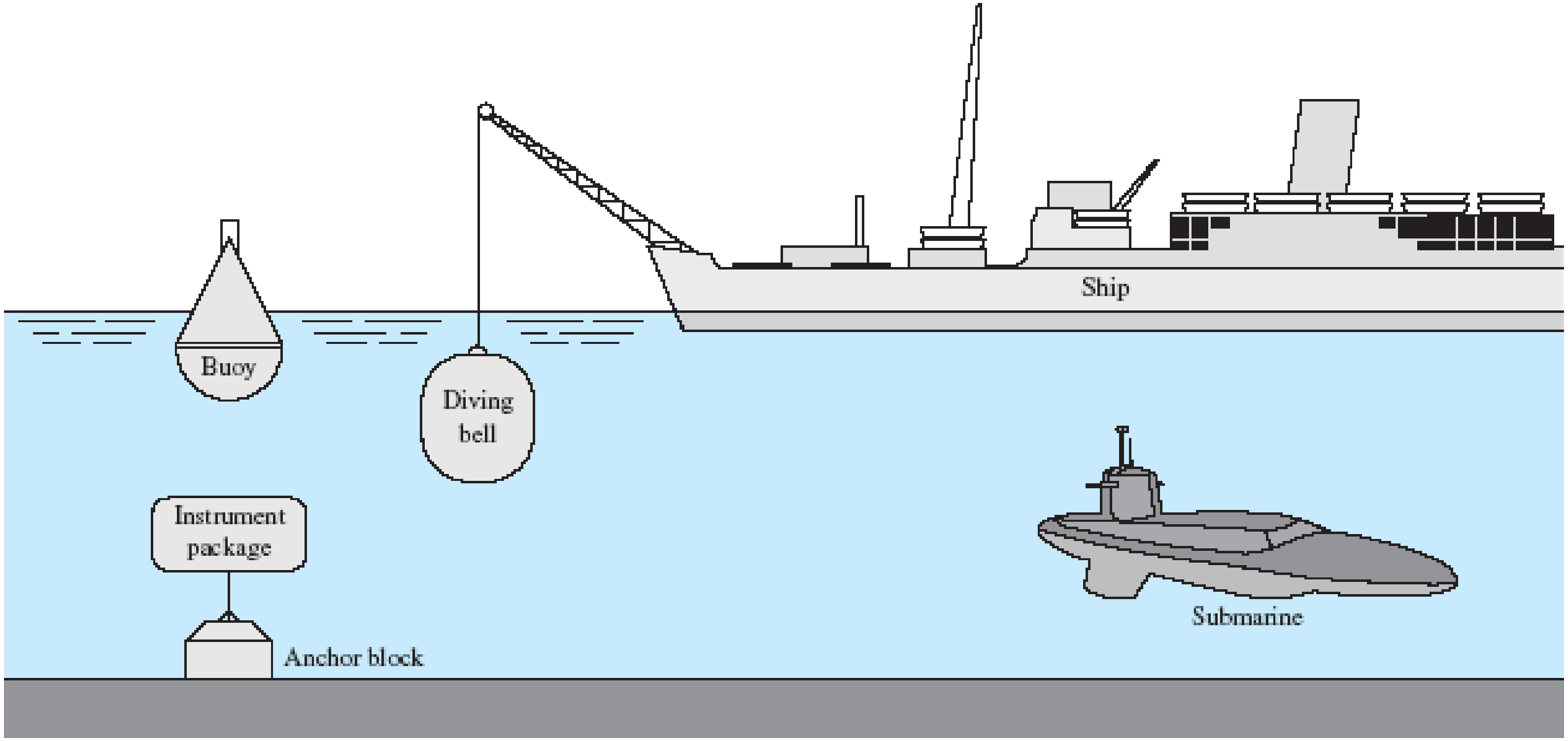
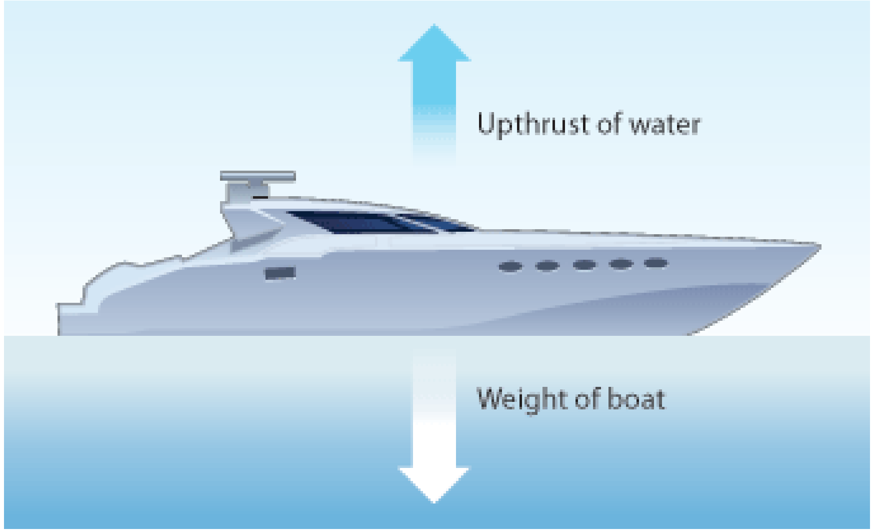
Buoyancy & Stability

