Mathematical Elements of CAD

Two Dimensional Transformations

- Transformation means changing some graphics into something else by applying rules. We can have various types of transformations such as:
  - ✓ Translation
  - ✓ Scaling
  - ✓ Rotation
  - ✓ Concatenation
- When a transformation takes place on a 2D plane, it is called 2D (Two-dimensional) transformation.
- Transformations play an important role in computer graphics to reposition the graphics on the screen and change their size or orientation.

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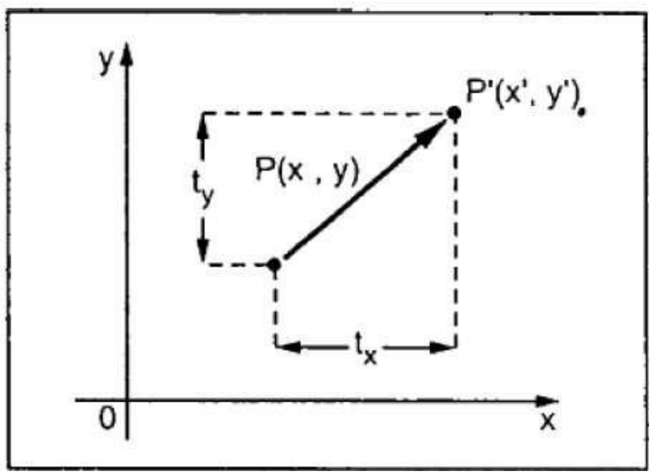
Two Dimensional Transformations

Translation

A translation moves an object to a different position on the screen. You can translate a point in 2D by adding translation coordinate (tx, ty) to the original coordinate (x, y) to get the new coordinate (x', y').



From the above figure, you can write that:

$$x' = x + tx$$

$$y' = y + ty$$

The above equations can also be represented using matrices

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

We can write it as:

$$P' = P + T$$

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Two Dimensional Transformations

Scaling

To change the size of an object, scaling transformation is used. In the scaling process, you either expand or compress the dimensions of the object. Scaling can be achieved by multiplying the original coordinates of the object with the scaling factor to get the desired result.

If we provide values less than 1 to the scaling factor S, then we can reduce the size of the object. If we provide values greater than 1, then we can increase the size of the object.

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Two Dimensional Transformations

Scaling

Let us assume that the original coordinates are (x, y), the scaling factors are (Sx, Sy), and the produced coordinates are (x', y'). This can be mathematically represented as shown below −

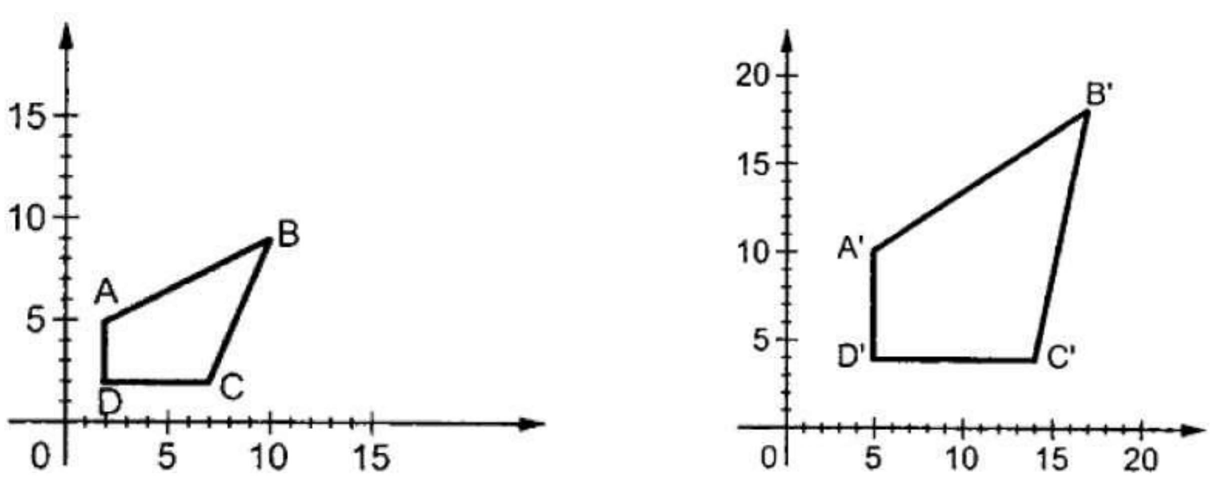
$$x' = x \cdot s_x \text{ and } y' = y \cdot s_y$$

The scaling factor s_x, s_y scales the object in x and y direction respectively. The above equations can also be represented in matrix form as below:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = P \cdot S$$

Where S is the scaling matrix. The scaling process is shown in the following figure.



