Stress Distribution in Soil

Stresses in Soil

- Structural loads are supported on soil via foundations.
- The loads produce stresses and resulting strains.
- Deformation in vertical direction occurring due to vertical stresses is called **settlement**.
- Stresses which produce excessive deformations are termed as **failure stresses**.
- Stresses also develop due to the soil layer above any point, known as **geostatic stress** or **over burden pressure**.
- Magnitude of geostatic stress at a point is affected by **groundwater table fluctuation**.
- Intensity of stress is not uniform but vary from point to point.
- For the design of structures such as retaining walls, sheet piles for braced excavations and waterfront structures and some types of pile foundations, stresses acting in the horizontal direction are more important.

Importance of Study of Stress in Soil

- The knowledge of stress distribution along a soil cross section is important to analyze the problems such as,
    - Settlement of foundations
    - Stability (bearing capacity) of foundations
    - Stability of slopes
    - Stability of retaining structures

Types of Stresses

- Soil mass is a skeleton of solid particles enclosing voids.
- Voids may contain water (saturated soil), air (dry soil) or both (partially saturated soil).
- When stress is applied, volume of soil reduces due to rearrangement of solid particles.
- Volume reduction brings particles close, forces acting at inter-particle contacts increase
- Forces acting between the particles remain unchanged if the rearrangement of particles does not occur.
- In fully saturated soil, reduction in volume is not possible, unless some water escapes
- Water within the voids can also withstand stresses by an increase in pressure when the soil is fully saturated.
- In a saturated soil mass, the following three types of stresses are generally considered while dealing with soil engineering problems.

Total Stress

- The stress developed at any point in a soil mass due any applied loading and/or weight of soil lying above that point is known as the total stress.
- Total stress at section XX in Fig:-1 & 2 is written as follows;

