Fluid Mechanics (CT-213)

- ***Course Instructor***

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Fluid Statics

Lecture -3

Force on a Plane Area

- No tangential force can exist within a fluid at rest. All forces are normal to the surface.
- If pressure is uniformly distributed over an area, the force is equal to pressure times the area and the point of application of force is at the centroid of the area.
- For submerged horizontal areas, the pressure is uniform.
- In case of gases, the pressure variation with vertical distance is very small due to low specific weight. Therefore when we compute the static fluid force exerted by a gas, we usually treat (p) as a constant. Thus for such cases:

$$F = \int p\,dA = p\int dA = pA \qquad (3.14)$$

Force on a Plane Area

- In the case of liquids the distribution of pressure is generally not uniform, so further analysis is necessary.
- Let us consider a vertical plane whose upper edge lies in the free surface of a liquid (Fig 3.15). Let this plane be perpendicular to the plane of the figure, so that *MN* is its edge.

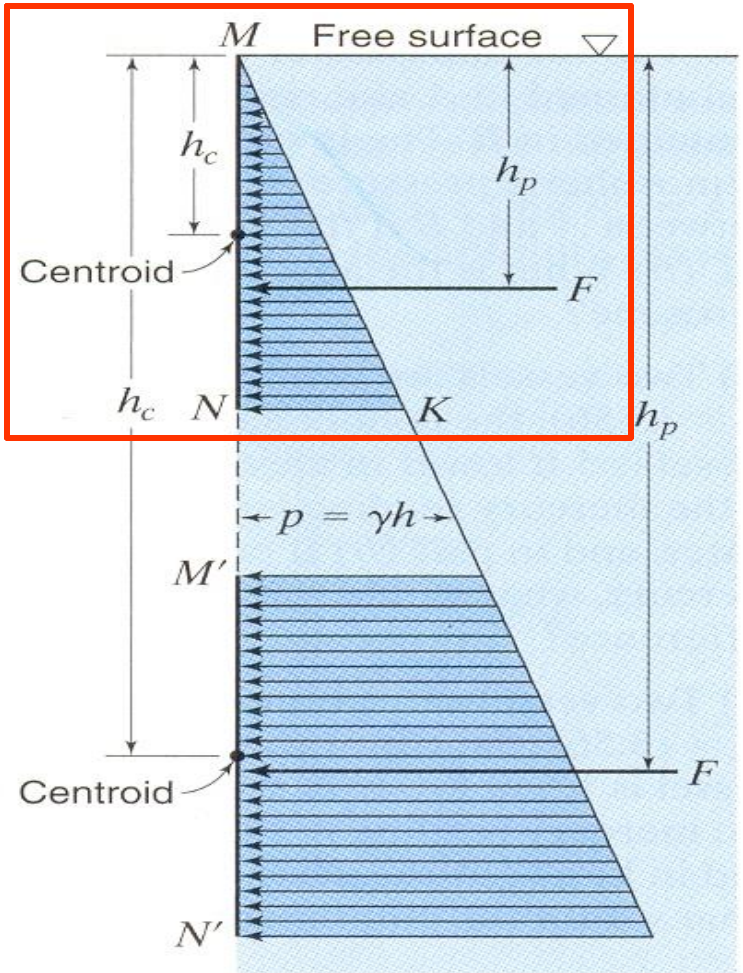


Fig-3.15

Force on a Plane Area

- The gage pressure will vary from zero at M to NK at N. The total force on one side of the plane is the sum of the products of the elementary areas and the pressure upon them.
- From the pressure distribution, we can see that the resultant of this system of parallel forces must act at a point below the centroid of the area, since the centroid of an area is the point where the resultant of a system of uniform parallel forces would act.

Force on a Plane Area

- If we lower the plane to position M'N', the change of pressure from M' to N' is less than it was from M to N. Hence the resultant pressure force will act nearer to the centroid of the plane surface.
- The deeper we submerge the plane, the smaller the pressure variation becomes, and the closer the resultant moves to the centroid.

