Shear Strength of Soil

SHEAR STRESS

- When a body is subjected to two equal and opposite forces acting tangentially across the resulting section as a result of which body tends to shear off the section, then the stresses induced is known as Shear stresses.
- And the resulting corresponding strain is known as Shear Strain.

SHEAR STRENGTH

- The Shear Strength of the soil is regarded as the resistance to deformation by continuous shear displacement of soil particles along surface of rupture.
- It can also be defined as the ability of soil to with stand Shear Stresses.

The shear strength of the soil is basically made up of,

- Frictional resistance to sliding between solid particles.

- Cohesion and Adhesion between soil particles.

- Interlocking of solid particles to resist deformation.

- Cohesion (C),(Stress Independent Component) is a measure of the forces that cement particles of soils.

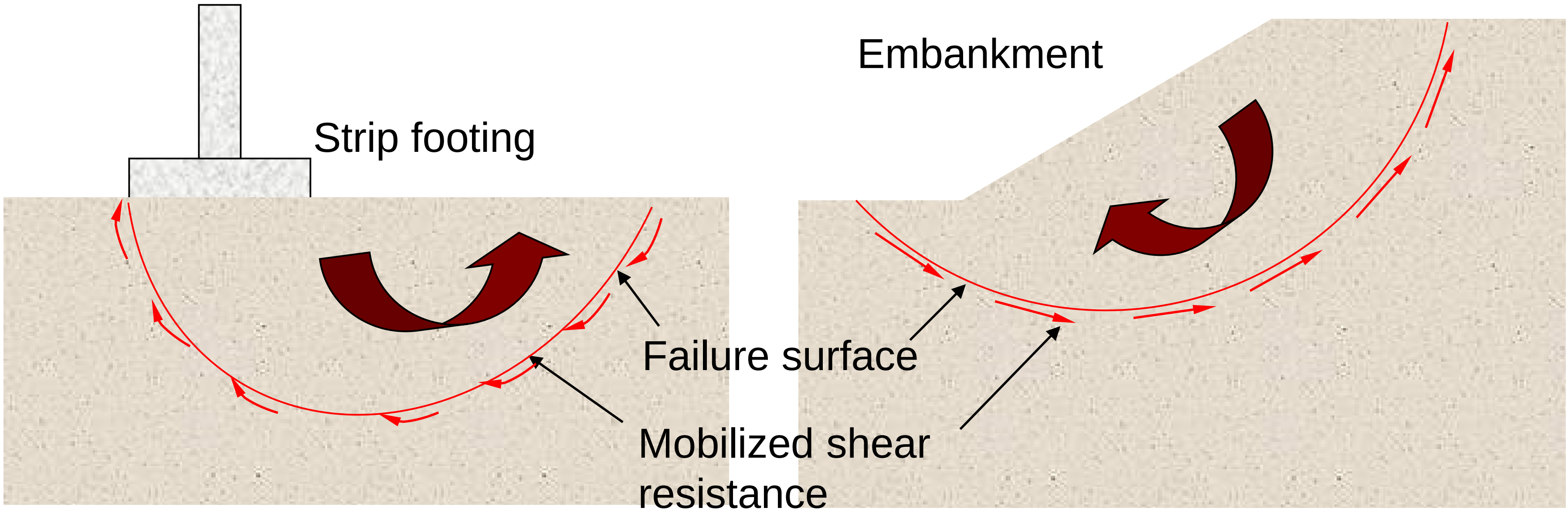
- Internal Friction angle (φ) (Angle of internal friction (friction angle) A measure of the ability of a unit of rock or soil to withstand a shear stress. It is the angle (φ), measured between the normal force (N) and resultant force (R), that is attained when failure just occurs in response to a shearing stress (S).), is the measure of the shear strength of soils due to friction.

SIGNIFICANCE OF SHEAR STRENGTH,

- Bearing capacity.
- Slope stability.
- Lateral earth pressure on earth-retaining structures.
- Friction developed by piles.

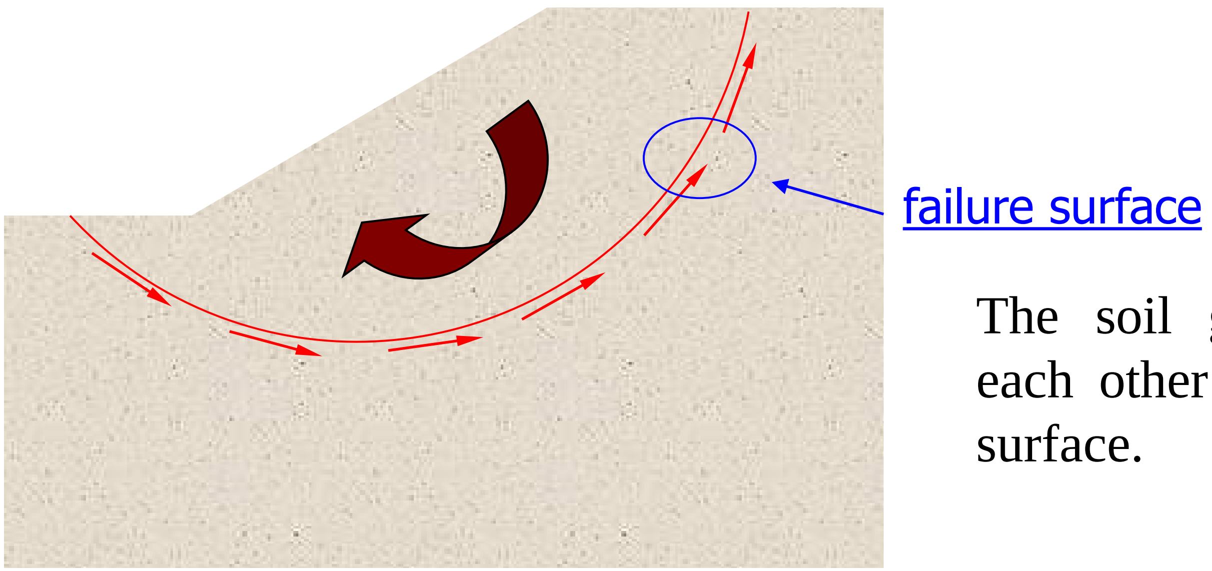
Shear failure of soils

Soils generally fail in <u>shear</u>



At failure, shear stress along the failure surface (mobilized shear resistance) reaches the shear strength.

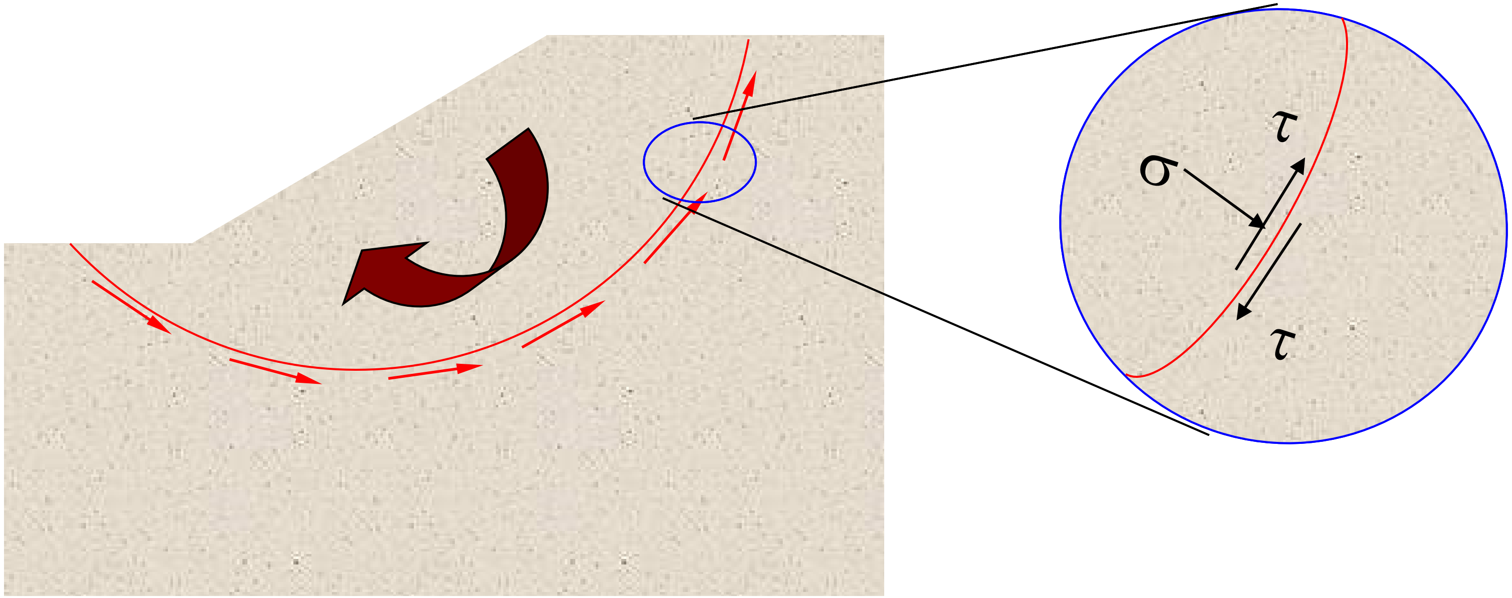
Shear failure mechanism



The soil grains slide over each other along the failure surface.

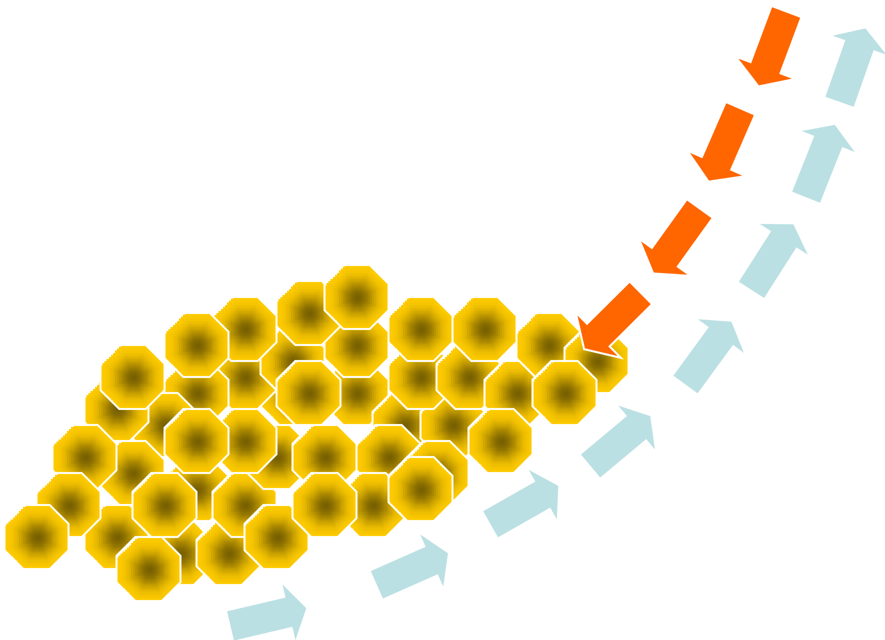
No crushing of individual grains.

Shear failure mechanism



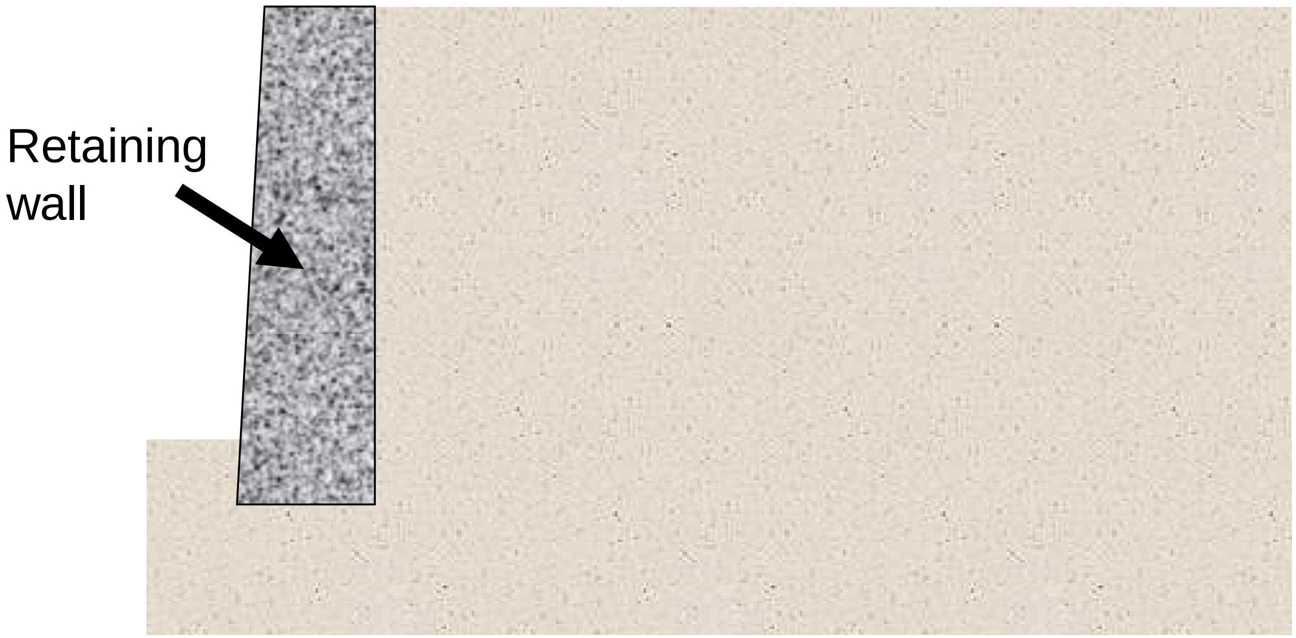
At failure, shear stress along the failure surface reaches the shear strength.