Examples of types of Buoyancy Problems:



Introduction:

- Whenever a body is placed over a liquid, either it sinks down or floats on the liquid.

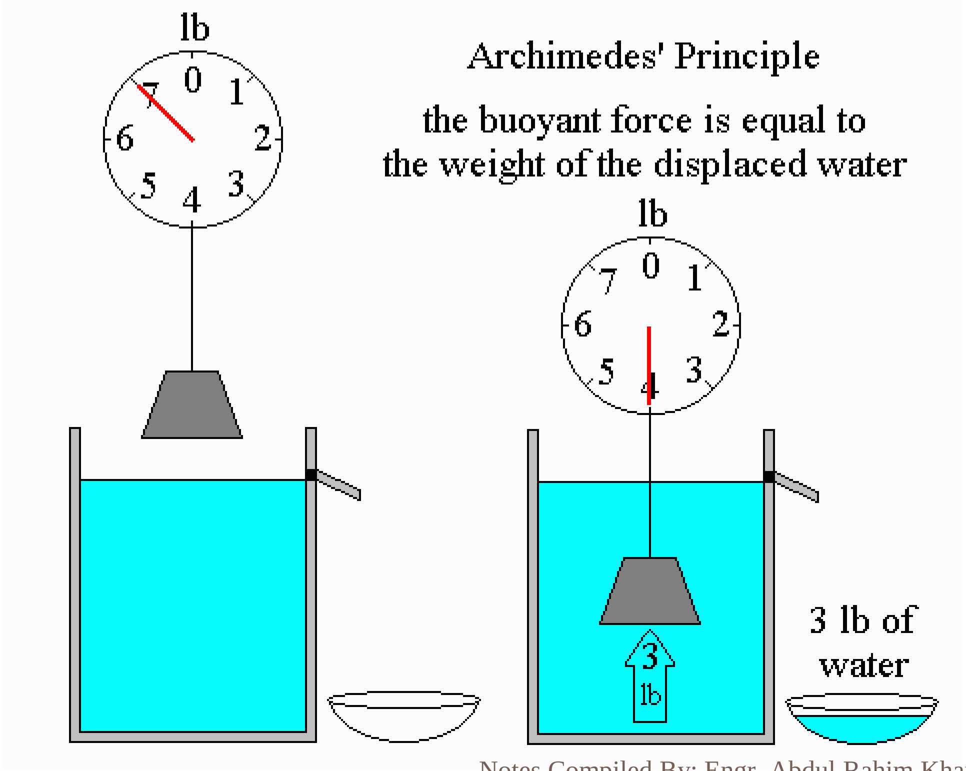


- Two forces involve are:
  1. Gravitational Force
  2. Up-thrust of the liquid

- If Gravitation force is more than Upthrust, body will sink.
- If Upthrust is more than Gravitation force, body will float.