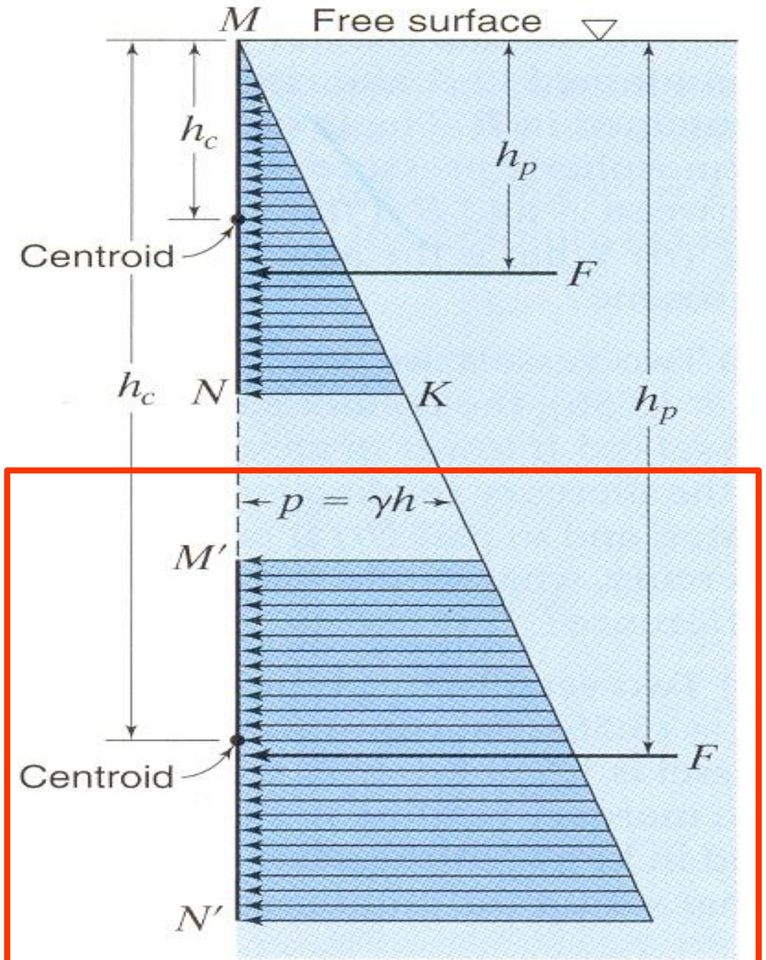


Fig-3.15

Force on a Plane Area (Sloping)

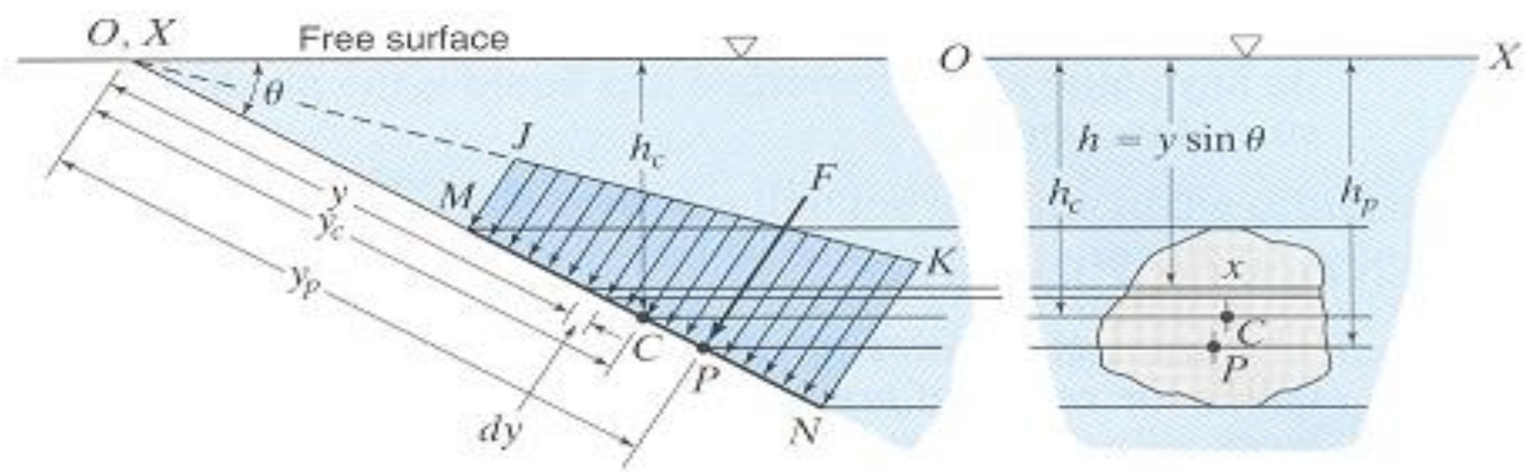


Fig-3.16

- In Fig. let *MN* be the edge of a plane area making an angle θ with the horizontal. To the right we see the projection of this area onto a vertical plane.