Shear Failure



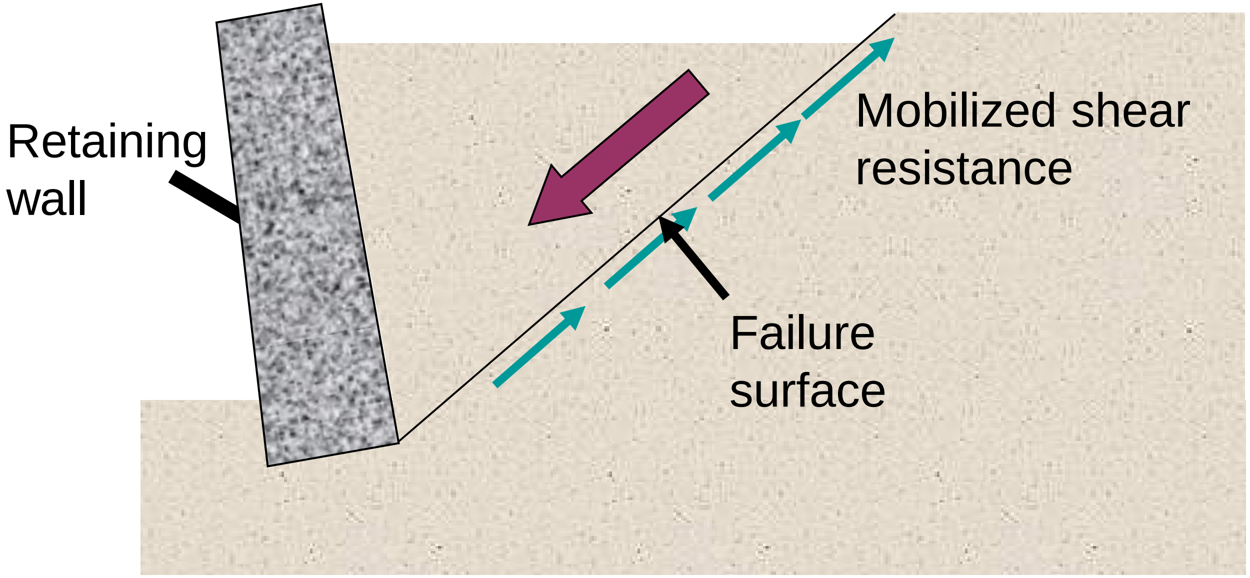
Shear failure of soils

Soils generally fail in shear



Shear failure of soils

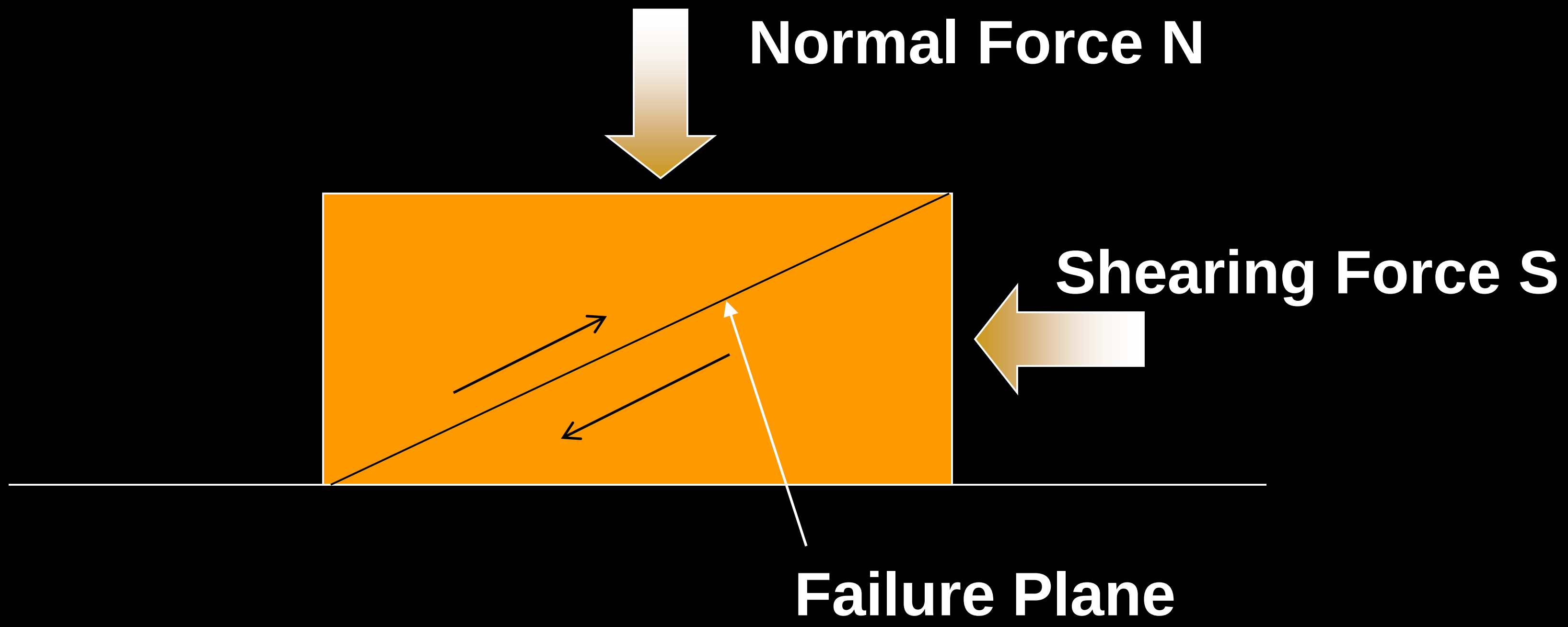
Soils generally fail in shear



At failure, shear stress along the failure surface (mobilized shear resistance) reaches the shear strength.

BASIC CONCEPT

- Consider a metal block B resting on a plan surface MN as shown in the figure.
- Block is acted upon by a force P_n normal to the plan surface and P_a , is tangential to plane surface.
- The force P_a is resisted by an equal and opposite force P_r .
- The magnitude of force P_n is kept constant, where as P_a is gradually increased from zero to higher value
- ø = angle of friction provided block and plane surface are of the same material.
- tan ø = Coefficient of friction.
- A = Contact area of Block with surface MN.



Normal Force N
Shearing Force S
Failure Plane

Neutral Stress/ Pore Water Pressure

- Now ,
- $Pa = P_n \tan ø$……(1)
- Where,
- P_a = Applied Force = Shear Force.
- P_n = Normal force.
- Dividing both sides of eq.1 by "A" we get,

- $Pa/A = P_n / A \tan ø$
- $T_f = ð_n \tan ø$

(3)

(4)

- Where,
- T_f = shearing stress at failure (Sliding).
- $ð_n$ = Normal Stress.

- If S = Maximum Shearing Stress at failure.

= Maximum resistance which a material is capable of developing under applied system of stresses.

- Than,
- S = $ð_n$ tan ø.
- With increase of normal stress $ð_n$ shearing stresses increases.

APPLICATION OF ABOVE INFORMATION OF SOIL

- **For purely granular soils,**
- $S = ð_n \tan ø$.
- **For soils other than purely granular soils.**
- $S = ð_n \tan ø + C$……(2)
- Where,
- S = Shearing strength
- $ð_n$ = Normal Stress.
- C = Stress developed due to cohesive forces between the soil grains.
- Cohesion is the internal property of the material.
- Cohesion is the shear strength of the material when applied normal stress is zero

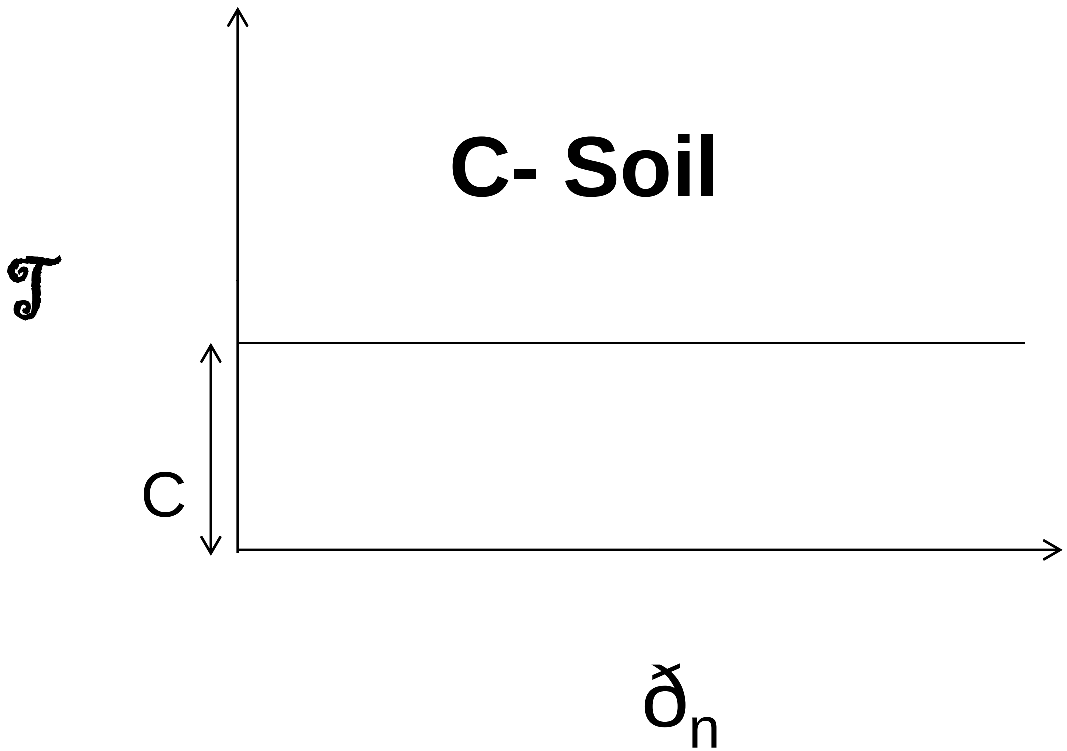
i.e

- $S = C$
- Equation 2 is called Coulomb's law.

Based on coulombs law we have three types of soils,

- C – Soil.
- C – ø Soil.
- ø Soil

- **C – Soil**
- These are purely cohesive soils or purely clays.
- In saturated conditions these may have some value of ø otherwise ø = 0
- i.e S = C
- Therefore C- Soil is represented by a straight line (In ðn VS T graph) having cohesion intercept with y- axis.

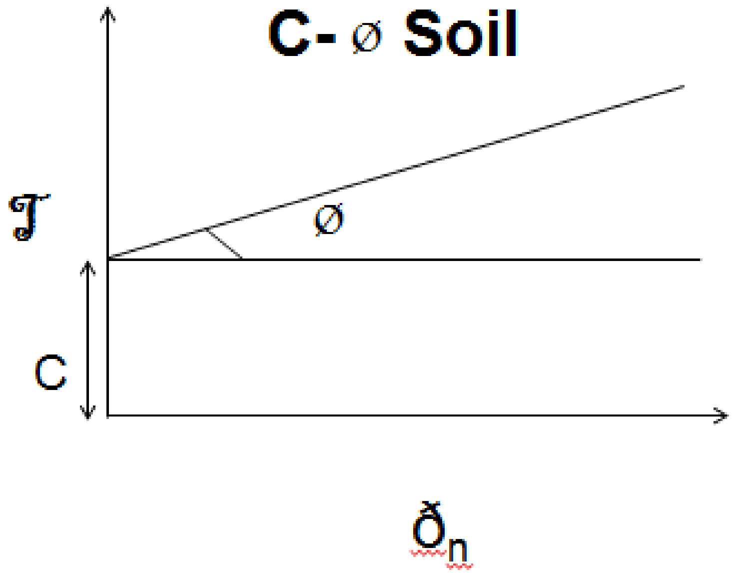


Fig#1

C – ø Soil

- These are silty soils and represent a line having cohesion intercept “C”, along with an angle of internal friction ø.

- The soil has properties intermediate between C and ø soils.

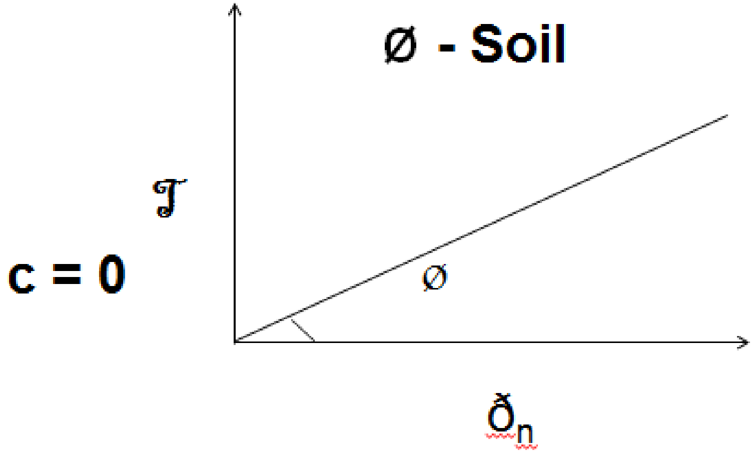


ø Soil

- These are sandy soils having cohesion intercept equal to zero.
- C = 0
- $S = ð_n \tan ø$.

- For dense sands,
- ø = 35 to 46 degree.
- For loose sands,
-
- ø = 28 to 34 degree.

- For well graded gravels,
-
- ø = up to 50 degree.



ANGLE OF REPOSE

- This applies to loose sands.
- If the dust free loose sand is poured on a horizontal surface from a small elevation, it will form a heap.
- It has been found by experiments that
- $\theta = \Phi$ for loose sands Φ can be found directly by heaping it.
- Whereas Φ is the angle of internal friction.

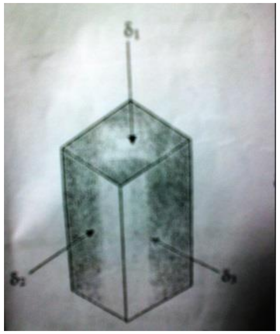
PRINCIPAL PLANE

- A plane that is acted upon by a normal stress only is known as a Principal plane.

- There is no tangential or shear stresses present.

PRINCIPAL STRESS

- The normal stress acting on a principal plane is referred to as a principal stress.
- At every point in a soil mass, the applied stress system that exists can be resolved into three stresses that are mutually orthogonal.
- The principal planes corresponding to these principal stresses are called the major, intermediate and minor principal planes.



STRESS AT A POINT (Unconfined Condition)

$$\left(+\uparrow\right) \qquad \sum F_v = 0 = -\sigma_1 \cdot A + \sigma_n \cdot \frac{A}{\cos\theta} \cdot \cos\theta + \tau_n \cdot \frac{A}{\cos\theta} \cdot \sin\theta \ldots\ldots\ldots(1)$$

$$\left(\xrightarrow{+}\right) \qquad \sum F_H = 0 = -\sigma_n \cdot \frac{A}{\cos\theta} \cdot \sin\theta + \tau_n \cdot \frac{A}{\cos\theta} \cdot \cos\theta \qquad \ldots\ldots\ldots(2)$$

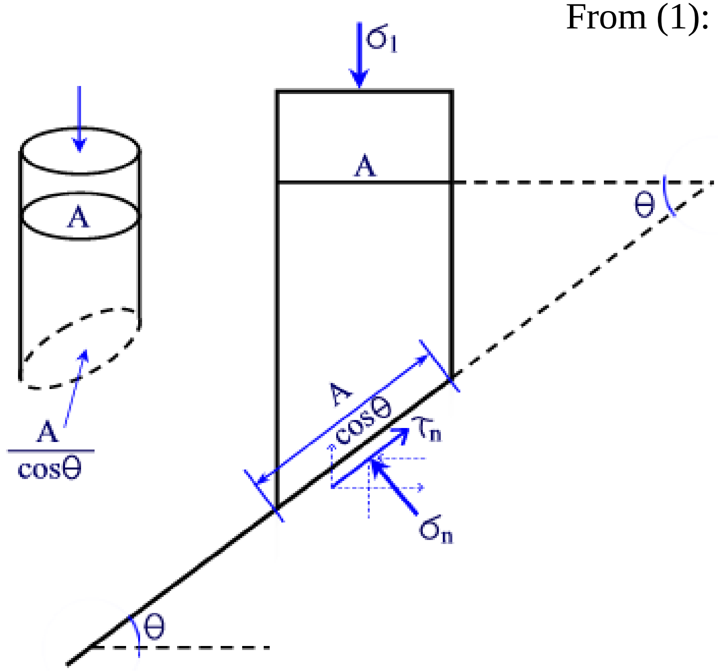
From (2): $\sigma_n A \tan\theta = \tau_n A$