Archimedes Principle:

- “Whenever a body is immersed wholly or partially in a fluid, it is buoyed up (i.e lifted up) by a force equal to the weight of the fluid displaced by the body.”



Buoyancy:

- A body in a fluid, whether floating or submerged, is buoyed up by a force equal to the weight of the fluid displaced.
- "The tendency of a fluid to uplift a submerged body, because of the up-thrust of the fluid, is known as force of buoyancy or simply buoyancy."
- The buoyant force acts vertically upward through the centroid of the displaced volume and can be defined mathematically by Archimedes' principle as follows:

$$F_d = \gamma_f V_d$$

F_d = Buoyant force

γ_f = Specific weight of fluid

V_d = Displaced volume of fluid

Center of Buoyancy:

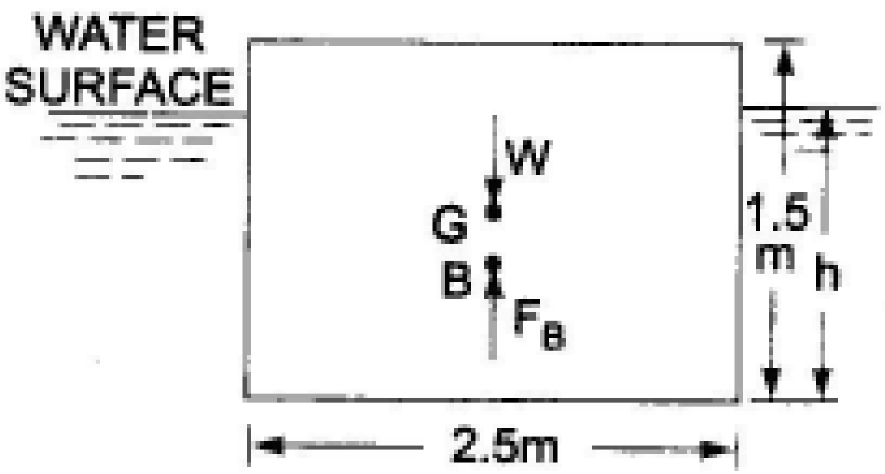
- It is defined as:

  "The point, through which the force of buoyancy is supposed to act."

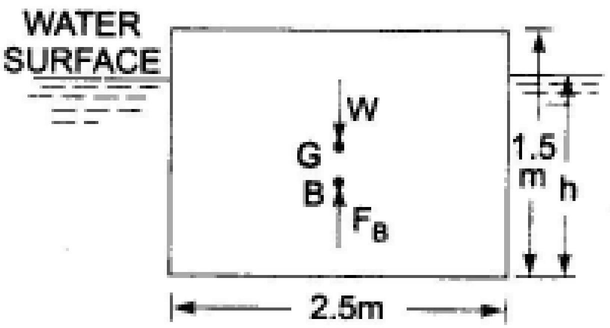
- As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.
- In other words, the centre of buoyancy is the centre of area of the immersed section.

Problem-1

Find the volume of the water displaced and position of centre of buoyancy for a wooden block of width 2.5 m and of depth 1.5 m, when it floats horizontally in water. The density of wooden block is 650 kg/m3 and its length 6.0 m.



Solution:



$\therefore$ Weight of block $= \rho \times g \times \text{Volume}$

$= 650 \times 9.81 \times 22.50 \text{ N} = 143471 \text{ N}$

For equilibrium the weight of water displaced = Weight of wooden block

$= 143471 \text{ N}$

$\therefore$ Volume of water displaced

$$= \frac{\text{Weight of water displaced}}{\text{Weight density of water}} = \frac{143471}{1000 \times 9.81} = \mathbf{14.625 \text{ m}^3}. \textbf{ Ans.}$$

($\because$ Weight density of water $= 1000 \times 9.81 \text{ N/m}^3$)