Force on a Plane Area (Sloping)

- The pressure distribution over the sloping area forms a ***pressure prism*** *(MNKJ* times width in Fig.), whose volume is equal to the total force *F* acting on the area.
- If the width (x) is constant then we can easily compute the volume of the pressure prism, using a mean pressure = 0.5 (MJ+NK), and so obtain F.
- If *x* varies, we must integrate to find *F*. Let *(h)* be the variable depth to any point and let *(y)* be the corresponding distance from *OX*, the intersection of the plane containing the area and the free surface.

Force on a Plane Area (Sloping)

- Choose an element of area so that the pressure over it is uniform. Such an element is a horizontal strip, of width *x*, so $dA = xdy$. As $p = \gamma h$ and $h = y \sin \theta$, the force dF on the horizontal strip is:

$$dF = pdA = \gamma h dA = \gamma y \sin \theta dA$$

$$\textit{Integrating} \quad \mathrm{F} = \int \mathrm{dF} = \gamma \sin \theta \int \mathrm{ydA} = \gamma \sin \theta \, \mathrm{y_c} A \qquad (3.15)$$

- Where yc is, by definition, the distance from OX along the sloping plane to the centroid C of the area A.

Force on a Plane Area (Sloping)

- If (hc) is the vertical depth to the centroid, then,

$$h_c = y_c \sin \theta$$

- And in general we have:

$$F = \gamma\, h_c A \quad (3.16)$$

- Thus we find the total force on any plane area submerged in a liquid by multiplying the specific weight of the liquid by the product of the area and the depth of its centroid. The value of F is independent of the angle of inclination of the plane so long as the depth of its centroid is unchanged.
- Since γhc is the pressure at the centroid, we can also say that the total force on any plane area submerged in a liquid is the product of the area and the pressure at its centroid.

Center of Pressure

- The point of application of the resultant pressure force on a submerged area is called the center of pressure. We need to know its location whenever we wish to work with the moment of this force.
- The most general way of looking at the problem of forces on a submerged plane area is through the use of the recently discussed pressure prism concept (Sec. 3.6 and Fig. 3.16). The line of action of the resultant pressure force must pass through the centroid of the pressure prism (volume). As noted earlier, this concept is very convenient to apply for simple areas such as rectangles. For example, if the submerged area in Fig. 3.15 is of constant width then we know that the centroid of the pressure prism on area MN is 2/3 MN below M.