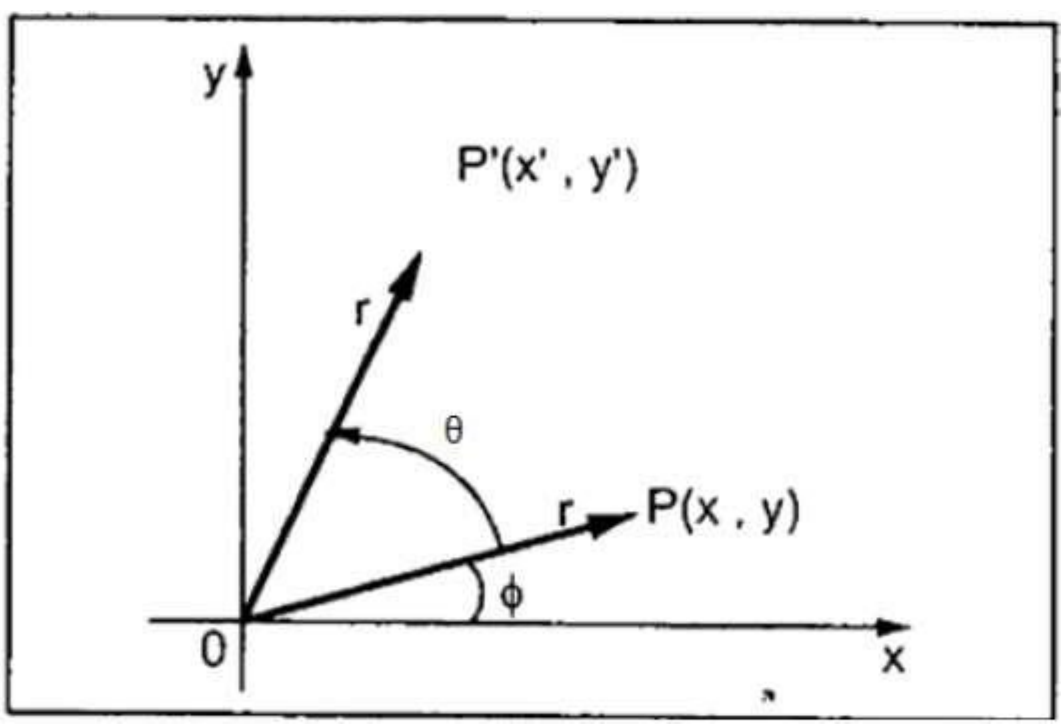
Mathematical Elements of CAD

Two Dimensional Transformations

Rotation

In rotation, we rotate the object at particular angle θ (theta) from its origin. From the following figure, we can see that the point P(x, y) is located at angle φ from the horizontal x coordinate with distance r from the origin.

Let us suppose you want to rotate it at the angle θ. After rotating it to a new location, you will get a new point P’ (x’, y’).



Mathematical Elements of CAD

Two Dimensional Transformations - Rotation

Using standard trigonometric the original coordinate of point P(x, y) can be represented as:

$$x=r\cos\phi......(1)$$

$$y=r\sin\phi......(2)$$

Same way we can represent the point P’ (x’, y’) as:

$$x'=r\cos(\phi+\theta)=r\cos\phi\cos\theta-r\sin\phi\sin\theta.......(3)$$

$$y'=r\sin(\phi+\theta)=r\cos\phi\sin\theta+r\sin\phi\cos\theta.......(4)$$

Substituting equation (1) & (2) in (3) & (4) respectively, we will get

$$x'=x\cos\theta-y\sin\theta$$

$$y'=x\sin\theta+y\cos\theta$$

Representing the above equation in matrix form,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = P \,.\, R$$

Where R is the rotation matrix

$$R=[\cos\theta-\sin\theta\sin\theta\cos\theta]$$

The rotation angle can be positive and negative.

For positive rotation angle, we can use the above rotation matrix. However, for negative angle rotation, the matrix will accordingly.