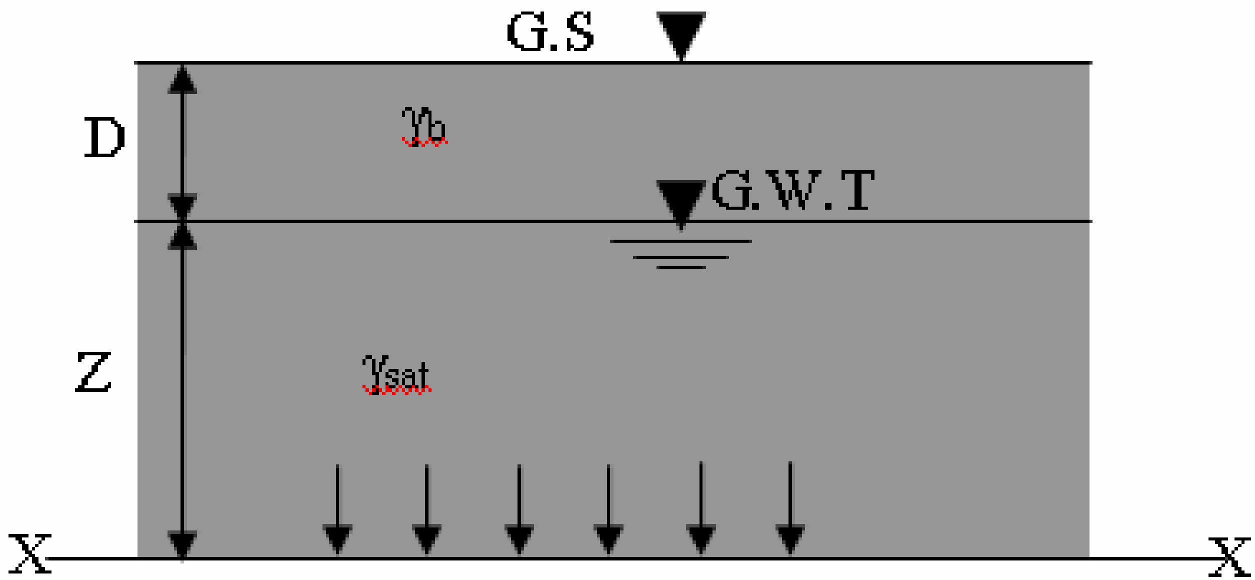


Fig:-1

$$\sigma = \gamma_b D + \gamma_{sat} Z \qquad (1)$$

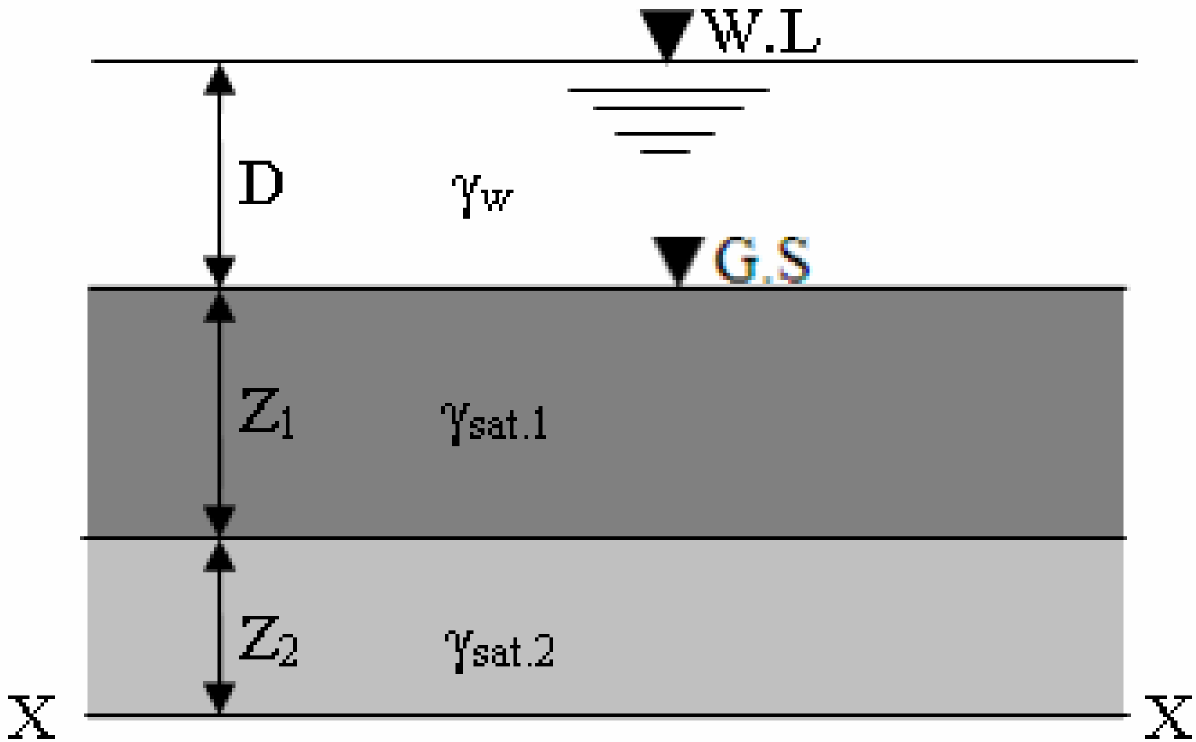


Fig:-2

$$\sigma = \gamma_w D + \gamma_{sat.1} Z_1 + \gamma_{sat.2} Z_2 \qquad (2)$$

Neutral Stress/ Pore Water Pressure

- It is the pressure of water filling the voids between solid particles.
- It is termed as neutral stress since it acts equally in all directions.
- The neutral stress reduces the inter-particle stress and the strength of soil is reduced.
- Neutral stress or hydrostatic pressure