Position of Centre of Buoyancy. Volume of wooden block in water

= Volume of water displaced

$2.5 \times h \times 6.0 = 14.625 \text{ m}^3$, where h is depth of wooden block in water

$$\therefore \quad h = \frac{14.625}{2.5 \times 6.0} = 0.975 \text{ m}$$

$$\therefore \quad \text{Centre of Buoyancy} = \frac{0.975}{2} = \mathbf{0.4875 \text{ m from base.}} \textbf{ Ans.}$$

Problem-2

A wooden block of 4m x 1m x 0.5m in size and of specific gravity 0.75 is floating in water. Find the weight of concrete of specific weight 24k kN/m3 that may be placed on the block, which will immerse the wooden block completely.

Solution:

Let W be the weight of Concrete required to be placed on wooden block.

Volume of wooden block $= 4 \times 1 \times 0.5 = 2\text{m}^3$

and its Weight $= 9.81 \times 0.75 \times 2 = 14.72$ kN

$\therefore$ Total weight of the block and concrete $= 14.72 + W$ kN

We know that when the block is completely immersed in water,

volume of water displaced $= 2\text{m}^3$

$\therefore$ Upward thrust when the block is completely immersed in water

$$= 9.81 \times 2 = 19.62 \text{ kN}$$

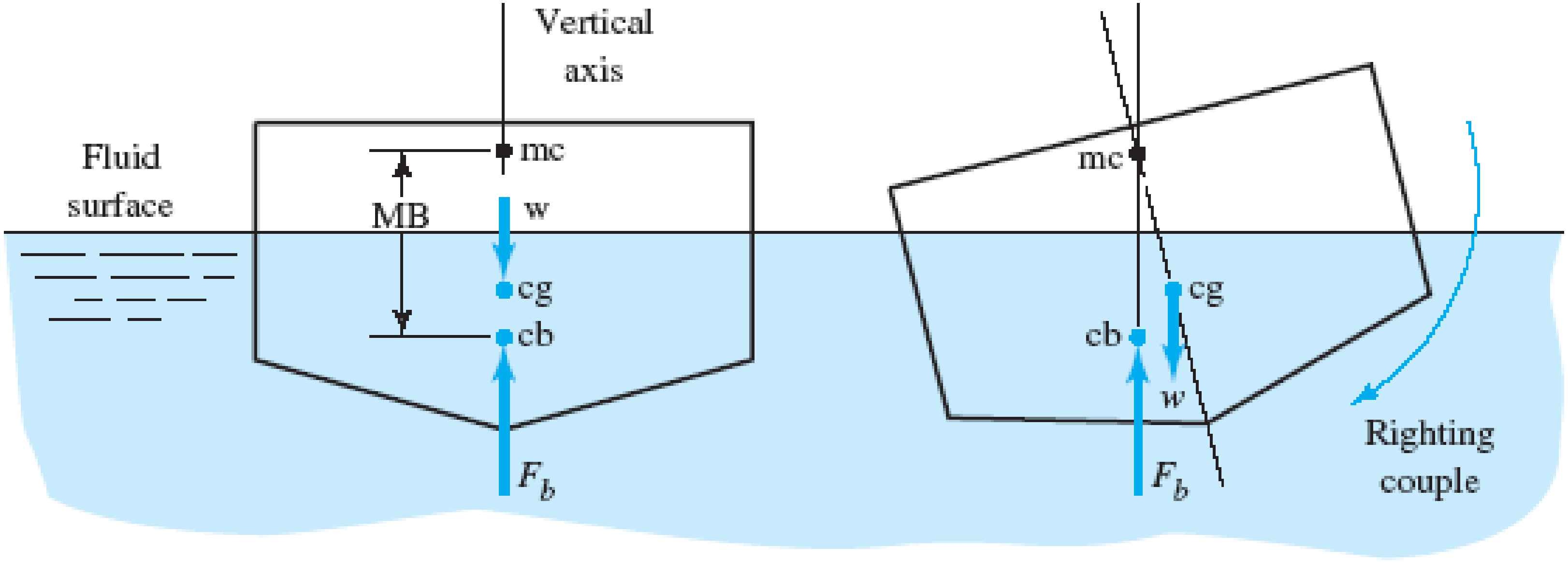
Now equating the total weight of block and concrete with upward thrust