From (1): $\sigma_1 \cdot A = \sigma_n A + \sigma_n A \tan^2\theta$

$$\sigma_1 = \sigma_n \left(1 + \tan^2\theta\right)$$

$$\sigma_1 = \sigma_n \left(\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}\right)$$

$$\sigma_1 \cos^2\theta = \sigma_n$$



$$\boxed{\sigma_n = \frac{\sigma_1}{2}\left(1+\cos 2\theta\right)}$$

$$\tau_n = \sigma_1 \cos^2\theta \tan\theta$$

$$\tau_n = \sigma_1 \sin\theta \cos\theta$$

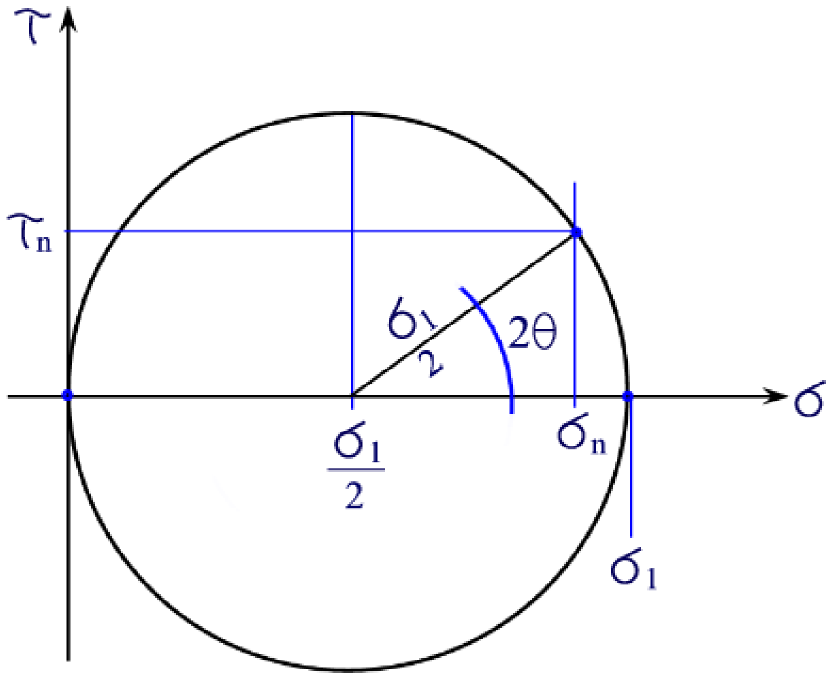
$$\boxed{\tau_n = \frac{\sigma_1}{2} \cdot \sin 2\theta}$$

STRESS AT A POINT (Unconfined Condition)

From the geometry of the circle,

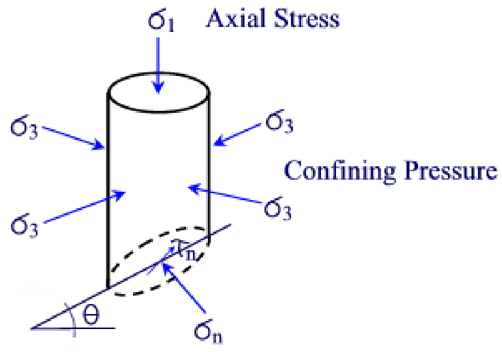
$$\sigma_n = \frac{\sigma_1}{2} + \frac{\sigma_1}{2} \cdot \cos 2\theta$$

$$\tau_n = \frac{\sigma_1}{2} \cdot \sin 2\theta$$



STRESS AT A POINT (Confined Condition)

- When the soil sample has confining pressure, σ_3 , as well as axial pressure, σ_1 ; the normal and shear stresses are:



$$\sigma_n = \frac{(\sigma_1 + \sigma_3)}{2} + \frac{(\sigma_1 - \sigma_3)}{2} \cos 2\theta$$

$$\tau_n = \frac{(\sigma_1 - \sigma_3)}{2} \sin 2\theta$$

Differential Stress:

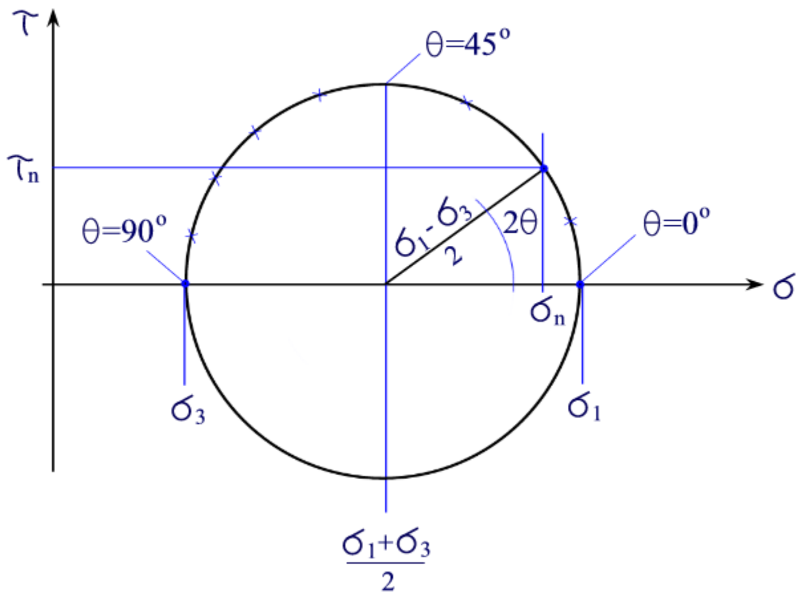
$$\sigma_d = \sigma_1 - \sigma_3$$

Deviatoric Stress:

$$\sigma_{dev} = \frac{\sigma_1 - \sigma_3}{2}$$

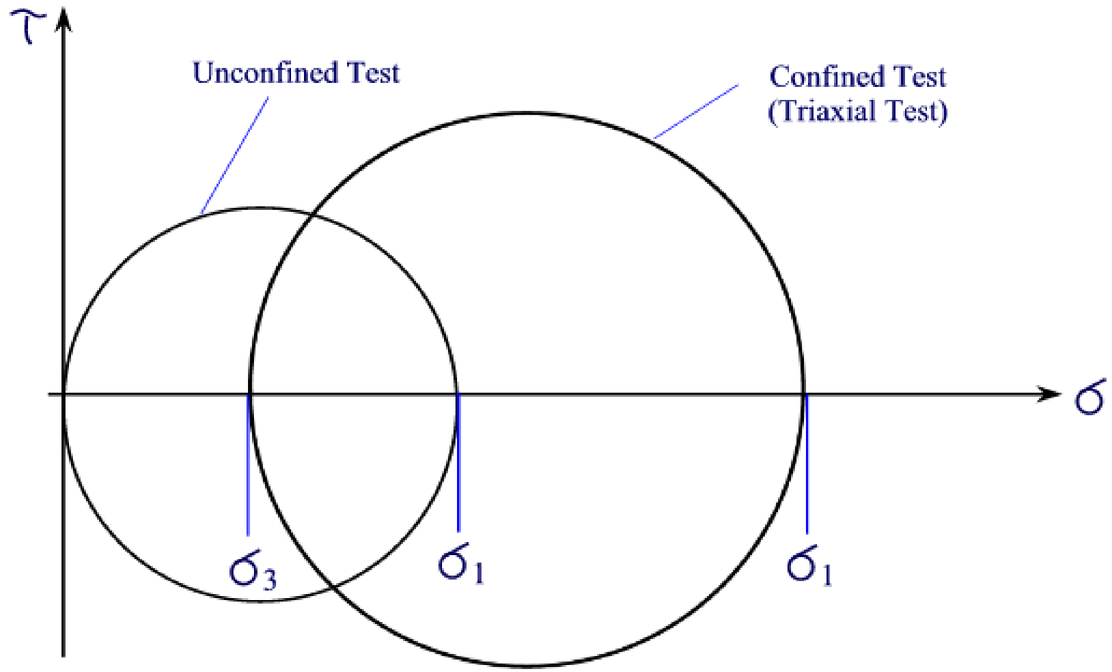
Mean Stress:

$$\sigma_m = \frac{\sigma_1 + \sigma_3}{2}$$



STRESS AT A POINT

- Shear strength comes from two sources
- 1. Cohesion- Cohesive shear strength, c
- 2. Friction - Angle of internal friction, φ
- or Angle of shearing resistance



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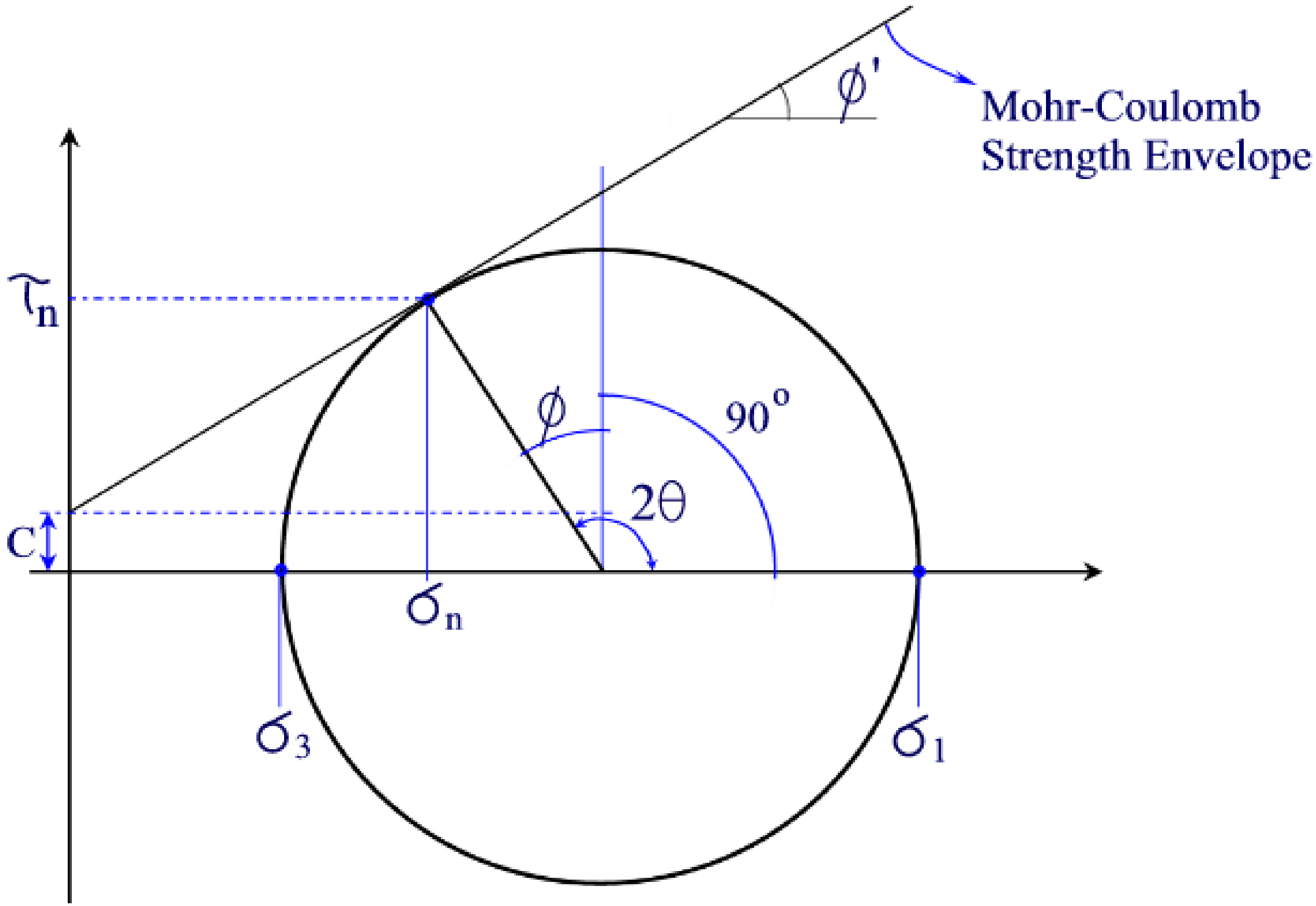
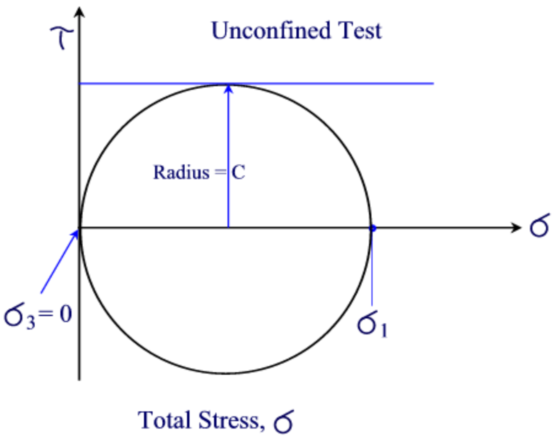
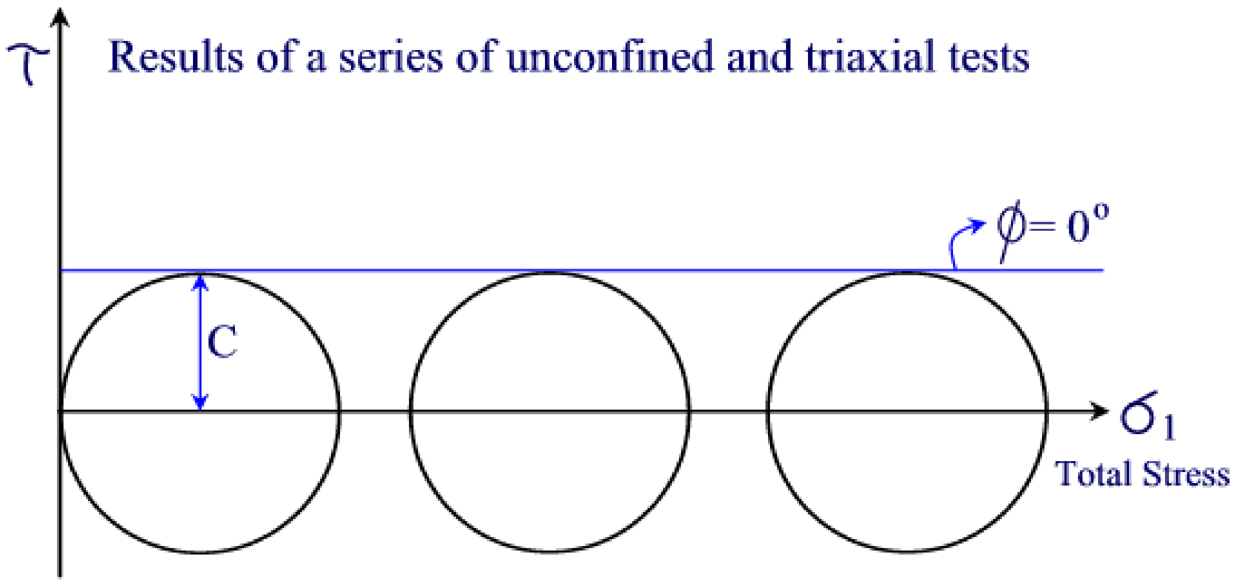