at sec. XX for Fig:-1 & 2

$$u = \gamma_w Z \qquad (3)$$

$$u = \gamma_w D + \gamma_w Z_1 + \gamma_w Z_2 \qquad (4)$$

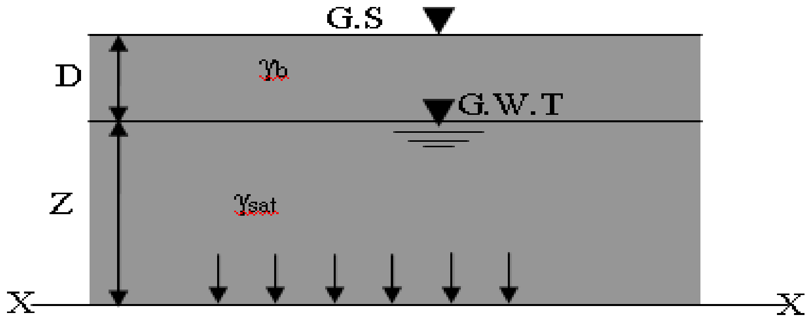


Fig:-1

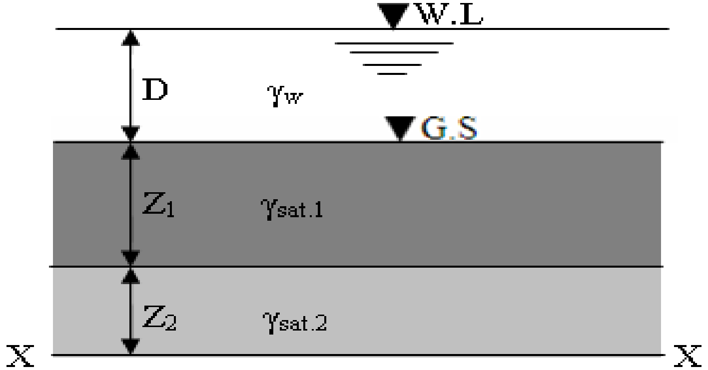


Fig:-2

Effective Stress

- The stress carried by solid particles at their points of contact (Fig.3).
- It is the sum of vertical components of forces developed at points of contact between the soil particles divided the cross- sectional are

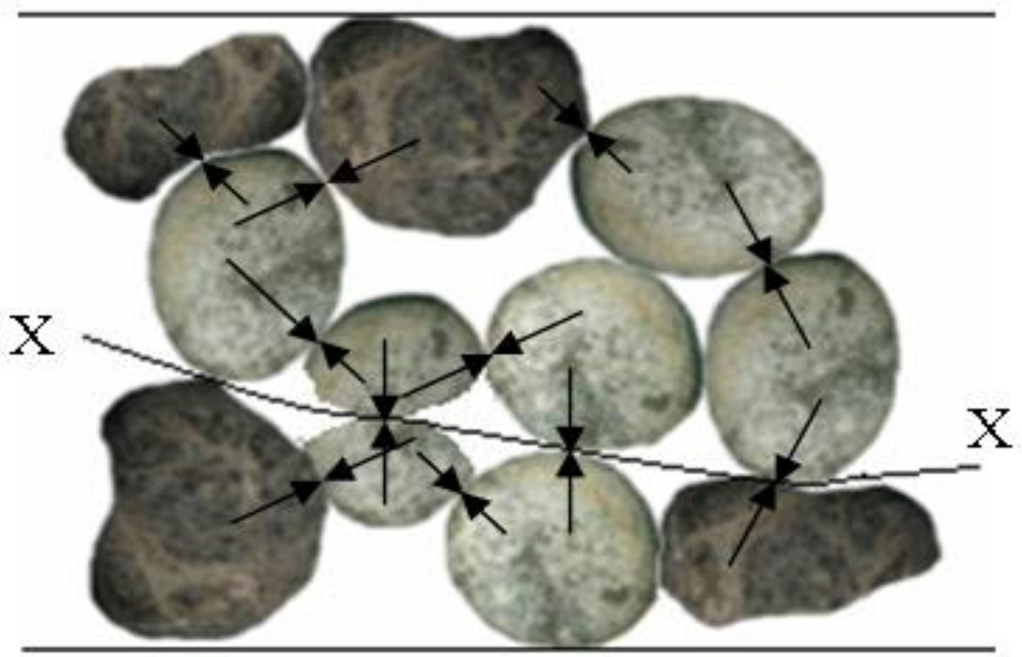


Fig:-3

- It is the effective stress due to which frictional resistance against particle movement such as rolling, slipping, sliding, etc., develops.
- When effective stress is zero, the soil is in a critical condition.
- Effective stress is expressed as follows;

$$\sigma' = \sigma - u \qquad (5)$$

- The effective stress at section XX for Fig. 1, is given below;

$$\sigma' = \gamma_b D + \gamma_{sat} Z - \gamma_w Z \qquad (6)$$

$$\sigma' = \gamma_b D + Z(\gamma_{sat} - \gamma_w) \qquad (7)$$

$$\sigma' = \gamma_b D + Z\gamma_{sub} \qquad (8)$$

- Where, γ_{sub} or γ' is submerged or buoyant unit weight of the soil.
- If water table rises to ground surface, the effective stress will be equal to "$\gamma_{sub}(Z+D)$",
- If water table lowers down to XX, then the effective stress will be equal to "$\gamma_b(Z+D)$".
- Hence it can be said that the lowering of water table causes an increase in the effective stress, because $\gamma_b > \gamma_{sub}$,