$$14.72 + W = 19.62$$

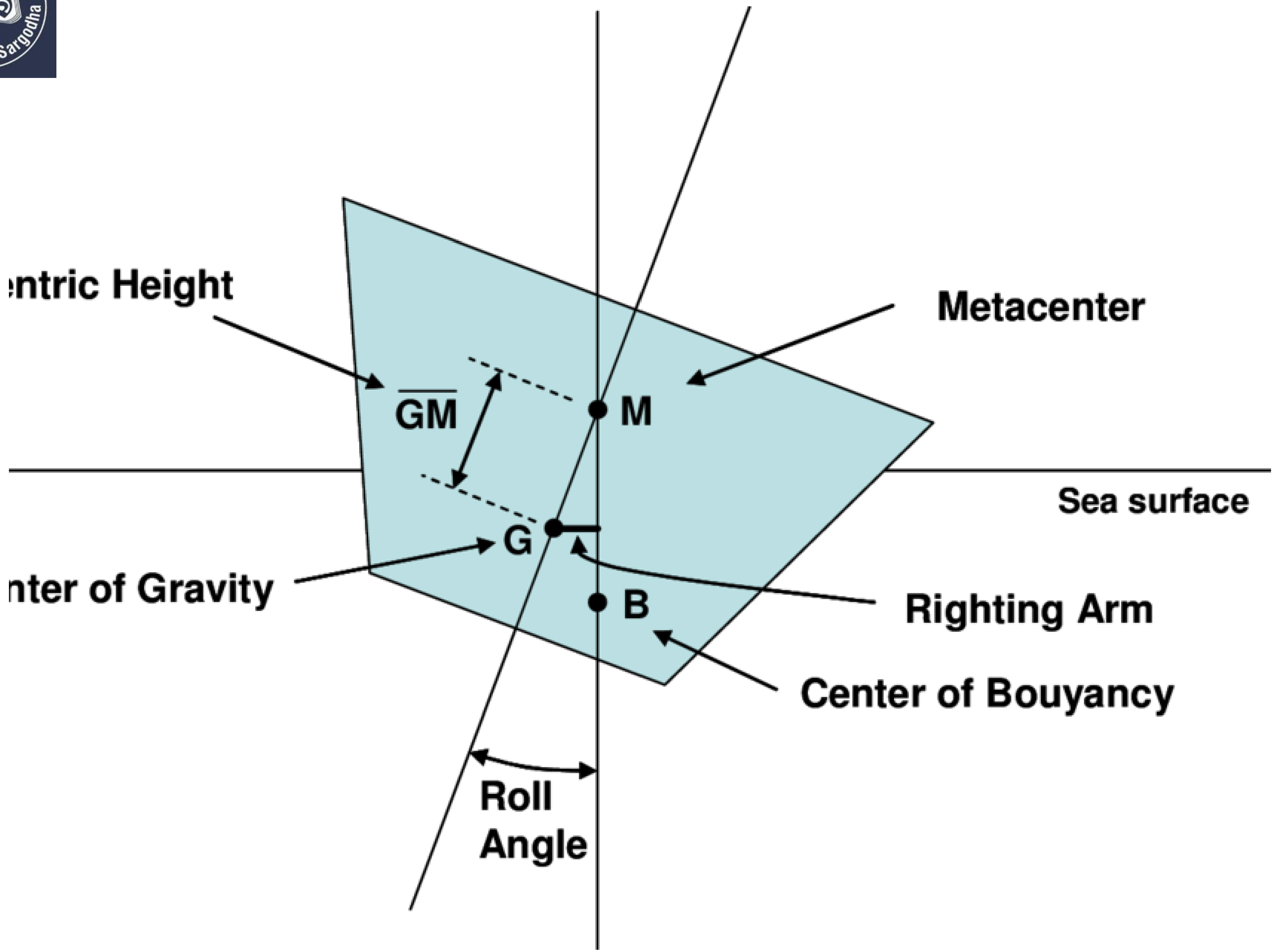
$$W = 4.9 \text{ kN}$$

Metacentre:

- “Whenever a body, floating in a liquid, is given a small angular displacement, it starts oscillating about some point. This point, about which the body starts oscillating, is called metacentre.”