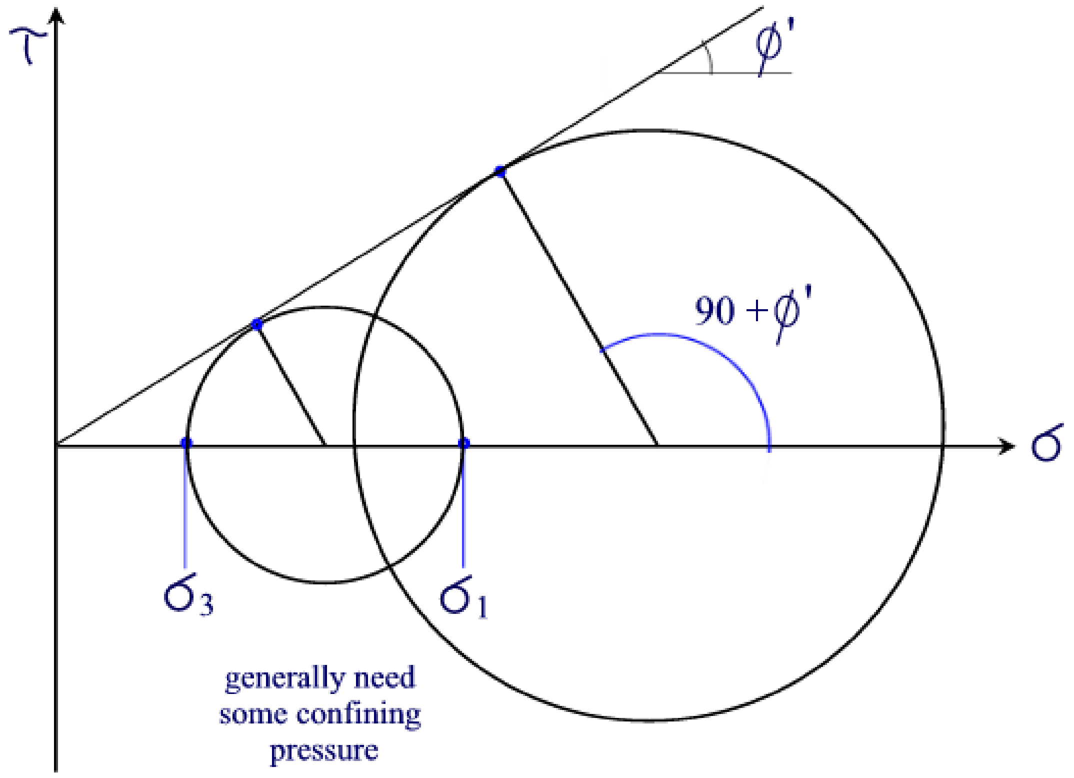
$$2\theta_f = 90 + \phi'$$

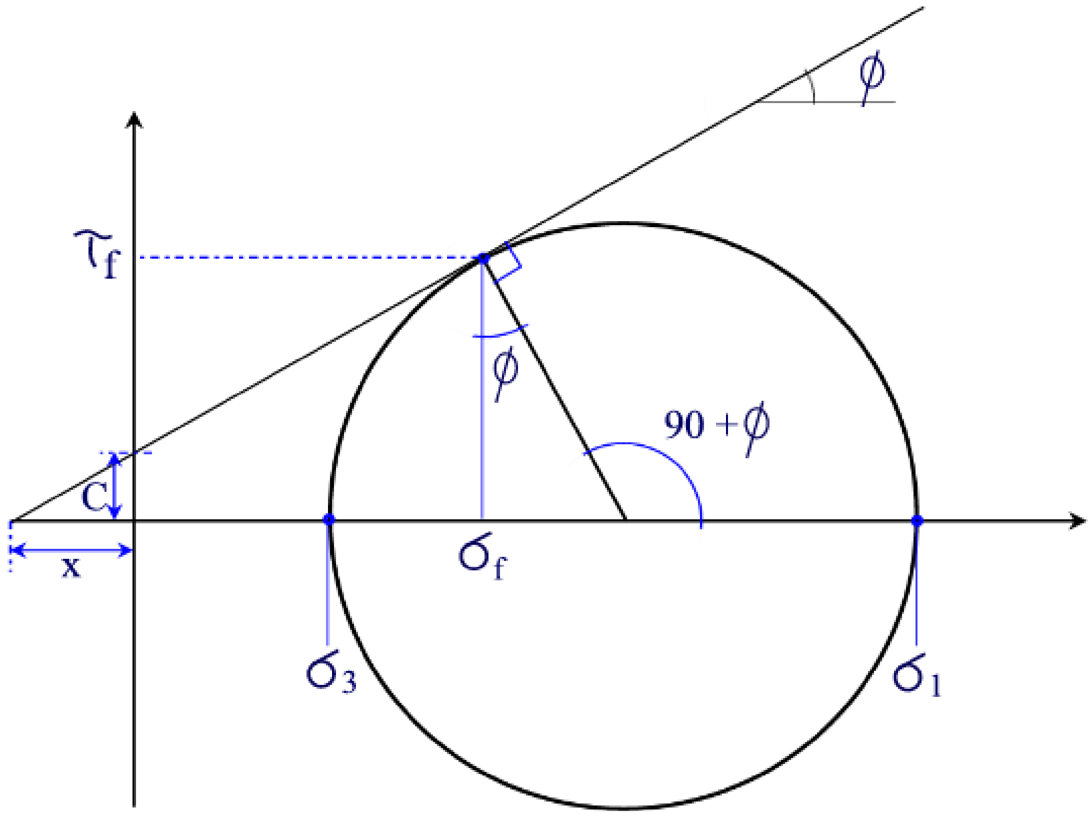
Strength of Clay





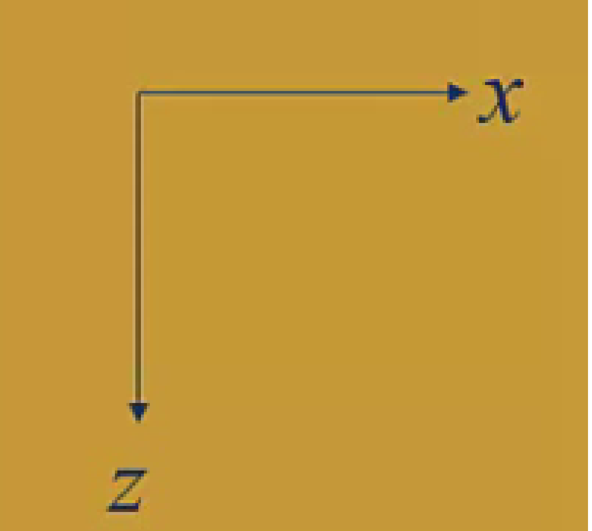
Strength of Sand

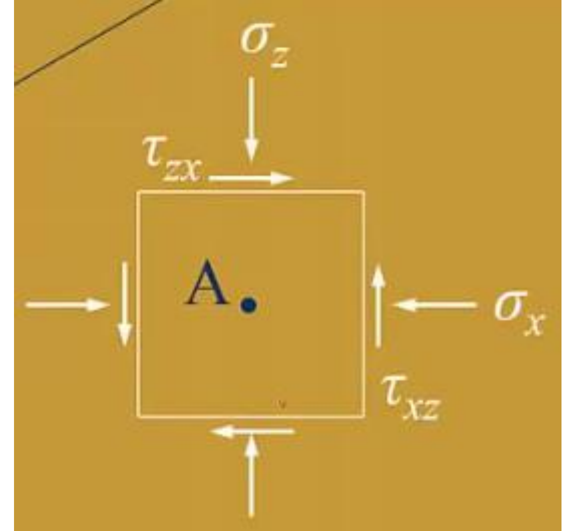


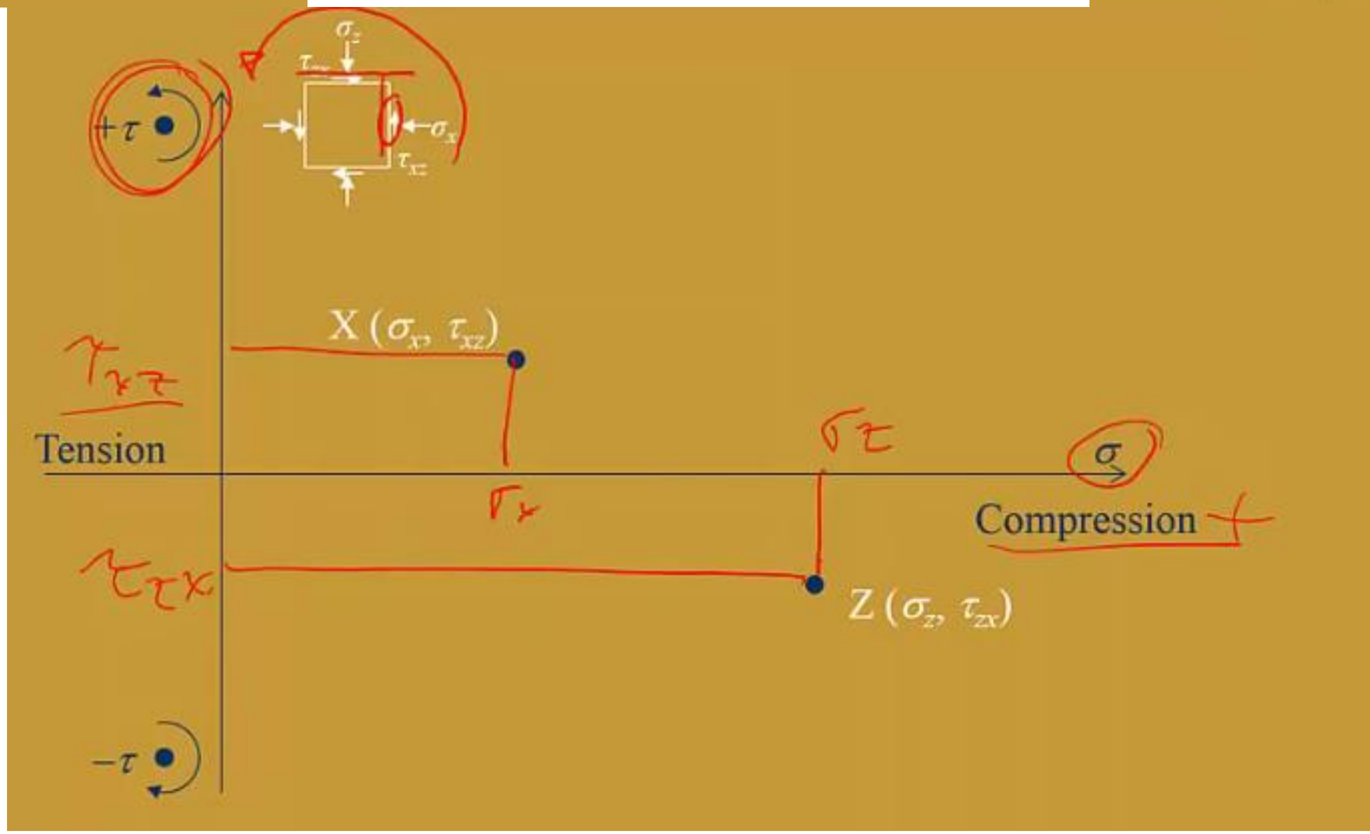


The radius of the circle is $r = (\sigma_1 - \sigma_3)/2$

The projection of the failure line is $x = c/\tan\varphi$







Notes Compiled By: Engr. Abdul Rahim Khan

$$\sigma_1 = \frac{\sigma_x + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$\sigma_3 = \frac{\sigma_x + \sigma_z}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

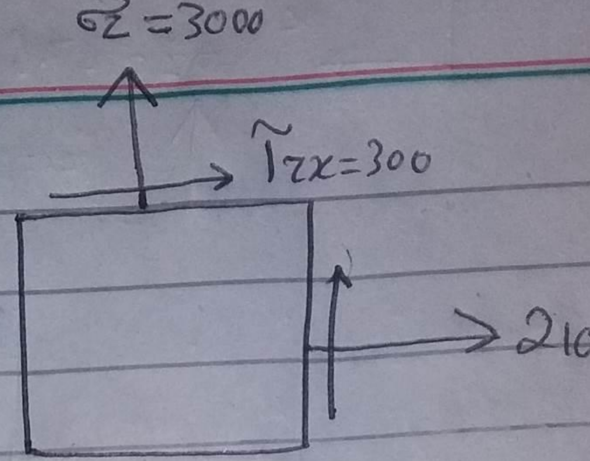
$$\theta = \frac{1}{2}\cos^{-1}\sqrt{\frac{1}{1 + \left[2\tau_{zx}/(\sigma_z - \sigma_x)\right]^2}}$$

$\sigma_x = 2100\ MPa$

$\sigma_z = 3000\ MPa$

$\tau_{zx} = 300\ MPa$

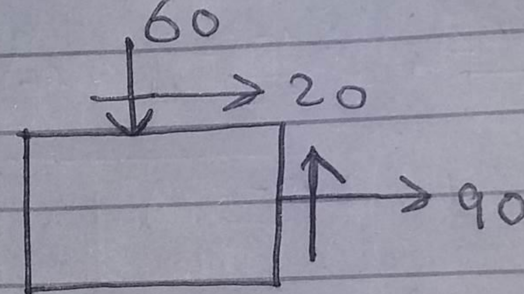
$\tau_{xz} = -300\ MPa$



$\sigma_x = 90$

$\sigma_z = -60$

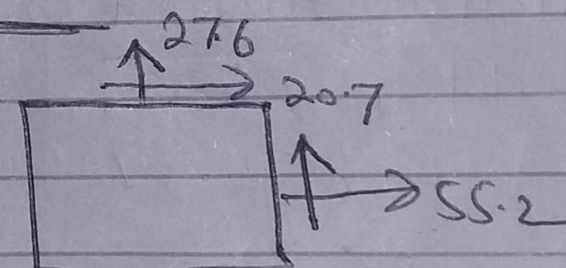
$\tau = 20$



$\sigma_x = 55.2$

$\sigma_z = 27.6$

$\tau = 20.7$



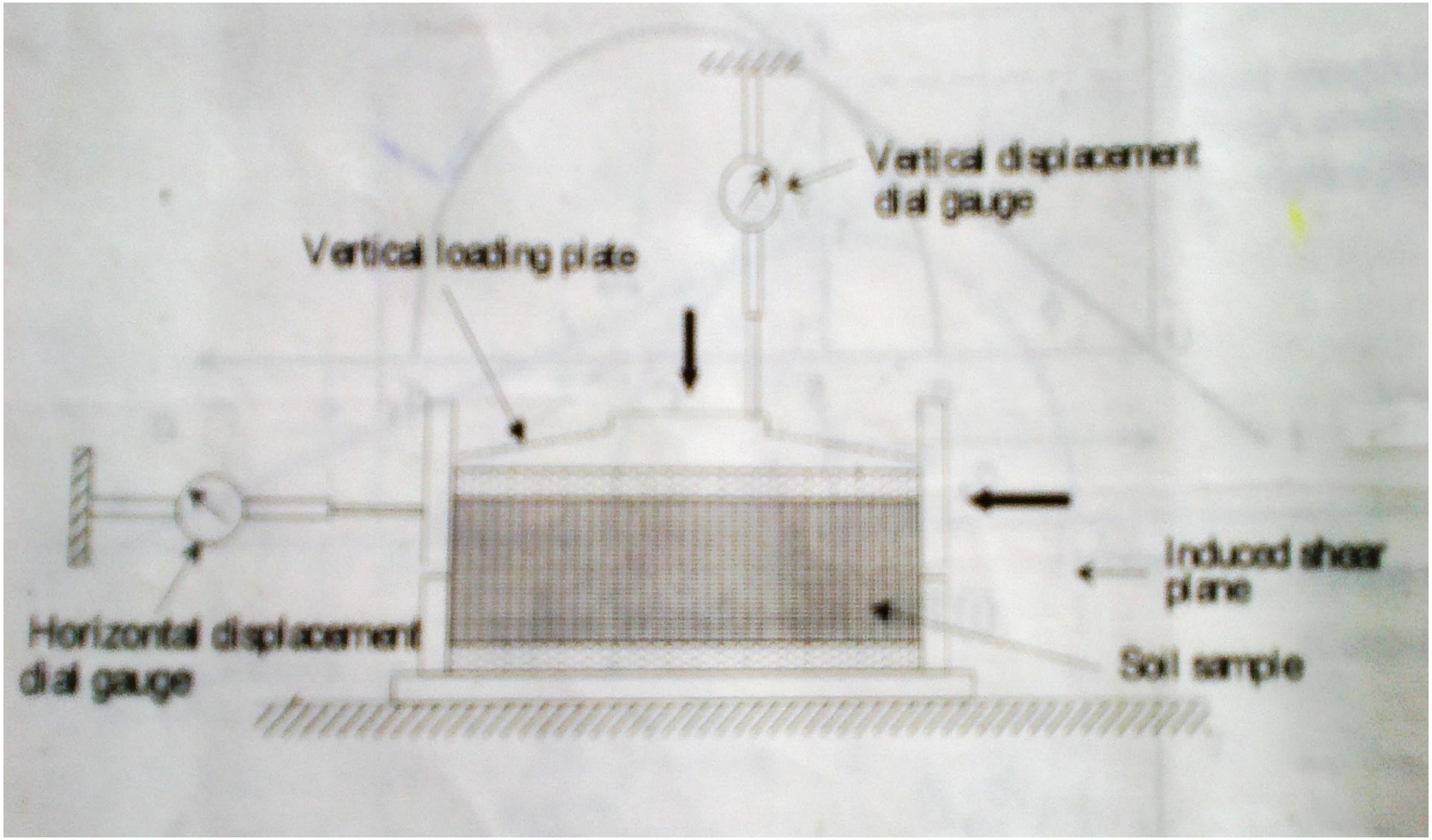
METHODS OF DETERMINING THE SHEAR STRENGTH

- Shear strength of soil can be determined using,
- Laboratory tests, and
- Field Tests
- **LABORATORY TESTING**
- Direct Shear test.
- Triaxial Compression Test.
- Unconfined Compression Test.
- Vane Shear Test
- **FIELD TESTING**
- Standard Penetration Test
- Cone Penetration Test
- Field Van Shear Test
- Pressuremeter Test
- Dilatometer Test