Mathematical Elements of CAD

Two Dimensional Transformations - Concatenation

Concatenation means "a series of interconnected things or the action of linking things together in a series"

When more than one transformation is linked in series, it is called Concatenation of Transformations

Let say if we wanted to scale as well as rotate the object, we could do the following as studied before (first scale then rotate):

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x' \\ y' \end{bmatrix}$$

But this is the same as:

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \cdot \begin{bmatrix} x' \\ y' \end{bmatrix} \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} s_x\cos\theta & -s_x\sin\theta \\ s_y\sin\theta & s_y\cos\theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$\uparrow$

Resultant Matrix

Hence, we can concatenate the Scaling and Rotation matrices, and then multiply the old point (x,y) by Resultant Matrix.

Mathematical Elements of CAD

Various Techniques of Design Optimization

Engineering is a profession whereby principles of nature are applied to build useful objects. A mechanical engineer designs a new engine, or a car suspension or a robot. A civil engineer designs a bridge or a building. A chemical engineer designs a distillation tower or a chemical process. An electrical engineer designs a computer or an integrated circuit.

For many reasons, not the least of which is the competitive marketplace, an engineer might not only be interested in a design which works at some sort of nominal level, but is the best design in some way. **The process of determining the best design is called optimization.** Thus we may wish to design the smallest heat exchanger that accomplishes the desired heat transfer, or we may wish to design the lowest-cost bridge for the site, or we may wish to maximize the load a robot can lift. So there can be various objectives of optimization like maximizing performance and efficiency, reducing cost etc.

Mathematical Elements of CAD

<u>**Various Techniques of Design Optimization**</u>

Often engineering optimization is done implicitly. Using a combination of judgment, experience, modeling, opinions of others, etc. the engineer makes design decisions which, he or she hopes, lead to an optimal design. Some engineers are very good at this. However, if there are many variables to be adjusted with several conflicting objectives and/or constraints, this type of experience-based optimization can fall short of identifying the optimum design.

The interactions are too complex and the variables too numerous to intuitively determine the optimum design.

In computer-based approach to design optimization, we use the computer to search for the best design according to criteria that we specify. The computer's enormous processing power allows us to evaluate many more design combinations than we could do manually. Further, we employ sophisticated techniques that enable the computer to efficiently search for the optimum. Often we start from the best design we have based on experience and intuition. We can then see if any improvement can be made.

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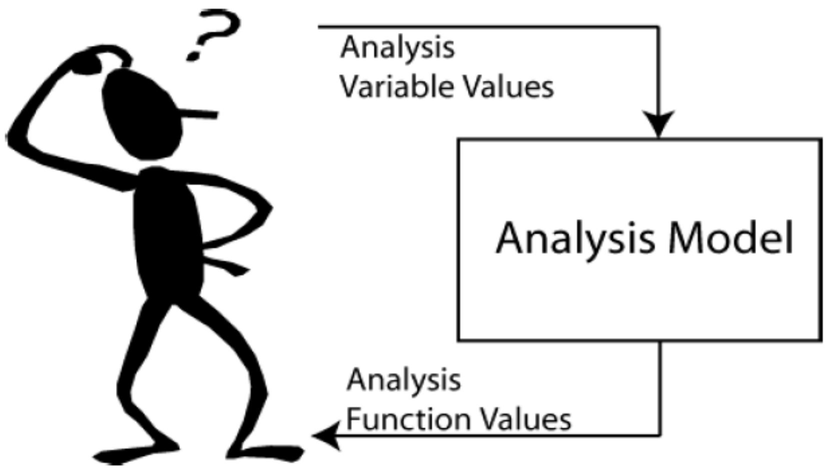
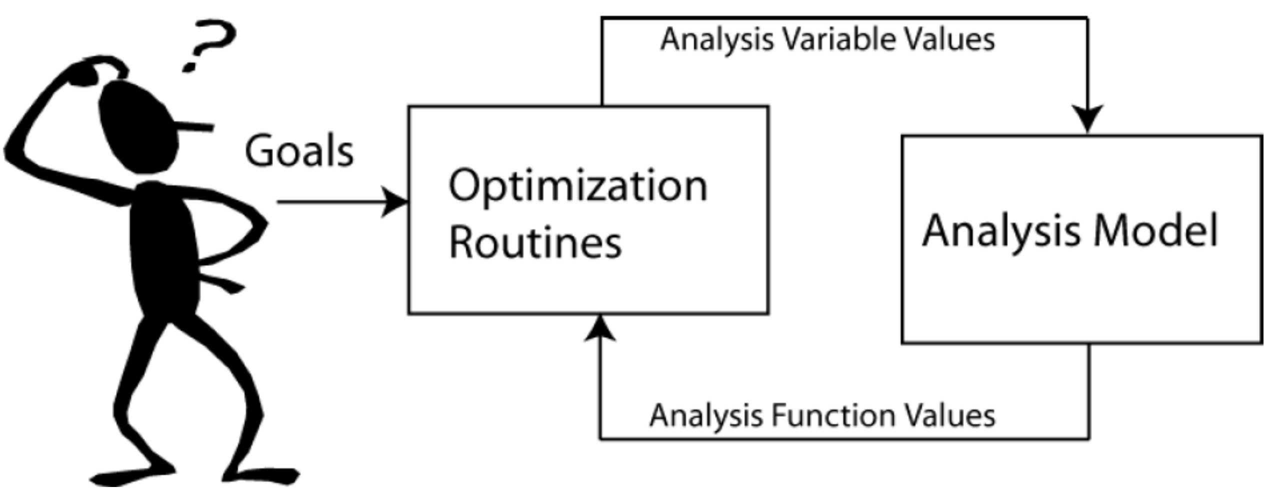
<u>Various Techniques of Design Optimization</u>