- The effective stress at section XX for Fig. 2 is as follows;

$$\sigma^{/} = \gamma_w D + \gamma_{sat.1} Z_1 + \gamma_{sat.2} Z_2 - (\gamma_w D + \gamma_w Z_1 + \gamma_w Z_2) \qquad (9)$$

$$= Z_1(\gamma_{sat.1} - \gamma_w) + Z_2(\gamma_{sat.2} - \gamma_w) \qquad (10)$$

$$= \gamma_{sub.1} Z_1 + \gamma_{sub.2} Z_2 \qquad (11)$$

- Equation 11 shows that effective stress is independent of depth of water above the ground surface.

Significance of Effective Stress

- Whenever load is applied to a soil stratum, major principal stresses on horizontal planes within the soil layer increase.
- For sustained load, soil settles and consolidates, reaching a new state of equilibrium due to changes in effective stress.
- Whatever the depth of water ‘D’ (Fig. 2), the effective stress at XX remains constant.
- If water level falls below ground level, effective stress will increase and consolidation will occur.
- The changes in effective stresses may be brought about by;
    - Increases in applied loading,
    - Lowering of the ground water table,
    - Reduction in the neutral stresses due to other reasons.

Stresses Due to Applied Loading

- Whenever loads are applied to soil, additional stresses are induced
- These stresses are distributed throughout the soil mass.
- The distribution of stress depends on the following properties.
    - Modulus of elasticity or the stress-strain behavior
    - Poisson’s ratio
    - Stratification
- For analysis of engineering problem, intensity of stress at different points within the influence zone is required.
- Methods to determine subsurface stresses are mostly based on the theory of elasticity.
- Stress-strain behavior of soil is very complex, simplifying assumptions are generally made the stresses thus computed are approximate.
- Fortunately, the results are good enough for the soil problems commonly encountered in practice.

BOUSSINESQ’S METHOD

- Boussinesq’s theory for stress distribution (1885) is based on the theory of elasticity.
- It is used to estimate stress in soil due to point load applied at the ground surface.
- Boussinesq’s made the following assumptions.
- The soil mass is an elastic, homogeneous, isotropic (which has the same value when measured in different directions.) and semi- infinite medium which extends infinitely in all directions from a level surface.
- The soil is weightless.
- The soil is not subjected to any other stress before the application of point load.
- The stress distribution due to point load is independent of the type of soil.
- The law of linear stress distribution is valid.
- There exists a continuity of stress.
- The change in soil volume due to loading is neglected.
- The stress distribution is symmetrical with respect to z-axis.

1. 1- Homogeneous- same material properties (e.g. e, m, γ, structure, etc.) through the soil mass
2. 2- Isotropic- same elastic properties in all directions
3. 3-Semi-infinite- a medium bounded on one side with a horizontal boundary. In case of soil, ground surface is the boundary and semi-infinite medium means the soil mass below the ground surface.

- Consider system as shown.