DIRECT SHEAR TEST

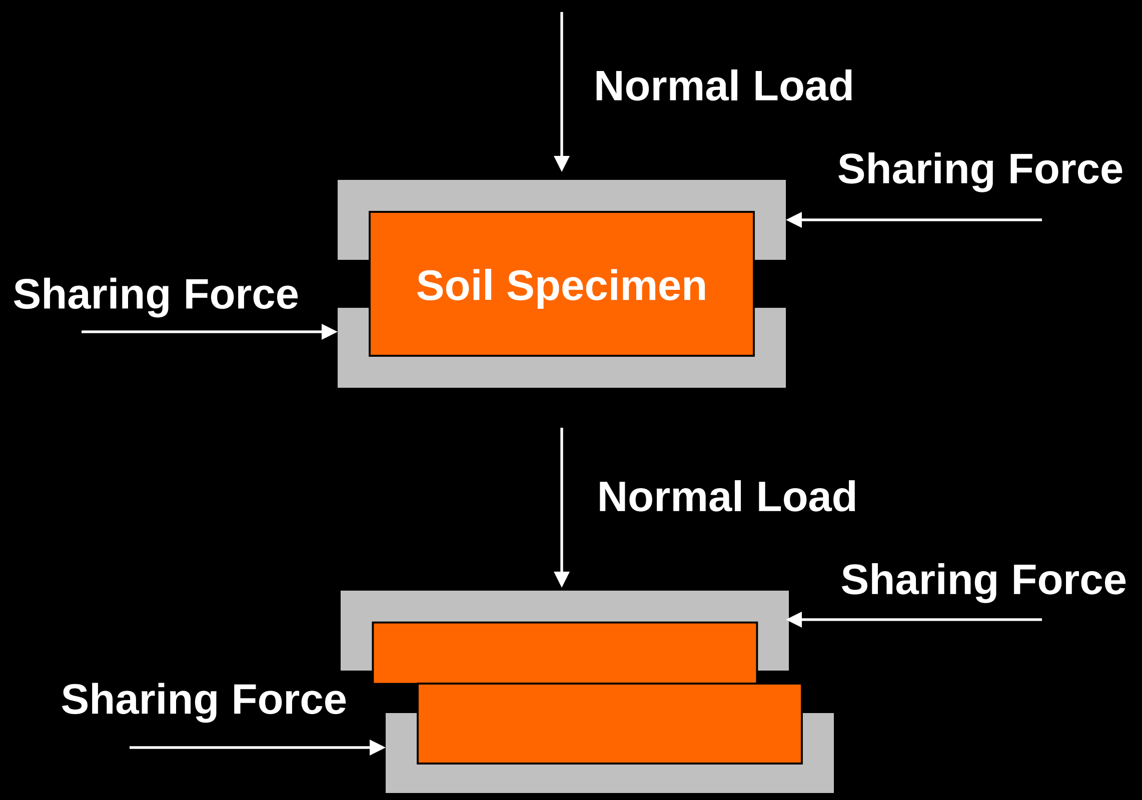
- A direct shear test is used to determine the shear strength of cohesive as well as cohesion less soil.
- It measures consolidated drained shear strength of soil.
- The specimen is deformed at a controlled rate of normal and shear stresses along a single plane.
- In this test soil samples are placed in a metal shear box of square or round shape spliced at mid height.
- Thus is this test , the samples is made to fail on a predetermined horizontal shear plane
- The shearing forces is determined by means of horizontal proving ring from which the peak shearing stress is determined.
- The horizontal and vertical deformation can be recorded using displacement dial gauges installed for this purpose.
- The normal force to the plane of shearing failure can be varied.

- The drainage of the sample can be controlled using solid or porous plates placed at the bottom and top of the sample.

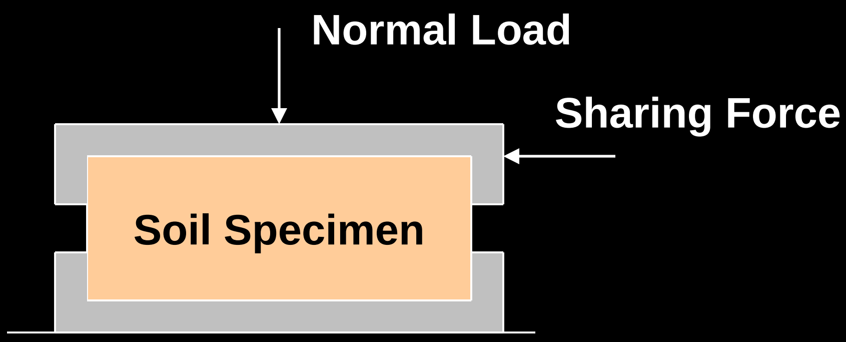


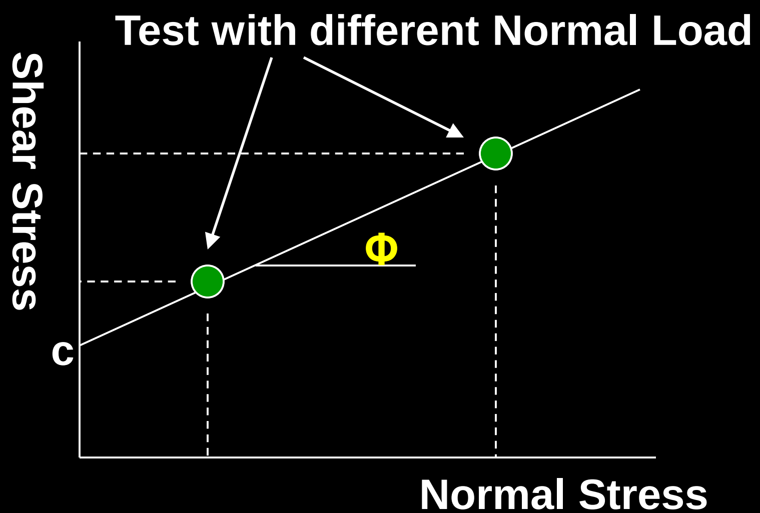
Diagrammatic sketch of DIRECT SHEAR TEST.

Direct Shear Test



Direct Shear Test





Φ angle of internal friction

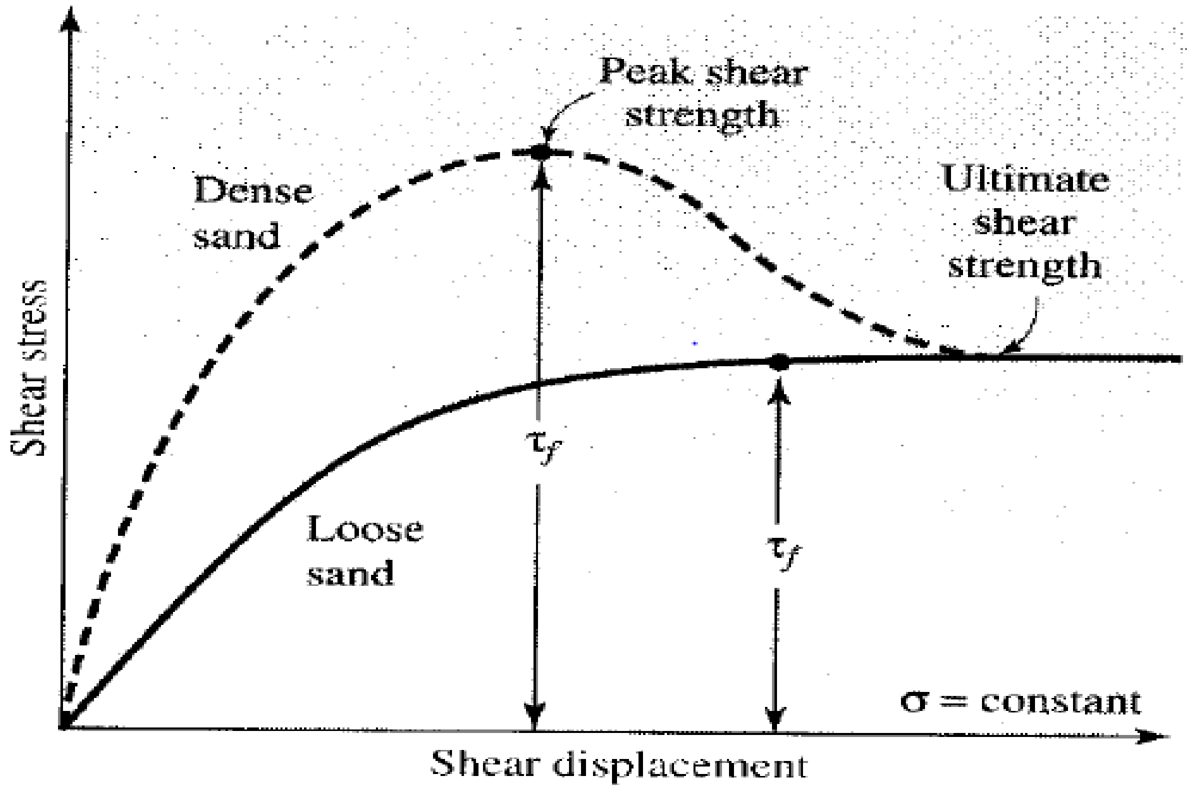
c: cohesion

Normal Stress = Normal load / the specimen's cross-sectional area

Shear stress = Shearing Force / the specimen's cross- sectional area

The graph can be used to determine the given soil's shear strength for any load

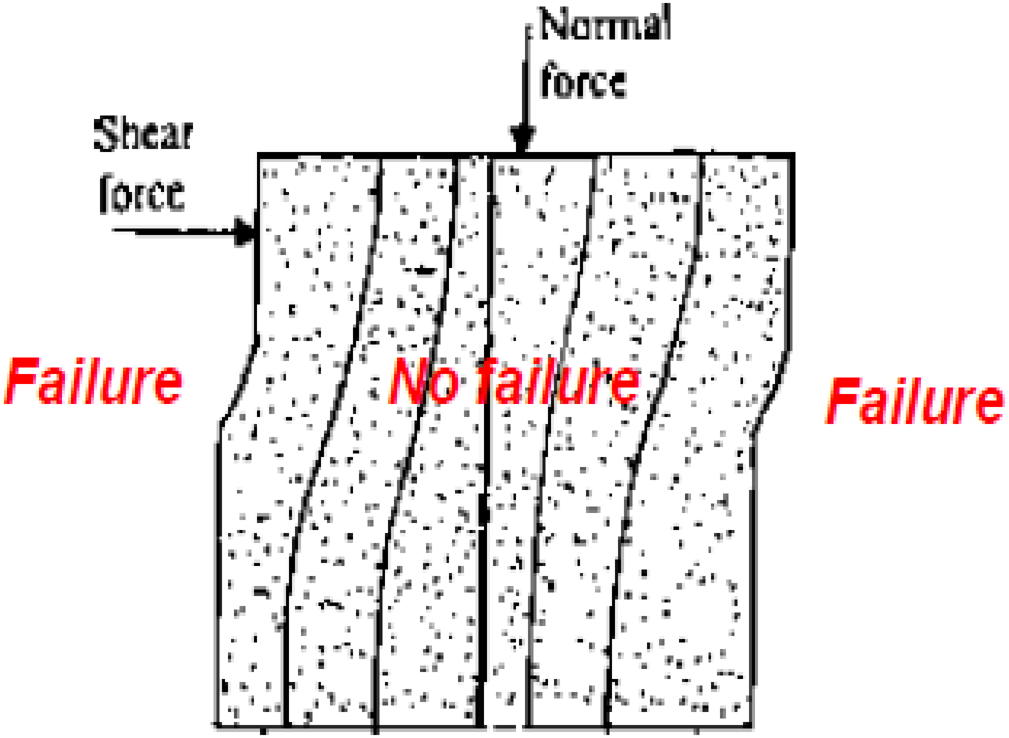
Problem: Shear failure is forced to occur along a predetermined plane, which is not necessarily the weakest plane of the soil specimen tested.



- In dense and medium sand shear stress increases with increase in shear displacement and reaches the maximum or peak value and then decreases to approximate constant value. This point when it become constant is known as ultimate shear strength. However in case of loose sand it increases to maximum value and become constant.

DIS-ADVANTAGES OF DIRECT SHEAR TEST

- The failure plan is predetermined which may not be the weakest plane.
- There is a little control over the drainage conditions.
- There is an unequal distribution of stress over the shear surface. The stress is greater at the edges than at the center. This type of stress distribution results in progressive failure.



Example- Problem

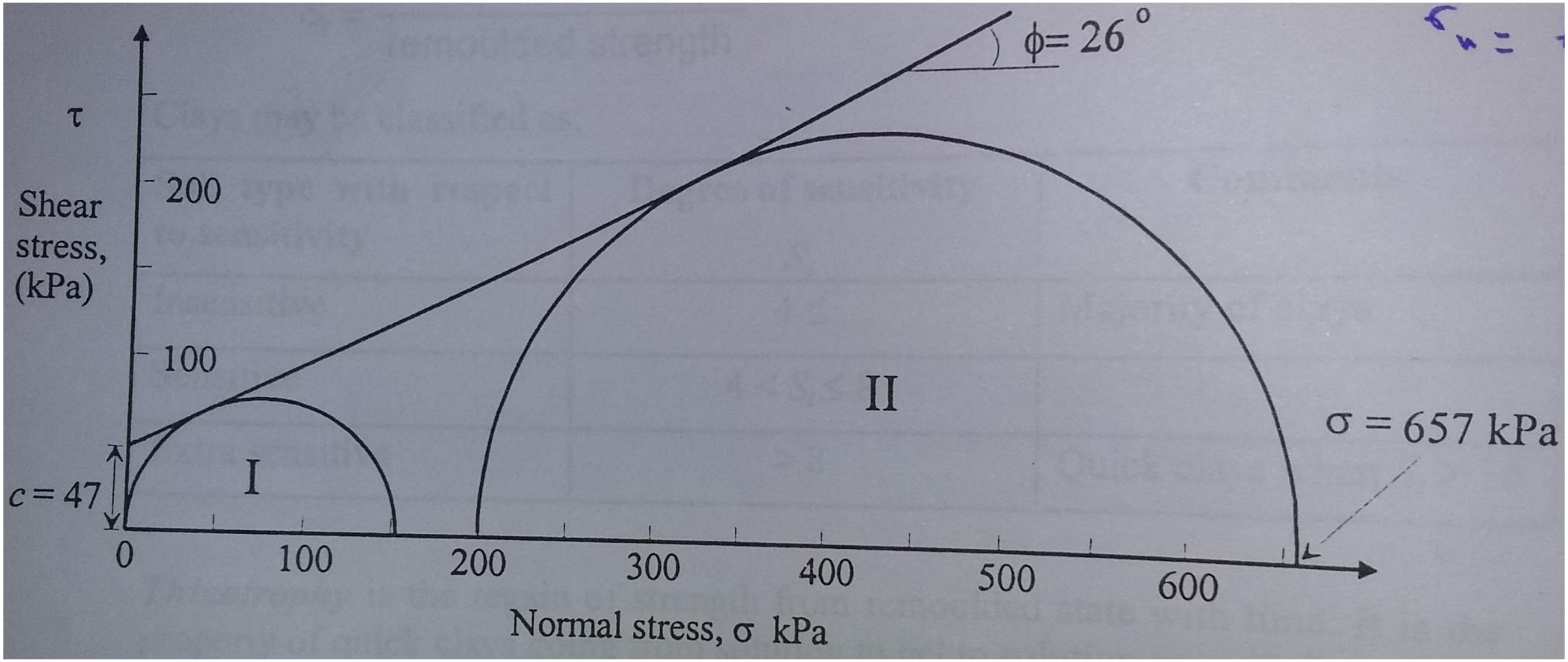
- Following are the data recorded during a series of direct shear tests performed on sandy clay. (1Div = 0.020KN)
- Size of direct shear box = 60 mm square
- Determine the C and Φ parameters.
- If Confining pressure is 200 kPa Find Total axial Stress to cause failure