Optimization by Trial-and-Error



Common "trial-and-error" iterative design process

Optimization with Computer Algorithms



Moving the designer out of the trial-and-error loop with computer-based optimization software

Mathematical Elements of CAD

Various Techniques of Design Optimization

In computerized design optimization, we define objective of optimization, design variables and constraints. Then algorithms search the direction within the space (space is defined by variables & constraints) which improves the objective and satisfies the constraints

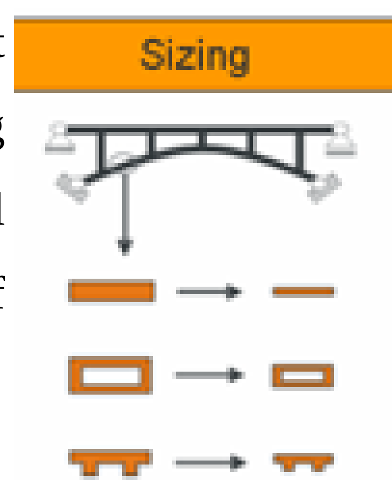
*Concept of **design space** is that for example, if we have four design variables, then we have a four dimensional design space that we can search to find the best design. Computers can search spaces with thousands of variables (dimensions).*

Mathematical Elements of CAD

Various Techniques of Design Optimization

Design optimization is more than reducing weight. It’s about maximizing performance and efficiency of material application while minimizing life-cycle cost. Finite Element technology provides us with tools for structural design optimization throughout several design phases. Techniques that attempt to improve or find the best structures can be classified into three categories:

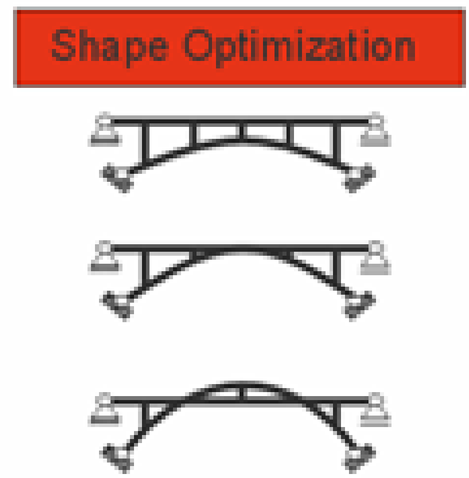
Sizing Optimization: The first category in structural design optimization techniques is sizing optimization. Without changing the general shape of the geometry an optimum relation between weight, stiffness and the dynamic behavior is found by optimizing sheet thicknesses. In early design phases a free sizing approach helps to get best indications for sheet partitioning and in later design phases the sizing optimization can lead to optimal thicknesses for all individual sheets in the structure. An example of sizing optimization:



Mathematical Elements of CAD

Various Techniques of Design Optimization

Shape Optimization: The second category in structural design optimization techniques is shape optimization. This method of optimization determines the optimal shape of the structure with the given sheet thicknesses by slightly changing the surface geometry to minimize stress peaks which in turn positively influences fatigue life. An example of shape optimization:



Topology Optimization: Topology optimization is a mathematical method that optimizes material layout within a given design space, for a given set of loads, boundary conditions and constraints with the goal of maximizing the performance of the system. Unlike sizing- and shape optimization, structures optimized through topology optimizations can attain any shape within the design space.