(a) Original position (b) Tilted position

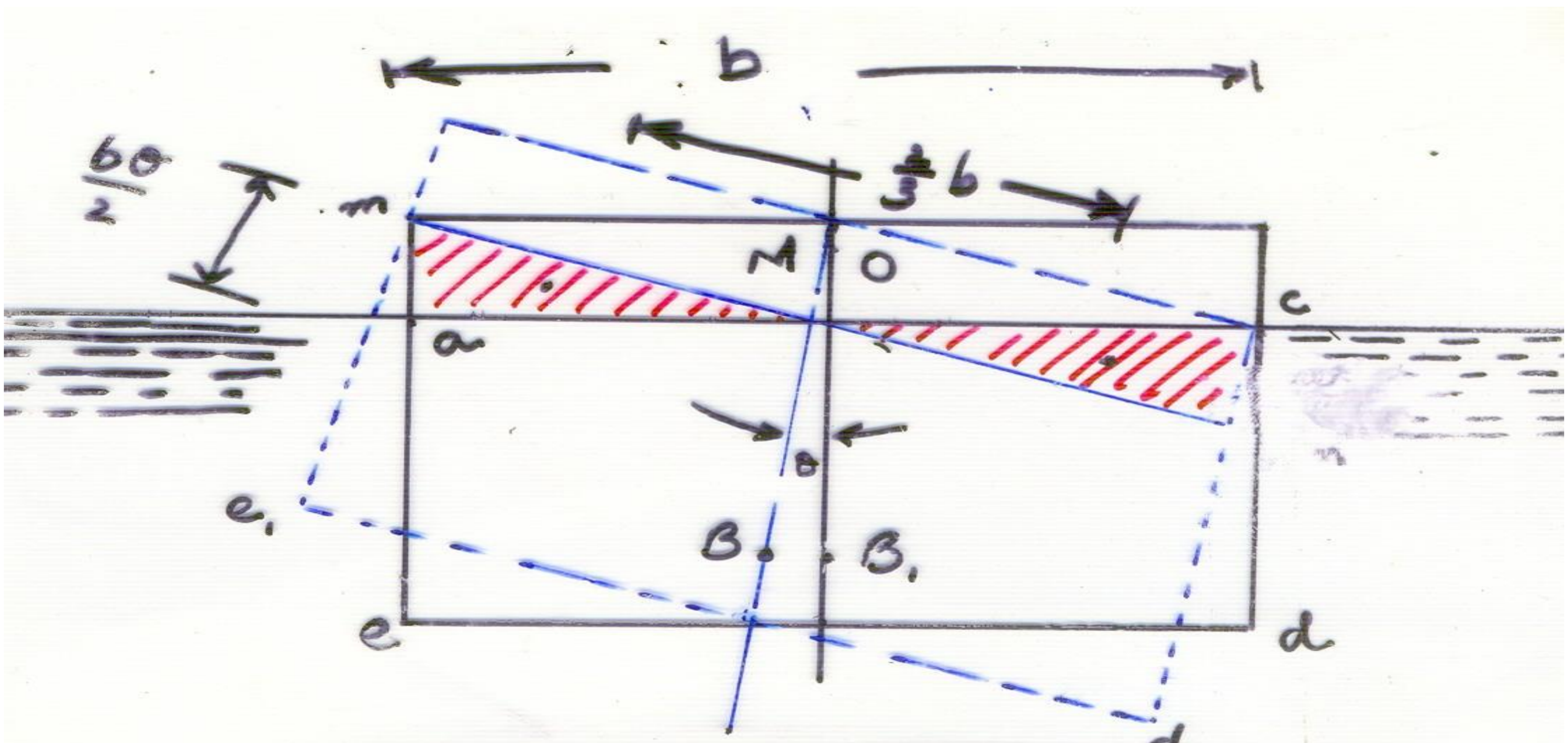

ntric Height
Metacenter
GM
M
Sea surface
G
nter of Gravity
Righting Arm
B
Center of Bouyancy
Roll
Angle