Vertical Load, P (KN)	Peak proving ring reading , R (Div)*
0.361	17
0.721	26
1.081	35
1.441	44

SOLUTION

Vertical Load P, (KN)	Normal Stress, σ=P/A (KPa)	Peak proving ring reading R, (div.)	Shear Stress, T= 0.020×R/A (KPa)
0.361	100.30	17	94.40
0.721	200.30	26	144.40
1.081	300.30	35	194.40
1.441	400.30	44	244.40

A= shear box area
= 0.06×0.06
= $0.0036\ m^2$

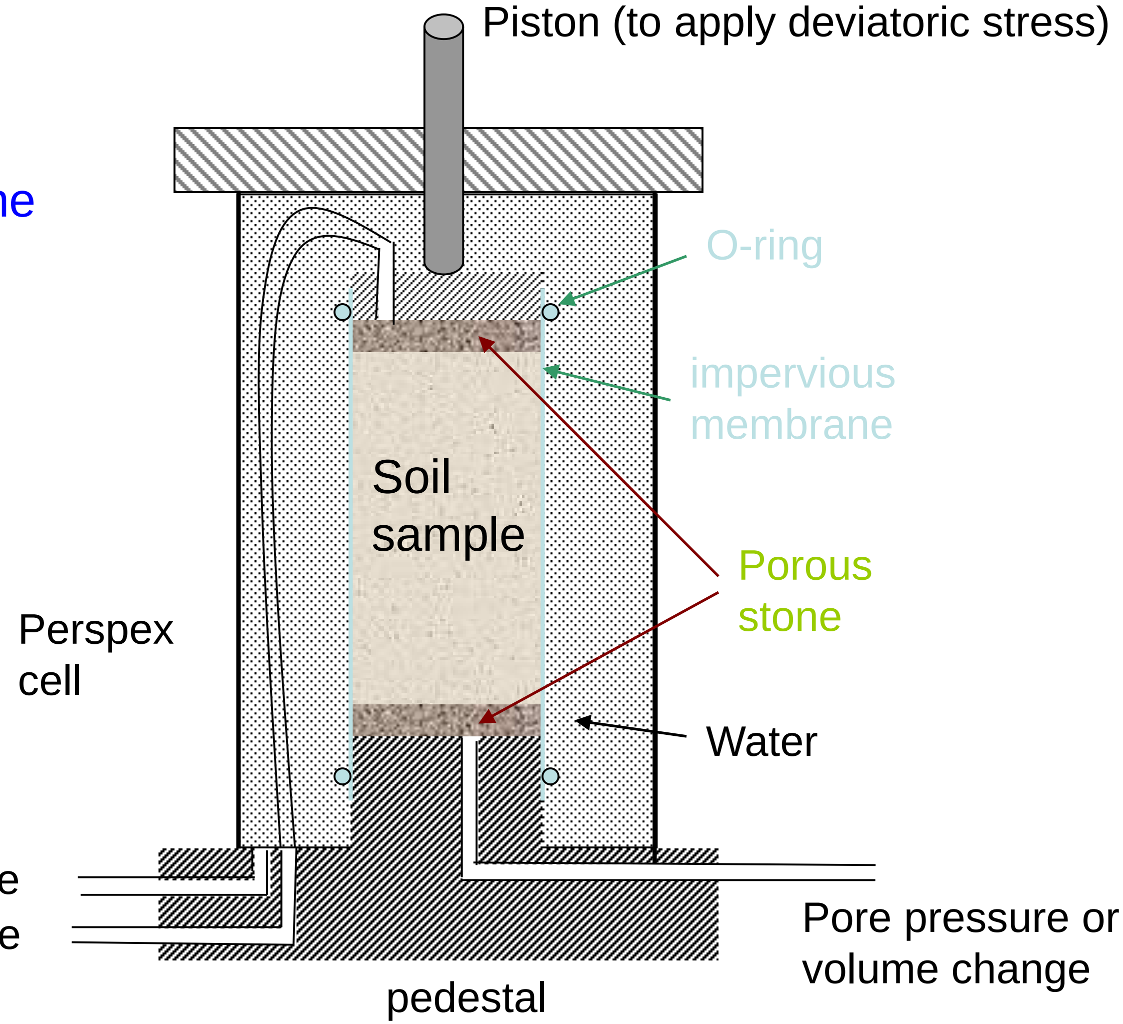

ϕ= 26°
τ
Shear stress, (kPa)
200
100
c = 47
I
II
σ = 657 kPa
0
100
200
300
400
500
600
Normal stress, σ kPa

TRIAXIAL COMPRESSION TEST

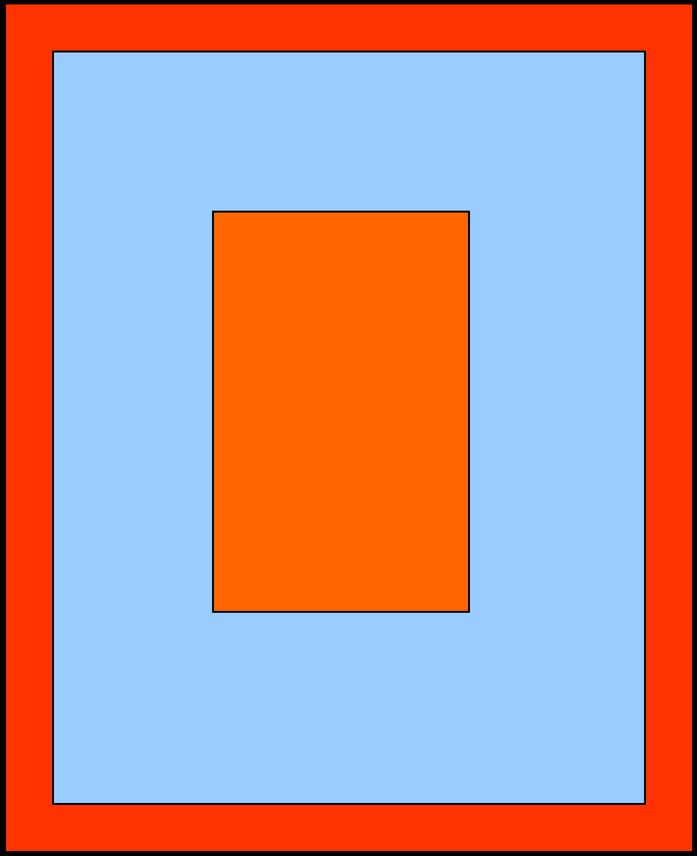
- Developed by Casagrande in an attempt to overcome some of the serious disadvantages of the direct shear test.
- Triaxial compression test is the most complex but, accurate shear test.
- In this test a cylindrical soil specimen of soil sample of height to diameter ratio 2 to 3 is loaded in all three dimensions.
- The analysis is reduced to two dimensions as a result of the lateral stresses (cell Pressure, $\sigma 3$) being equal in all directions.

Triaxial Shear Test

Soil sample at failure

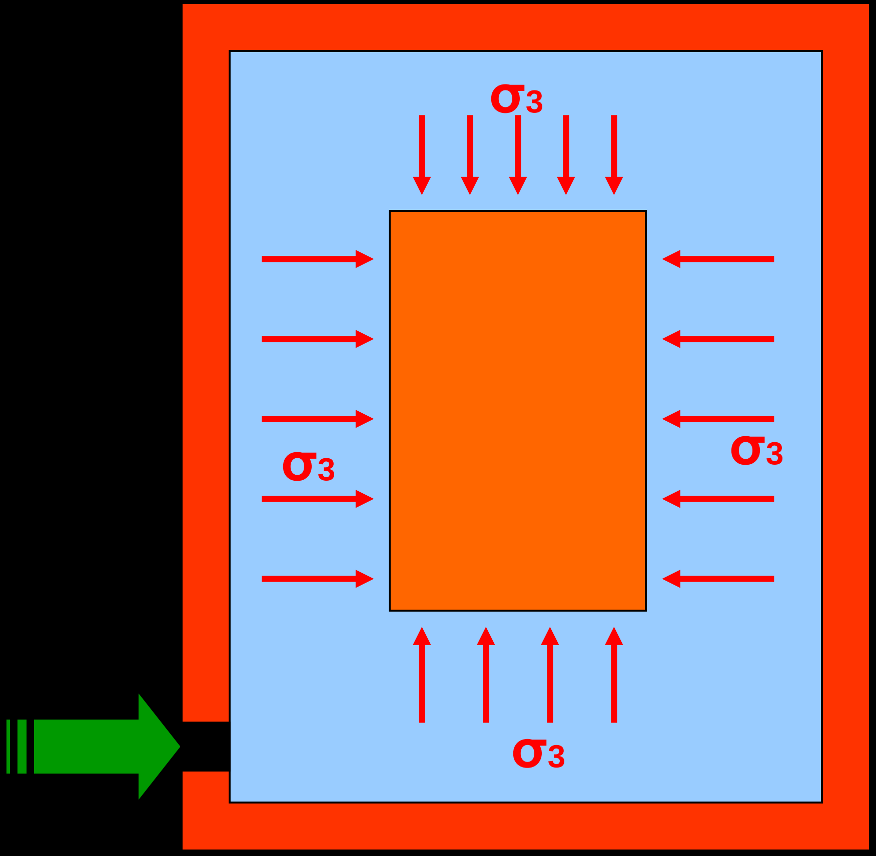


Triaxial Compression Test



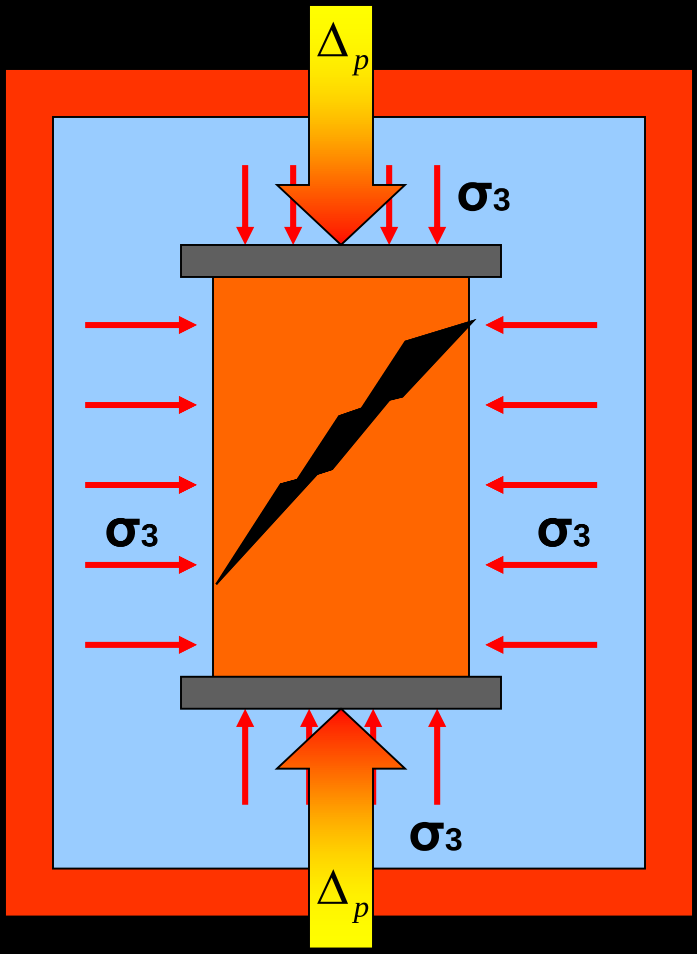
Enclose the specimen in a chamber filled with water

Triaxial Compression Test



Apply a specific pressure using water

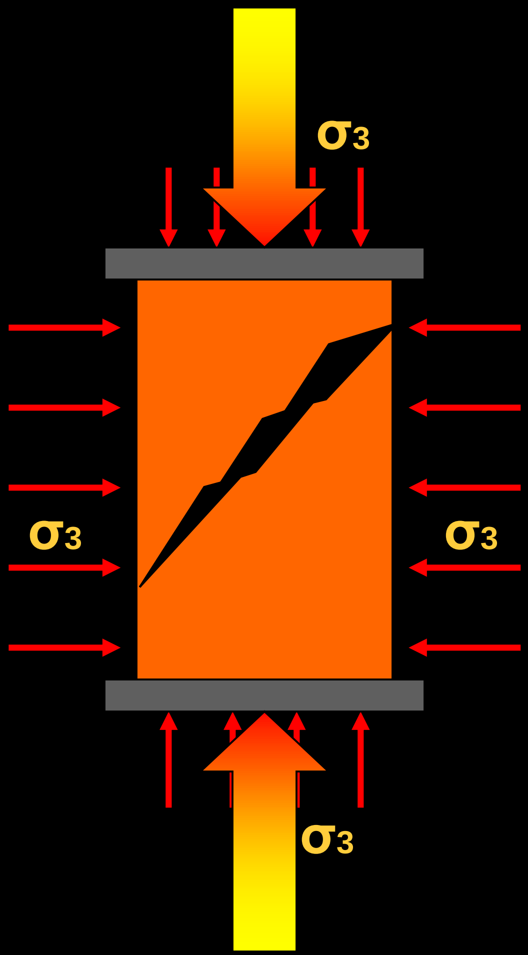
Triaxial Compression Test



Apply a vertical load and increase until the specimen fails

Use different lateral pressure, conduct the same experiment

Triaxial Compression Test

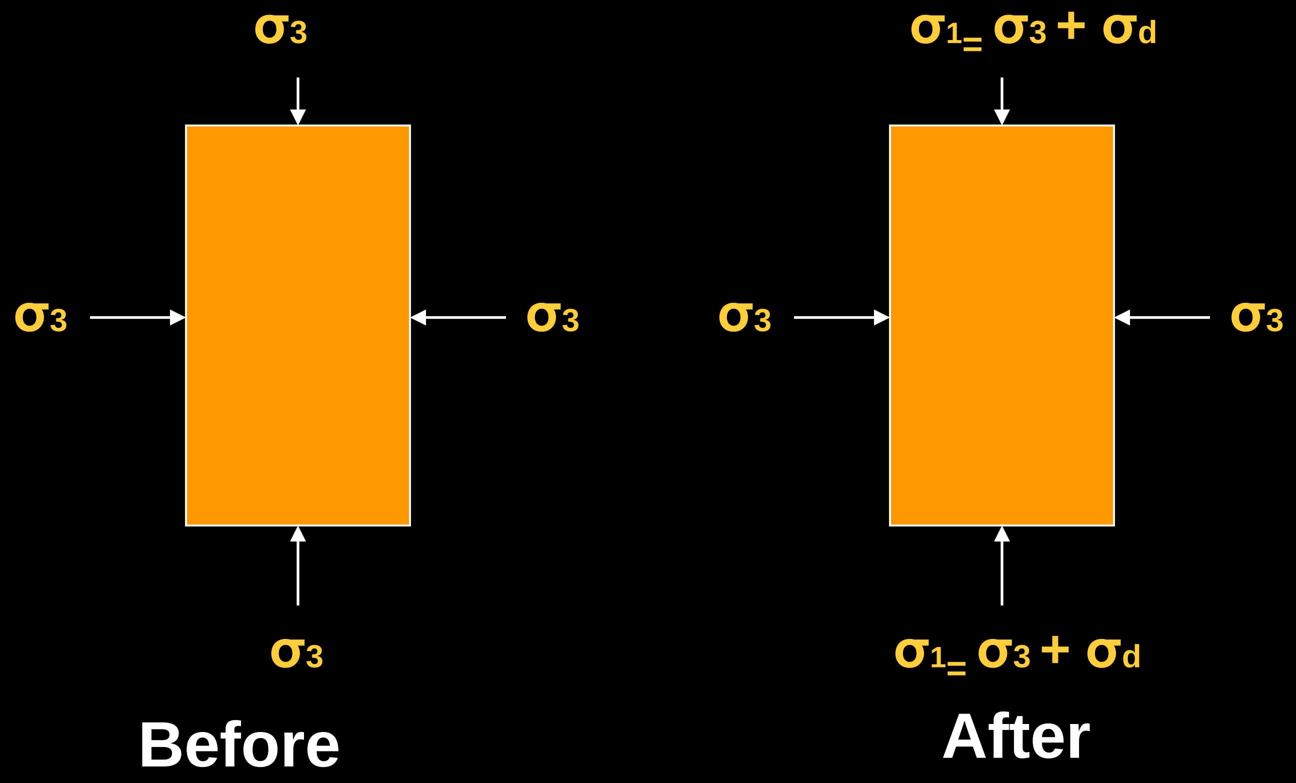


σ_3 : Minor principal stress

$\sigma_d = \Delta\sigma$: Deviator stress = Since the cell pressure σ_3 was acting all around , the additional vertical stress of