Center of Pressure

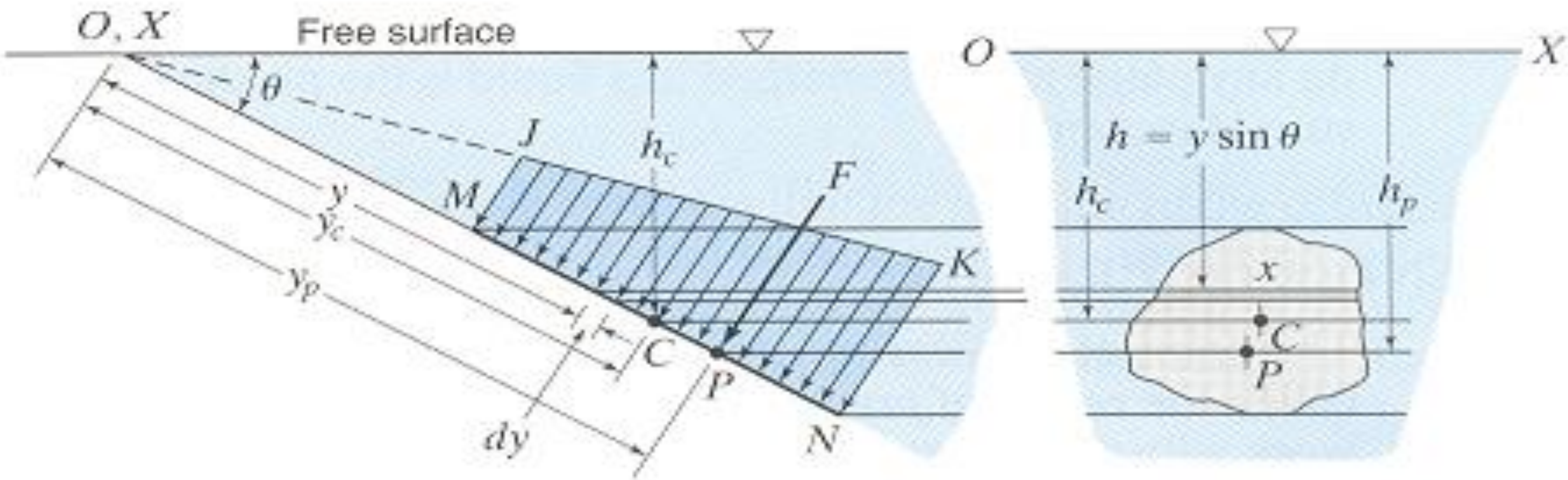


Fig-3.16

Center of Pressure

- If the shape of the area is not so regular, i.e., if the width x in Fig. 3.16 varies, then we must take moments and integrate. Taking OX in Fig. 3.16 as an axis of moments, the moment of an elementary force $dF = \gamma y \sin \theta \, dA$ is:

$$ydF = \gamma\, y^2 \sin\theta \, dA$$

- and if (y_p denotes the distance to the center of pressure, using the basic concept that the moment of the resultant force equals the sum of the moments of the component forces,

$$y_p F = \gamma \sin\theta \int y^2 dA = \gamma \sin\theta \, \mathrm{I_o}$$

- Where we recognize that I_o is the moment of inertia of the plane area about axis OX.

Center of Pressure

- If we divide this last expression by the value of *F* given by Eq. (3.15), we obtain

$$y_p = \frac{\gamma \sin \theta I_o}{\gamma \sin \theta y_c A} = \frac{I_o}{y_c A} \qquad (3.17)$$

- The product $y_c A$ is the static moment of area *A* about *OX*. Therefore Eq. (3.17) tells us that we can obtain the distance from the center of pressure to the axis where the plane (extended) intersects the liquid surface by dividing the moment of inertia of the area *A* about the surface axis by its static moment about the same axis.

Center of Pressure

- We may also express this in another form, by noting from the parallel axis theorem that:

$$I_o = A{y_c}^2 + I_c$$

- Where Ic is the moment of Inertia of an area about its centroidal axis. By substituting for Io into Eq. 3.17.