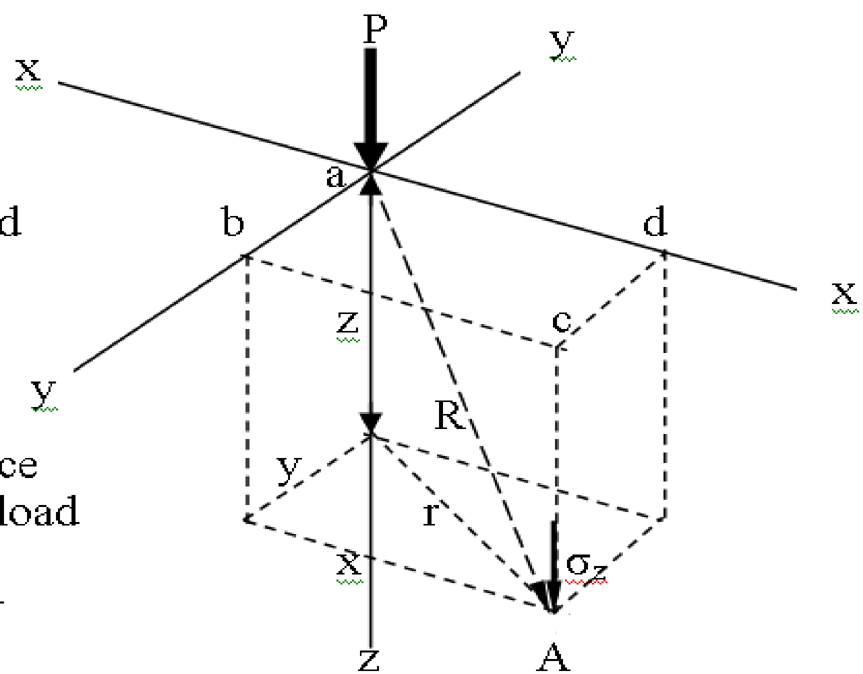
Fig:-4

The plane (abcd) represents the ground surface.

$r^2 = x^2 + y^2$

r, is the radial distance from the axis of the load

z, is the depth below the ground surface



$$\sigma_z = \frac{p}{z^2} k_b \qquad (12)$$

- Where,
- P, is the applied loading
- Z, is the depth where stress is required
- r, is the radial distance and
- **k_b is Boussinesq's stress coefficient and is given below**

$$k_b = \frac{3}{2\pi\left(1+\frac{r^2}{z^2}\right)^{\frac{5}{2}}} = \frac{0.478}{\left(1+\frac{r^2}{z^2}\right)^{\frac{5}{2}}} = \frac{0.478z^5}{\left(r^2+z^2\right)^{\frac{5}{2}}} \qquad (13)$$

- kb can be determined from the given values of r/z from the Eq-13. When the point 'A' lies on the z-axis i.e., r = 0
- kb = 0.478 (14)
- And 'σz' for such a condition is maximum

Boussinesq's equation has the following important points worth noting,

- The vertical stress does not depend on the modulus of elasticity (E) and the poisson's ratio (mu).
- It only depends on magnitude of load and the coordinates (r , z).
- At the ground surface (z = 0), the vertical stress is theoretically infinite. However, in actual case only finite stress develops.
- The vertical stress (σ_z) decreases rapidly with an increase in r/z ratio.
- Theoretically, the vertical stress would be zero only at an infinite distance. But actually the vertical stress becomes negligible when r/z = 5 or more.
- In actual practice, foundation loads are never applied on the ground surface.
- The field observations reveal that the actual stresses are generally smaller than the theoretical values calculated by Boussinesq's equation, especially at shallow depths. Thus, the Boussinesq's equation, gives conservative values and is commonly used in soil engineering problems.

Limitations of Boussinesq's Equation

- The application of Boussinesq's equation can be justified when there is a stress increase (a loading case, which is in soil). The real requirement of the equation is that the soil should have a constant stress-strain ratio, which is almost true for load ranges applied to the soil.
- For the unloading case, when there is a stress decrease the relation between stress and strain is not linear and, therefore, the equation is not applicable.

- For practical cases, the Boussinesq's equation can be safely used for homogeneous deposits of clay, man-made fills and for limited thickness of uniform sand deposits. In deep sand deposits, the modulus of elasticity increases with increase in depth and, therefore, Boussinesq's equation does not give satisfactory results.
- The point loads applied below ground surface induce somewhat smaller stresses than those caused by surface loads, and, therefore, the Boussinesq's equation is not reliably applicable. However, the equation is frequently used for shallow footings, in which z is measured below the base of the footing.