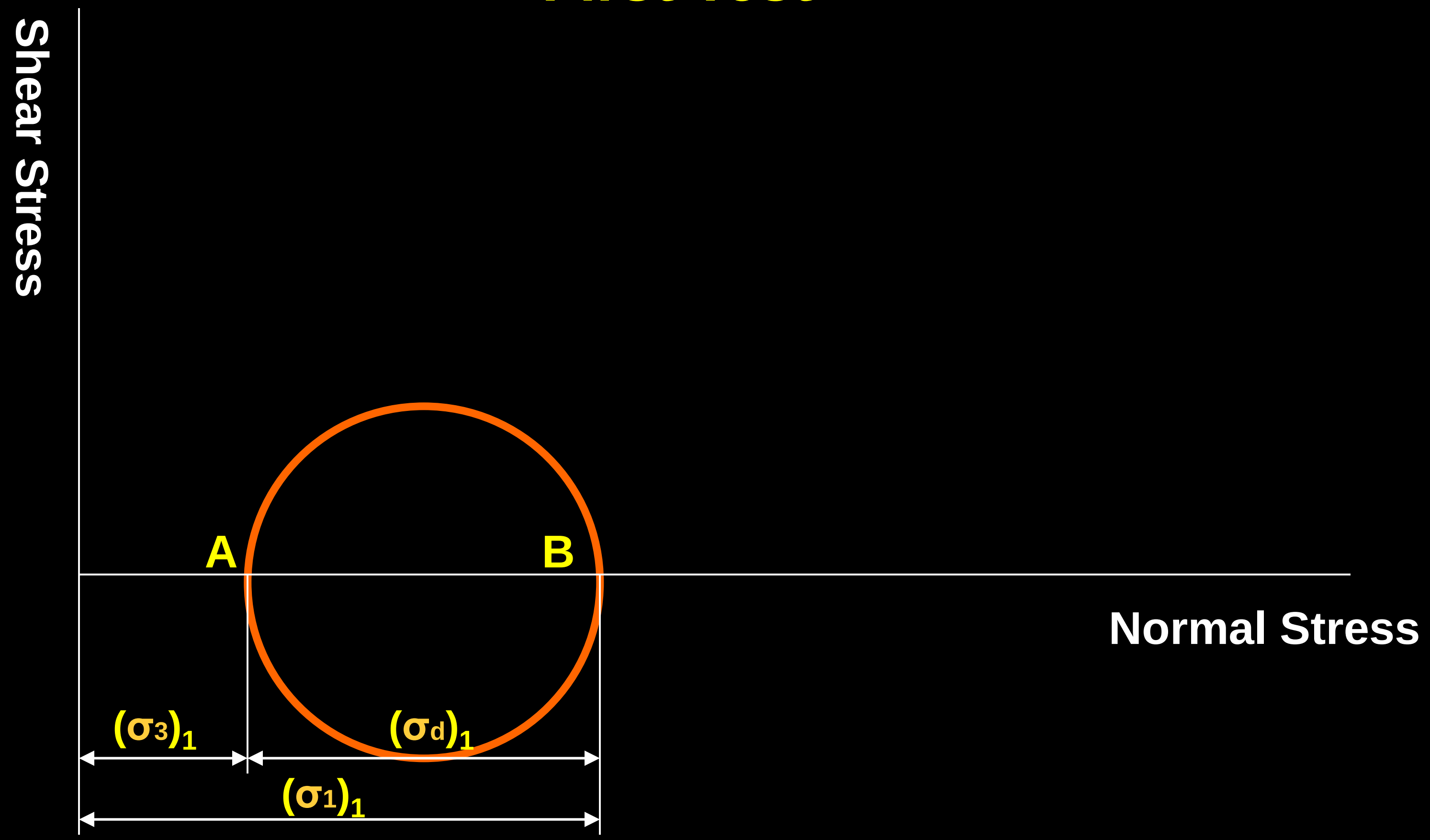
(σ_1 -σ_3) which causes tha failure of sample and such additional stress is known as deviator stress.

Triaxial Compression Test



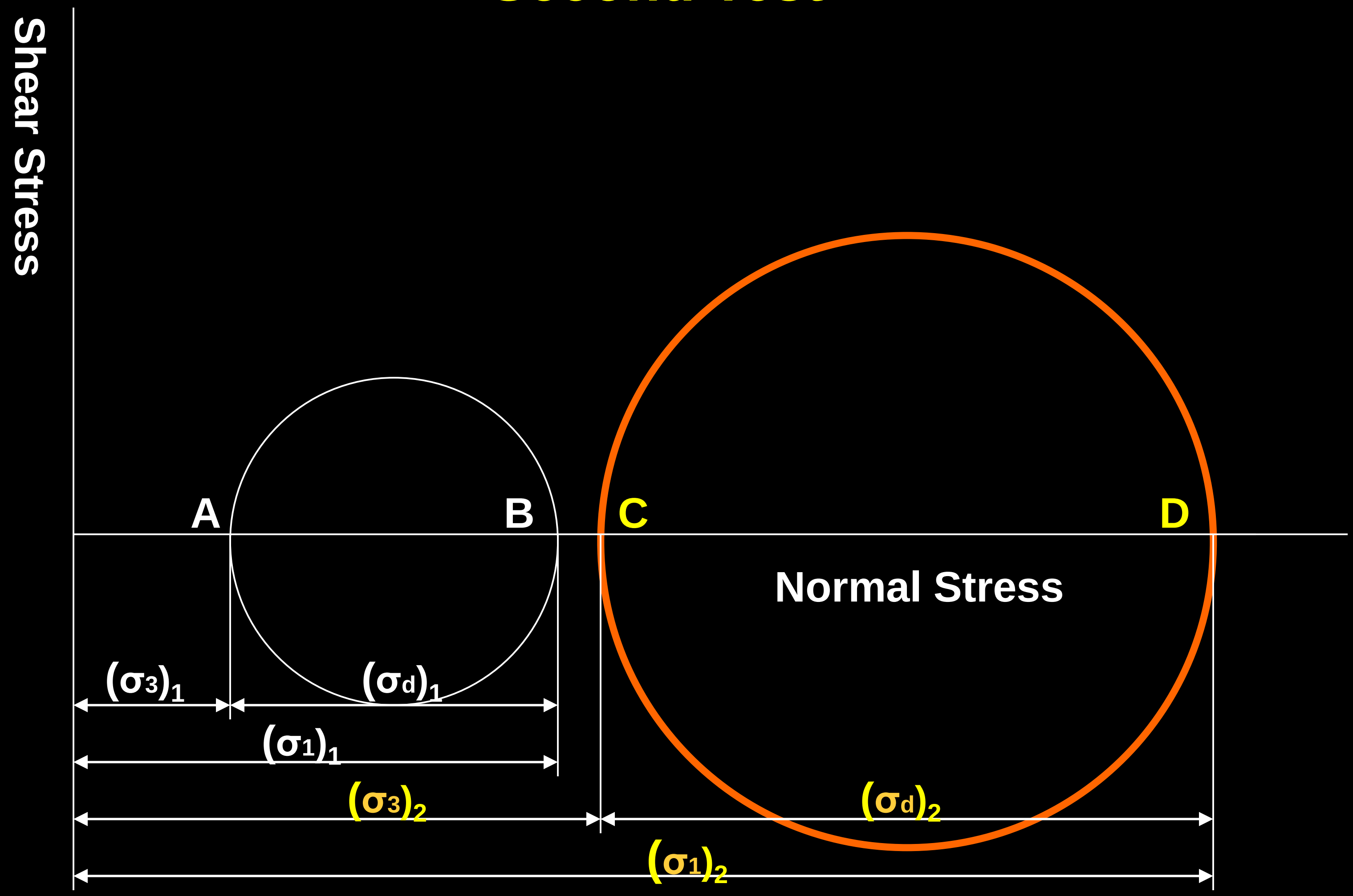
Triaxial Compression Test

First Test



Triaxial Compression Test

Second Test



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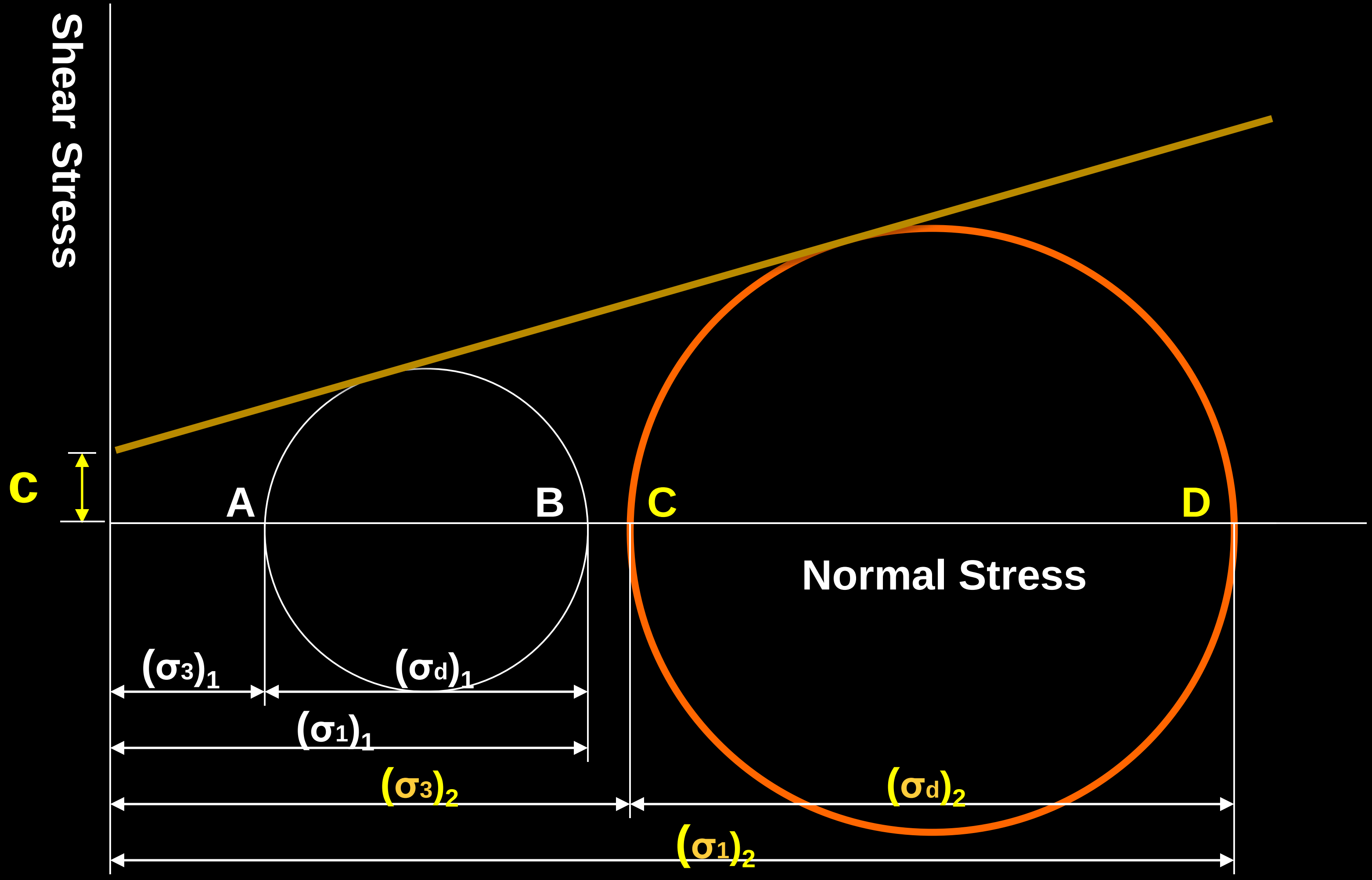
Second Test



Example - Problem

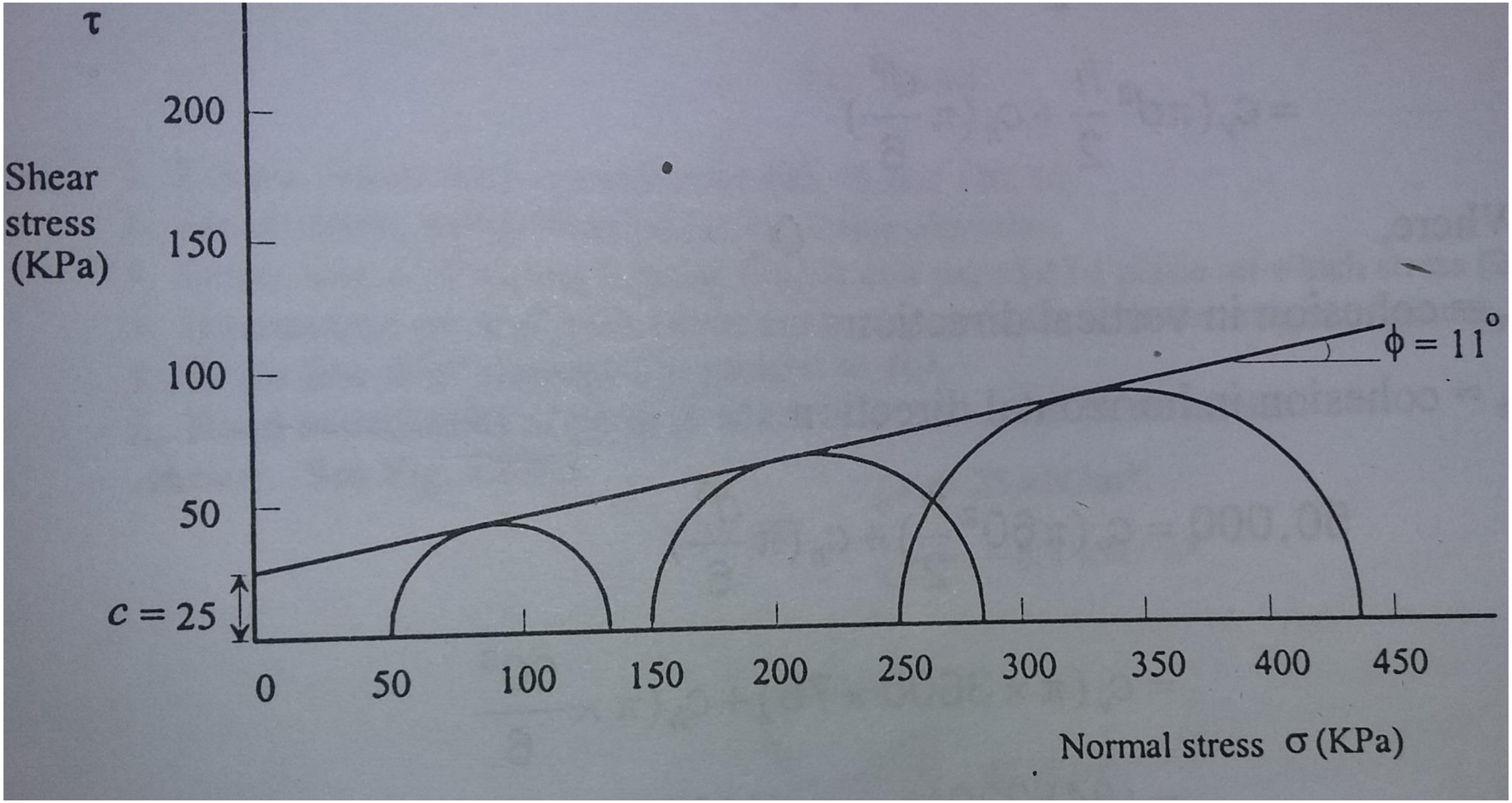
- The following data were recorded during triaxial tests performed on undisturbed soil samples: (Sample size; Dia = 37.5mm; L=75mm) (Load calibration factor, 1 Division = 1.4 $\times 10^3$.
- Compute c and Φ.