$$y_p = \frac{A{y_c}^2 + I_c}{y_c A}$$

So,

$$y_p = y_c + \frac{I_c}{y_c A} \qquad (3.18)$$

Center of Pressure

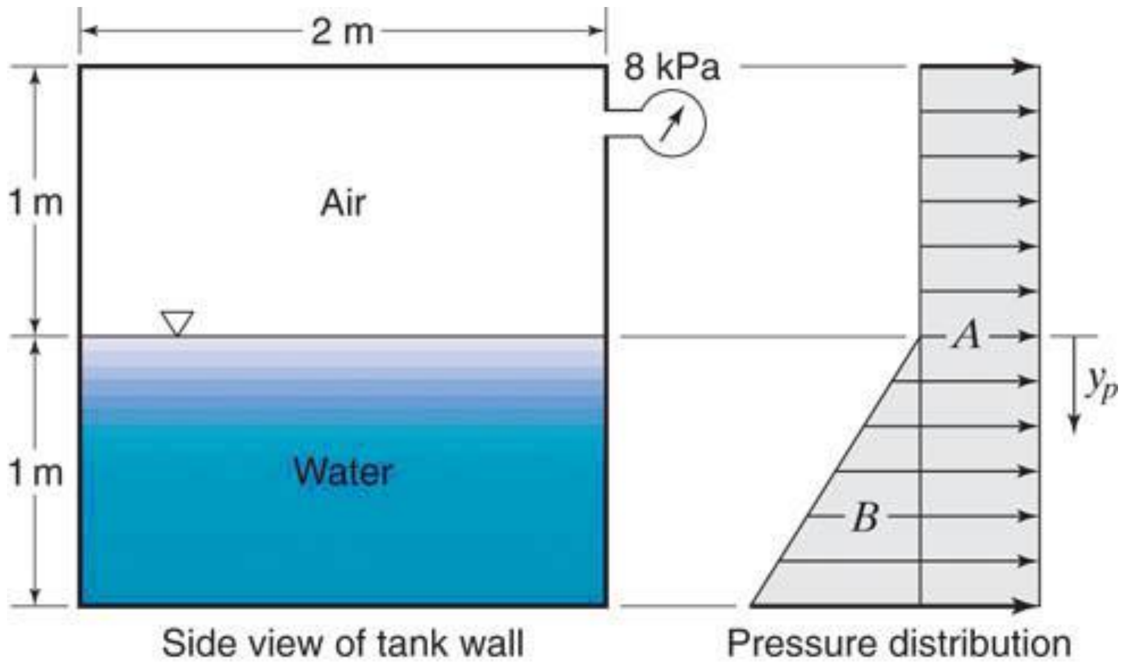
- From this equation we see that the location of center of pressure P is independent of the angel θ that is, we can rotate the plane area about axis OX without affecting the location of P. Also, we see that P is always below the centroid C and that as the depth of immersion is increased, yc increases and therefore P approaches C.
- We can find the lateral position of the centre of pressure P by considering that the area is made up of a series of elemental horizontal strips. The centre of pressure for each strip is at the midpoint of the strip. Since the moment of the resultant force F must be equal to the moment of the distributed force system about an axis, say, the y-axis.

$$X_P F = \int x_p \, p dA \qquad (3.19)$$

- Where Xp is the lateral distance from the selected y axis to the center of pressure P of the resultant force F, and xp is the lateral distance to the center of any elemental horizontal strip of area dA on which the pressure is p.

Problem

The cubic tank shown in Fig. is half full of water. Find (a) the pressure on the bottom of the tank, (b) the force exerted by the fluids on a tank wall, and (c) the location of the center of pressure on a wall.



Solution:

$$(a) \quad p_{bott} = p_{air} + \gamma_{water} h_{water} = 8 + 9.81x1$$

$$= 17.81\text{kN/m}^2 = 17.81kPa$$

(b) The force action on the tank end is divided into two components, labeled A and B on the pressure distribution sketch. Componet A has a uniform pressure distribution, due to the pressure of the confined air, which acts throughout the water:

$$F_A = p_{air} A_{air} = 8x4 = 32kN$$

For component B i.e., the varying water pressure distribution on the lower half of the tank wall, the centroid C of the area of application is at

$h_c = y_c = 0.5x1 = 0.5m$ below the water top surface,

$$F_B = \gamma_{water} h_c A_{water} = 9.81x0.5x2 = 9.81kN$$

Total Force $= F_A + F_B = 32 + 9.81 = 41.8kN$

The location of centers of pressure of the component forces, as distance y_p below the water top surface, are

$$(y_p)_A = 0m$$

below the water top surface, to the centroid of the 2m square area for the uniform air pressure.

$$(y_p)_B = \frac{2}{3} h_{water} = \frac{2}{3}(1) = 0.667m$$

below the water top surface for the varying pressure on the rectangular wetted wall area.

We can also find by using this equation :

$$y_p = y_c + \frac{I_c}{y_c A}$$

$$y_c = 0.5m, \qquad I_c = \frac{bh^3}{12} = 2x1^3/12 = 0.1667m^4 \quad A = bh = 2m^2$$