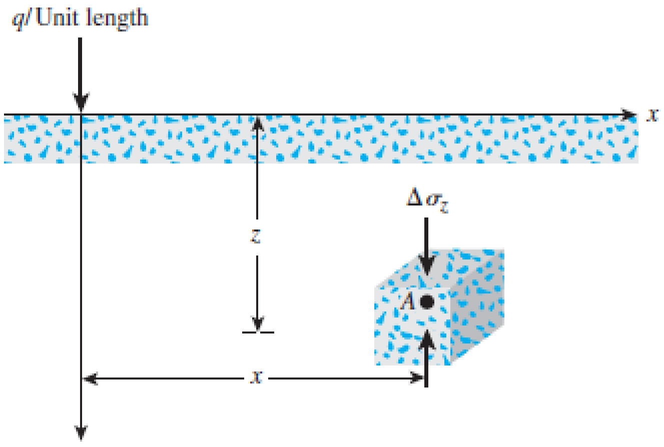
r/z	I_1	r/z	I_1
0	0.4775	0.9	0.1083
0.1	0.4657	1.0	0.0844
0.2	0.4329	1.5	0.0251
0.3	0.3849	1.75	0.0144
0.4	0.3295	2.0	0.0085
0.5	0.2733	2.5	0.0034
0.6	0.2214	3.0	0.0015
0.7	0.1762	4.0	0.0004
0.8	0.1386	5.0	0.00014

Vertical Stress Caused by a Vertical Line Load

Following Fig shows a vertical flexible line load of infinite length that has an intensity q/unit length on the surface of a semi-infinite soil mass.

The vertical stress increase, $\Delta\sigma_z$ inside the soil mass can be determined by using the following relation

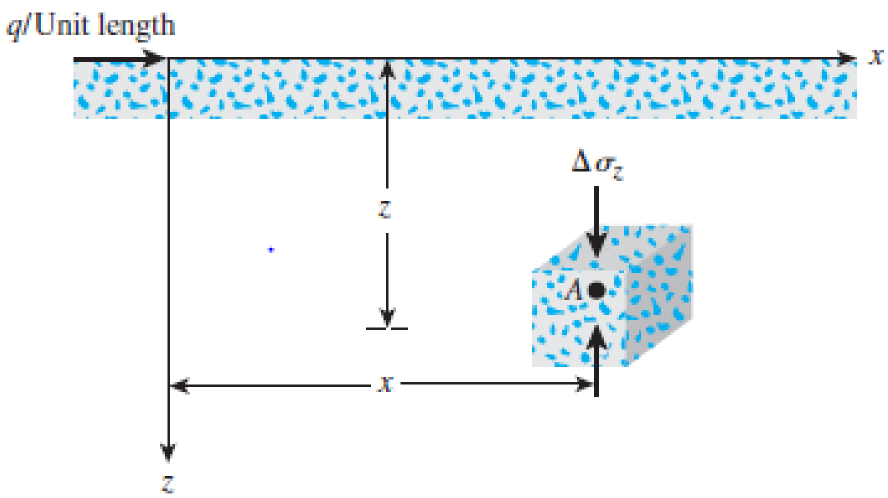
$$\Delta\sigma_z = \frac{2qz^3}{\pi\left(x^2 + y^2\right)^2}$$



Vertical Stress Caused by a Horizontal Line Load

shows a horizontal flexible line load on the surface of a semi-infinite soil mass. The vertical stress increase at point A in the soil mass can be given as

$$\Delta\sigma_z = \frac{2qxz^2}{\pi\left(x^2 + y^2\right)^2}$$



Vertical Stress under Center of a Uniformly Loaded Circular Area

Boussinesq's method is applied to find vertical stress under the center of a uniformly loaded flexible circular area.

Let *'q'* be the pressure intensity on circular area of radius '*R*'.

Total load on the elementary area $d_A = d_p = q\,(r\,d\alpha.)\,d_r$ (1)