Metacentric Height:

- "The distance between centre of gravity of a floating body and the metacentre (i.e distance between cg and m in Fig.) is called metacentric height."
- Metacentric height of a floating body is a direct measure of its stability.
- *More the metacentric height of a floating body, more it will stable and vice versa.*
- Some values of metacentric height:
  - Merchant Ships = upto 1.0m
  - Sailing Ships = upto 1.5m
  - Battle Ships = upto 2.0m
  - River Craft = upto 3.5m

Analytical Method for Metacentric Height:

- Considering a ship floating freely in water. Let the ship be given a clockwise rotation through a small angle θ (in radians) as shown in Fig. The immersed section has now changed from *acde* to acd_1e_1.



Analytical Method for Metacentric Height:

- The original centre of buoyancy B has now changed to a new position B_1. It may be noted that the triangular wedge *ocn* has gone under water. Since the volume of water displaced remains the same, therefore the two triangular wedges must have equal areas.
- A little consideration will show, that as the triangular wedge *oam* has come out of water, thus decreasing the force of buoyancy on the left, therefore it tends to rotate the vessel in an anti-clockwise direction.
- Similarly, as the triangular wedge *ocn* has gone under water, thus increasing the force of buoyancy on the right, therefore it again tends to rotate the vessel in an anticlockwise direction.

Analytical Method for Metacentric Height:

- It is thus obvious, that these forces of buoyancy will form a couple, which will tend to rotate vessel in anticlockwise direction about O. If the angle (θ), through which the body is given rotation, is extremely small, then the ship may be assumed to rotate about M (i.e., metacentre).
- Let l =length of ship

  b=breadth of ship

  θ=Very small angle (in radian) through which the ship is rotated

  V=Volume of water displaced by the ship

Analytical Method for Metacentric Height:

From the geometry of the figure, we find that

$$am = cn = b\theta/2$$

--Volume of wedge of water *aom*

$$= \tfrac{1}{2}\,(b/2 \text{ x } am) \text{x} l$$

$$= \tfrac{1}{2}\,(b/2 \text{ x } b\theta/2) l \qquad (am = b\theta/2)$$

$$= b^2\theta l/8$$

--Weight of this wedge of water

$$= \gamma b^2\theta l/8 \qquad (\gamma = \text{sp. Wt. of water})$$

--And arm L.R. of the couple = 2/3 b

--Moment of the restoring couple

$$= (\gamma b^2\theta l/8) \text{ x } (2/3\ b) = \gamma b^3\theta l/12 \qquad \ldots(i)$$

Analytical Method for Metacentric Height:

--And moment of the disturbing force

$$= \gamma \,.\, V \text{ x } BB_1 \qquad \ldots\text{(ii)}$$

--Equating these two moments (i & ii),

$$\gamma b^3\theta l/12 = \gamma \text{ x } V \text{ x } BB_1$$

--Substituting values of:

$lb^3/12 = I$ (i.e. moment of inertia of the plan of the ship) and

$BB_1 = BM \text{ x } \theta$ in the above equation,

$$\gamma \,.\, I \,.\, \theta = \gamma \text{ x } V \,(BM \text{ x } \theta)$$

$$BM = I/V$$

BM= Moment of inertia of the plan/ Volume of water displaced

Analytical Method for Metacentric Height:

--Now metacentric height,

$$GM = BM \pm BG$$

Note: +ve sign is to be used if G is lower than B and,

–ve sign is to be used if G is higher than B.

Problems:

1. A rectangular block of 5m long, 3m wide and 1.2m deep is immersed 0.8m in the sea water. If the density of sea water is 10kN/m2, find the metacentric height of block.