Test #	Cell Pressure, σ_3	Peak Proving ring dial reading, R (Div.)
1	50	66
2	150	106
3	250	147

SOLUTION

Cell Pressure, σ_3	Peak Proving ring dial reading, R (Div.)	Additional Vertical Stress (Deviator Stress), (Kpa)	Total Vertical Stress, $\sigma_1 = \sigma_3$ + deviator stress
50	66	83.7	133.4
150	106	134.4	284.4
250	147	186.4	436.4

Sample X-sectional area, $A = (\pi/4)D^2 = (\pi \times 37.5^2)/ 4 \times 10\text{^}6\ m^2$
$= 1.104 \times 10^{-3}\ m^2$


τ
Shear
stress
(KPa)
200
150
100
50
c = 25
0
50
100
150
200
250
300
350
400
450
φ = 11°
Normal stress σ (KPa)

COMPARISON OF DIRECT SHEAR TEST AND TRIAXIAL TEST

Direct Shear	Triaxial
Soil sample is made to fail along predetermined plane which may not be the weakest plane	Sample is free to fail along the weakest plane. Failure plane is not predetermined
There is little control over drainage conditions. Arrange for pore water measure is not provided	There is proper control over drainage conditions. Arrangement for pore water measurement
Undrained test on sand can not be performed	Any type of test can be performed on any type of soil
There is unequal shear stress distribution over the sliding plane	The stress distribution is relatively uniform
Effective stress can not be computed	Effective stress at various stages can be computed

LABORATORY SHEAR TEST CONDITION

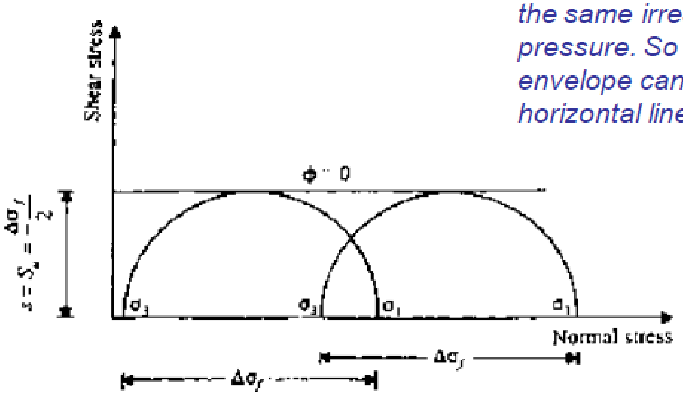
- With respect to loading and drainage conditions, the laboratory tests can be divided into the following three main categories;
- Un-consolidated Undrained (UU Test) or Quick Test.
- Consolidated Undrained (CU test) or Consolidated Quick test.
- Consolidated- Drained (CD Test) or Slow Test

Unconsolidated Undrained (UU) Triaxial Test

In UU test, *drainage is not allowed at any stage*. First, the confining pressure σ_3 is applied, after which the deviator stress is increased until failure occurs.

total major principal stress $= \sigma_3 + \Delta\sigma_f = \sigma_1$
total minor principal stress $= \sigma_3$

For a saturated soil $\Delta\sigma_f$ is practically the same irrespective of the confining pressure. So the total-stress failure envelope can be assumed to be a horizontal line, and ϕ = 0.



Un-Consolidated Un-Drained Test (QUICK TEST)

- In this test drainage is not allowed at any stage.
- First Confining Pressure is applied after which deviator stresses is increased until the failure occur.
- To ensure that during testing the void ratio of the soil sample would change as little as possible, the shearing force is applied rapidly.
- The entire test is completed within a period of about 5 to 10 minutes and pore water pressure usually not measured.
- Hence the test yields total stress parameters (C_u , Φ_u).
- We mostly have to deal with a relatively quick shear loading where the excess pore pressure has no time to dissipate, or there is no time to adjust or equalize the pore pressure.
- The most unfavorable conditions are to be considered, the sudden loading of soil mass till failure.
- Accordingly the shear test in the laboratory is to be performed quickly.
- Typical examples of the use of this test are the determination of shear strength;
- In temporary excavation.
- Calculation of bearing capacity of cohesive soil used in the design of foundations.
- Slope stability analysis of earth dams during construction.

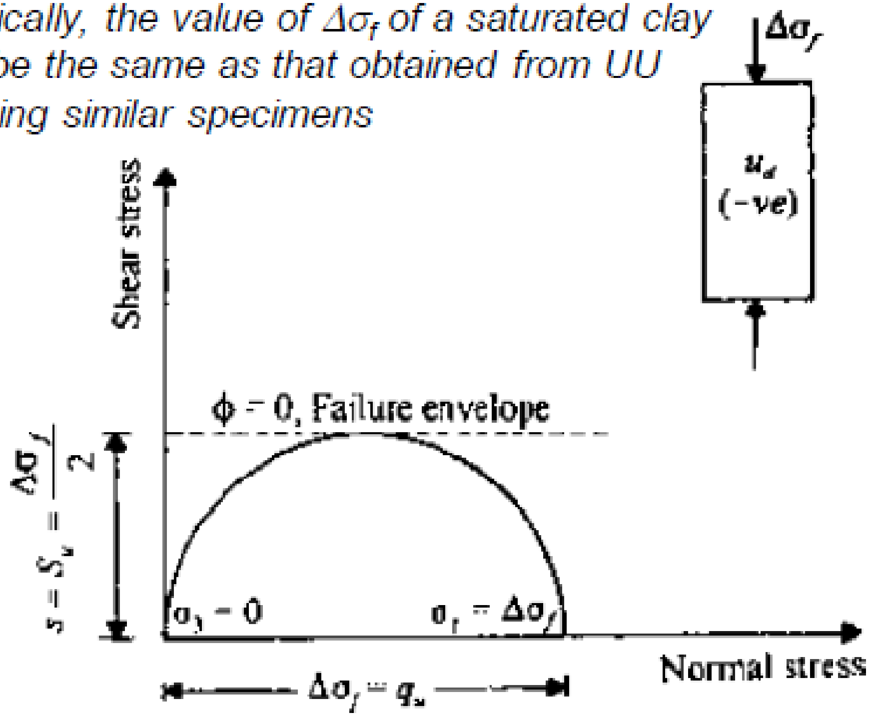
UNCONFINED COMPRESSION TEST

(Special case of the UU triaxial test in which $\sigma_3 = 0$)

For such conditions, for saturated clays, the *PWP* at the beginning of the test is negative (capillary pressure).

Axial stress on the specimen is gradually increased until the specimen fails. At failure, $\sigma_3 = 0$ and $\sigma_1 = \Delta\sigma_f = q_u$

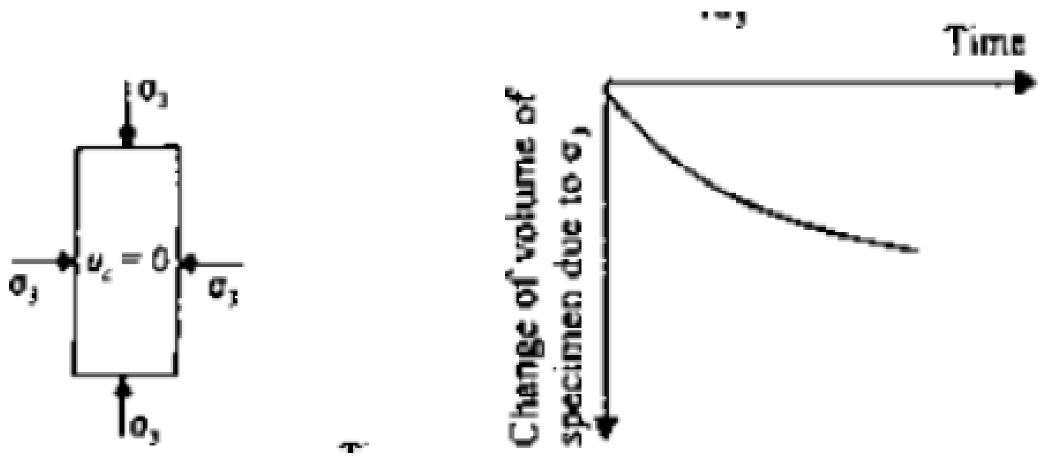
Theoretically, the value of $\Delta\sigma_f$ of a saturated clay should be the same as that obtained from UU tests using similar specimens



Consistency	q_u (kN/m^2)
Very soft	0–24
Soft	24–48
Medium	48–96
Stiff	96–192
Very stiff	192–383
Hard	>383

CU TRIAXIAL TESTS IN CLAY

In *CU* triaxial test, the soil specimen is first consolidated by a chamber-confining pressure σ_3; full drainage from the specimen is allowed.



After complete dissipation of excess *pwp*, u_c, generated by the confining pressure, the deviator stress, $\Delta\sigma$ is increased to cause failure of the specimen. During this phase of loading, the drainage line from the specimen is closed. Since drainage is not permitted, the *pwp* due to deviator stress, u_d in the specimen increases. Simultaneous measurements of $\Delta\sigma$ and u_d are made during the test.

- In practice CU strengths are used for stability problems where soils have first become fully consolidated and are at equilibrium with the existing stresses system.
- Then for some reasons additional stresses are applied quickly, with no drainage occurring.
- Examples; stability of slopes of earthen dams during rapid draw down.

CONSOLIDATED DRAINED TEST (SLOW TEST)

- In CD test, soil consolidates under normal load and the drainage is allowed during consolidation.
- On completion of consolidation, drainage is allowed while the normal stress is increased at a rate such that no pore pressure can developed.
- The resulting shear strength parameters are in terms of effective stresses only.
- Slow test on clays may take 4 to 6 weeks to complete and this test is usually used in research. This test is not very popular in clays.
- Since non-cohesive soils are free drainage and consolidate quickly, therefore these are tested usually under CD conditions.

VANE SHEAR TEST

- This is also a rapid test, used either in field or laboratory to determine the un-drained shear strength of soft cohesive sensitive clays.
- Sensitive clays are those soft clays which loose part of their shear strength when disturbed.
- According to codes this test is applicable in clays having un-drained shear strength less than 100 KN/m^2.
- In this test a cruciform vane shown in figure.

- A torque is applied to the shaft of the vane until failure occurs due to shearing on the cylinder of diameter d, and height h.
- Vane blades are pushed into the soil and rotated at a constant rate of 1 degree per minute by a worm gear and wheel arrangement.
- The undrained shear strength of clay (Cu) is computing using;
- Torque = T = C_v (πdh) d/2 + C_h ($\pi d^2/4$) 1/3 (d × 2)
- (Sides of cylinder) (Ends of Cylinder)

- If Cv = Ch
- $$Cu = \frac{T}{\pi d^2 (h/2 + d/12)}$$
- Where,
- T = Maximum Torque at failure in kg-cm or ft-lb.
- h = height of vane in cm or ft.
- d = diameter of vane in cm or ft.

Example Problem (Vane Shear Test)

- In a vane shear test a torque of 50Nm is needed to cause failure in a clay soil. The vane is 150mm long and has a diameter of 60mm. Compute the cohesion (c) of the soil.
- When a vane of length 200mm and diameter 90mm is used in the same soil, the failure torque was recorded as 140Nm. Calculate the ratio of shear strength of the clay in vertical direction to that in horizontal direction.
- **Solution**
- Original Test;
- $C = T/ [\pi d^2 (h/2 + d/6)]$
- $= 50\times10^3/[\pi (60^2) (150/2 + 60/6)]$
- $= 52KPa.$
- For both tests;
- $T = Cv (\pi dh) d/2 + Ch (\pi d^2/4) (d/3)\times 2$
- $= Cv (\pi d^2 (h/2) + Ch (\pi d^3/6)$
- Where;
- Cv = Cohesion in vertical direction.
- Ch = Cohesion in horizontal direction.
-

M

- $50{,}000 = Cv\,(\pi\times60^2\times h/2) + Ch\,(\pi\times d^3/6)$
- $= Cv\,(\pi\times3600\times75) + Ch\,(\pi\times60^3/6)$
- $= 848230\,Cv + (113097)\,Ch$
- $5 = 84.823\,Cv + 11.3097\,Ch$…….(1)

- Similarly;
- $140{,}000 = Cv\,(\pi\times90^2\times200/2) + Ch\,(\pi\times90^3/6)$
- $14 = Cv\,(254.5) + Ch\,(38.2)$
- $14 = Cv\,(254.5) + Ch\,(38.2)$………(2)
- Computing equation (1) & (2).
- $Ch = 0.233\ N/mm^2$
- $Cv = 0.028\ N/mm^2$
- Therefore ; $Ch / Cv = 8.32$
-

FACTORS AFFECTING SHEAR STRENGTH OF SOIL

- **In case of Sands,**

Following factors affect the shear strength of sands,

- Void Ratio (e = Vv / Vs).
- Grain size and grain size distribution.(GSD)
- Grain Surface Roughness.
- Presence of water
- Intermediate principle stress
- Over consolidation
- **In case of Clays,**
- Shear strength of clays greatly influenced by drainage conditions during the testing and history of deposition of the clays i.e. over consolidated or normally consolidated clays.
- For study of shear strength of clays, we divide clays into,
- Normally consolidated clays (NCC).
- Over Consolidated Clays. (OCC)
- Sensitive Clays

SENSITIVE CLAYS

- Marine or lake clays and organic silts with higher water content may have no measurable remoulded strength.
- Disturbance during sampling may cause considerable decrease in shear strength and may convert the deposit into viscous fluid.
- These are known as Sensitive or Quick clays.
- The ratio of the undisturbed shear strength of cohesive soil to the remoulded strength at the same water content is defined as sensitivity;

- S_t = Undisturbed Strength / Remoulded Strength

THANK YOU!