Where,

d_A = length x width of elementary area = $(d_r)\,(r\,d\alpha)$

Vertical stress, $d\sigma z$, for point 'A' at a depth 'z', due dp

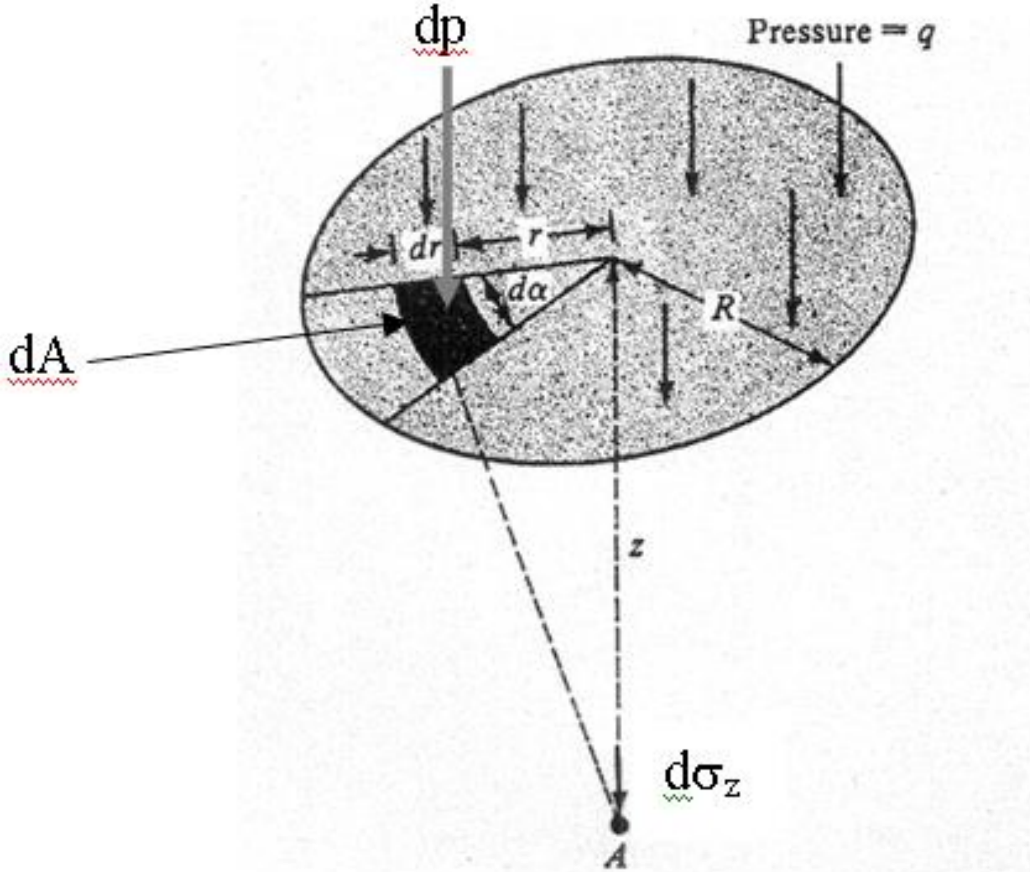
$$d\sigma_z = \frac{3(qr\,\mathrm{dr}\,\mathrm{d}\alpha)}{2\pi}\,\frac{z^3}{(r^2+z^2)^{5/2}} \qquad (2)$$

- The stress at A due to entire loaded area can be found by integrating Eq-2

$$\sigma_z = \int d\sigma_z = \int_{\alpha=0}^{\alpha=2\pi} \int_{r=0}^{r=R} \frac{3q}{2\pi} \frac{z^3 r}{(r^2+z^2)^{5/2}} dr\, \mathrm{d}\alpha \qquad (3)$$

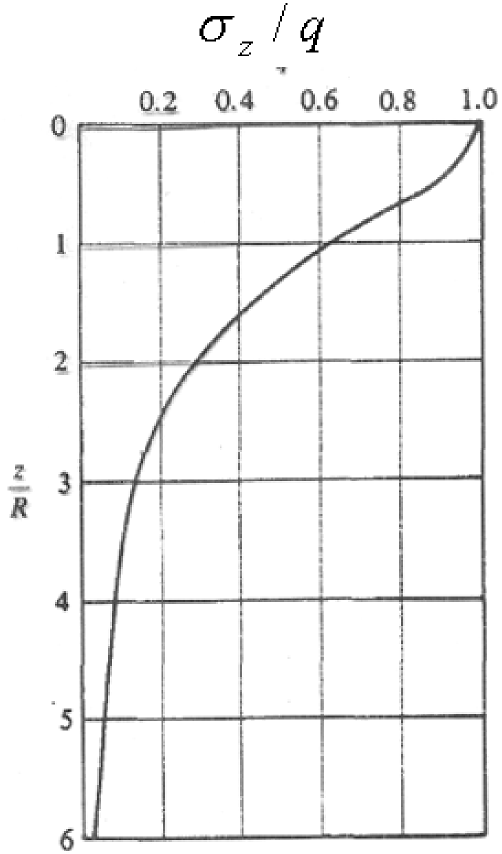
$$\sigma_z = q\left\{1 - \frac{1}{[(R/z)^2+1]^{3/2}}\right\}$$

$$\frac{\sigma_z}{q} = \left\{1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}\right\} \qquad (4)$$