...s: l = 5m, b = 3m, d = 1.2m

Depth of immersion = 0.8 m, w = 10 KN/m3

Center of buoyancy = $OB = \frac{h}{2} = \frac{0.8}{2} = 0.4\ m$

Center of gravity = $OG = \frac{d}{2} = \frac{1.2}{2} = 0.6m$

B $BG = OG - OB$

$= 0.6 - 0.4 = 0.2\ m$

$B.M = \frac{I}{V}$

$I = \frac{lb^3}{12} = \frac{5\,(3)^3}{12} = 11.25\ m^4$

$V = l \times b \times h = 5 \times 3 \times 0.8 = 12m^3$

$B.M = \frac{11.25}{12} \Rightarrow 0.94m$

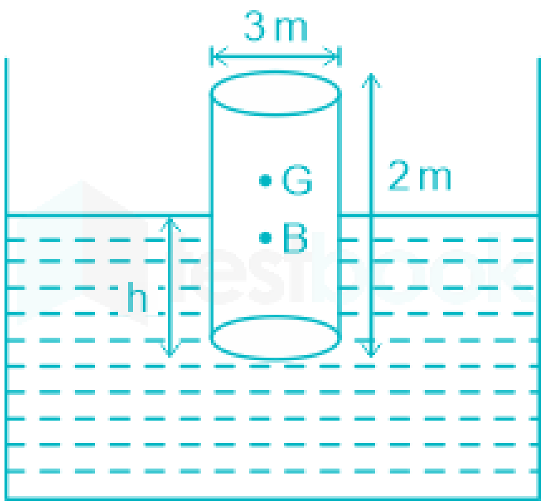
$GM = B.M - BG$

$= 0.94 - 0.2$

$\boxed{GM = 0.74m}$

Problems:

1. A solid cylinder of 3m diameter and 2m height is made up of a material of Sp. Gravity 0.7and floats in water. Find its metacentric height.



Find h using W = F_B (From the condition of floatation)

$\rho_c gV = \rho_F gV_d$

$$0.7 \times 10^3 \times 9.81 \times \frac{\pi}{4} \times d^2 \times 2 = 10^3 \times 9.81 \times \frac{\pi}{4} \times d^2 \times h$$

h = 1.4 m

Now, Metacentric height (GM) = BM - BG

BG = (2/2) - (1.4/2) = 0.3 m

$$BM = \frac{I}{V} = \frac{\pi d^4}{64} \times \frac{4}{\pi d^2 \times 1.4}$$

BM = 0.4017 m

GM = 0.4017 - 0.3 = 0.1017 m

Conditions of Equilibrium of a Floating Body:

- A body is said to be in equilibrium, when it remains in steady state, While floating in a liquid following are the three conditions of equilibrium of a floating body:

1. Stable Equilibrium
2. Unstable Equilibrium
3. Neutral Equilibrium

1. Stable Equilibrium:

- A body is said to be in a stable equilibrium, if it returns back to its original position, when given a small angular displacement.
- This happens when metacentre (M) is higher than centre of gravity (G) of the floating body.

2. Unstable Equilibrium:

- A body is said to be in a Unstable equilibrium, if it does not return back to its original position, when given a small angular displacement.
- This happens when metacentre (M) is lower than centre of gravity (G) of the floating body.

3. Neutral Equilibrium:

- A body is said to be in a neutral equilibrium, if it occupies a new position and remains at rest in this new position, when given a small angular displacement.
- This happens when metacentre (M) concides with centre of gravity (G) of the floating body.

Problems:

3. A rectangular timber block 2m long, 1.8m wide and 1.2m deep is immersed in water. If the specific gravity of the timber is 0.65, prove that it is in stable equilibrium.
4. A cylindrical buoy of 3m diameter and 4m long is weighing 150N. Show that it cannot float vertically in water.
5. A solid cylinder of 360mm long and 80mm diameter has its base 10mm thick of specific gravity 7. The remaining part of cylinder is of specific gravity 0.5. Determine, if the cylinder can float vertically in water.