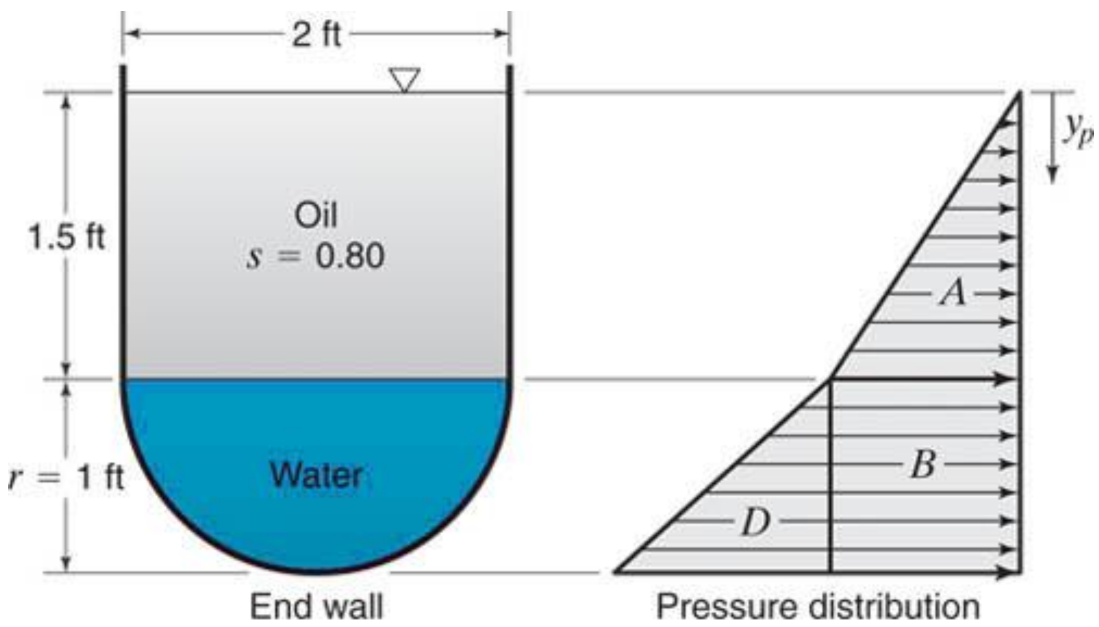
$$y_p = 0.667m$$

Taking moments : $\quad F(y_p) = F_A(y_p)_A + F_B(y_p)_B$

$y_p = 0.1565m$ below the water top surface.

Problem

Water and oil in an open storage tank are in contact with the end wall as shown in Fig. S3.7. (a) Find the pressure at the bottom (lowest point) of the tank caused by the liquids. Also find (b) the total force exerted on the end wall by the liquids.



Solution:

(a) $P_{bott} = \gamma_{oil}\, h_{oil} + \gamma_{water}\, h_{water}$

$= (0.8x62.4 Ib/ft^3)\ (1.5ft)\ + (62.4 Ib/ft^3)\ (1.0\ ft)$

$= 137.3\ lb/ft^3\ =$ **0.953psi**

(b) The force acting on the end consists of three components, labeled *A*, *B* and *D*, on the pressure distribution sketch. Note that component *B* has a *uniform* pressure distribution, due to the oil *(A)* above, which acts throughout the liquid below.

As a preliminary, we note for the semicircular end area ($r = 1$ ft) that

(i) $A = \pi r^2/2 = \pi 1^2/2 = 1.571\ ft^2$;

(ii) from Appendix A, Table A.7, the centroid is $4r/3\pi = 0.424$ ft from the center of the circle, i.e., below the water top surface.

Solution:

For component *A*, i.e., the varying oil pressure distribution on the l.5-ft height of the end wall, the centroid C of the area of application is at:

$h_c = y_c$ = 0.5(1.5 ft) = 0.75 ft below the free oil surface,

so, from Eq. (3.16),

$F_A = \gamma_{oil}\, h_c\;\; A_{oil} = (0.8 \times 62.4)0.75(1.5 \times 2) = 112.3\text{Ib}$

For component *B*, the force F_B on the water-wetted area of the end wall due to the uniform pressure produced by the 1.5-ft depth of oil above is

$F_B = pA = \gamma\, h\, A = (0.8 \times 62.4\,)1.5(\pi 1^2/2) = 117.6$ lb

For component *D*, i.e., the varying pressure distribution due to the water (only) on the water-wetted area of the end wall, the centroid C is at

$h_c = y_c$ = 0.424 ft below the water top surface,

so: $F_D = \gamma\, h_c\;\; A = 62.4(0.424)\, \pi 1^2/2\; = 41.61\text{b}$

Solution:

The total force F on the end of the tank is therefore

$$\boldsymbol{F = F_A + F_B + F_D =} \textbf{272 lb}$$