Vertical stress under center of uniformly loaded flexible circular area

The variation of σ_z/q with z/R. is given in Table, and also shown in Figure. The value of σ_z decreases rapidly with depth; and, at $z = 5R$, it is about 6% of q,



Variation of σ_z/q with z/R

z/R	σ_z/q
0	1
0.02	0.9999
0.05	0.9998
0.10	0.9990
0.2	0.9925
0.4	0.9488
0.5	0.9106
0.8	0.7562
1.0	0.6465
1.5	0.4240
2.0	0.2845
2.5	0.1996
3.0	0.1436
4.0	0.0869
5.0	0.0571

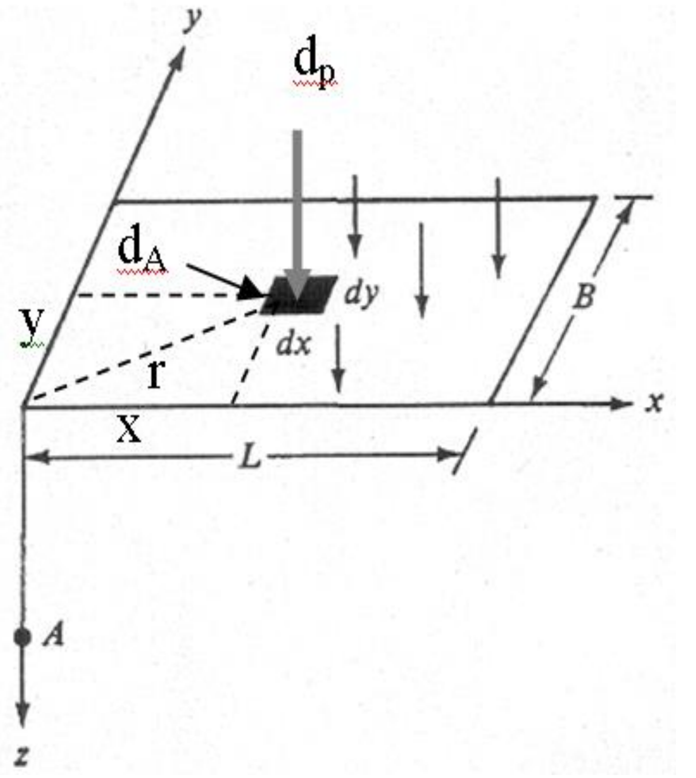
Intensity of stress under the center of a uniformly loaded flexible area

Vertical Stress Due to Rectangular Loaded Area

Boussinesq's Theory is used to find vertical stress under a rectangular loaded area.
Area (L x B) at the ground surface is subjected to an applied pressure = q

Consider a small elementary area dA = dx dy

The load on the area dA = dp = q dx dy



Vertical stress below corner of a uniformly loaded rectangular area

Vertical Stress Due to Rectangular Loaded Area

- The Vertical stress, $d\sigma_z$, for point 'A' below the corner of the rectangular area at a depth 'z', due d_p can be obtained by replacing P by d_p and r^2 by $x^2 + y^2$ in the Bousinesq's Eq. Thus.

$$d\sigma_z = \frac{3q\,\mathrm{dx}\,\mathrm{dy}\,z^3}{2\pi\,(x^2 + y^2 + z^2)^{5/2}} \qquad (5)$$

- The stress σz at A due to the entire loaded area can be determined by integrating the above equation.

$$\sigma_z = \int d\sigma_z = \int_{y=0}^{B} \int_{x=0}^{L} \frac{3qz^3(dx\,\mathrm{dy})}{2\pi(x^2 + y^2 + z^2)^{5/2}} = qI_3 \qquad (6)$$

$$I_3 = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right] \qquad (7)$$

$$m = \frac{B}{z} \qquad n = \frac{L}{z}$$

Table 5.2 Variation of Influence Value I [Eq. (5.6)]*

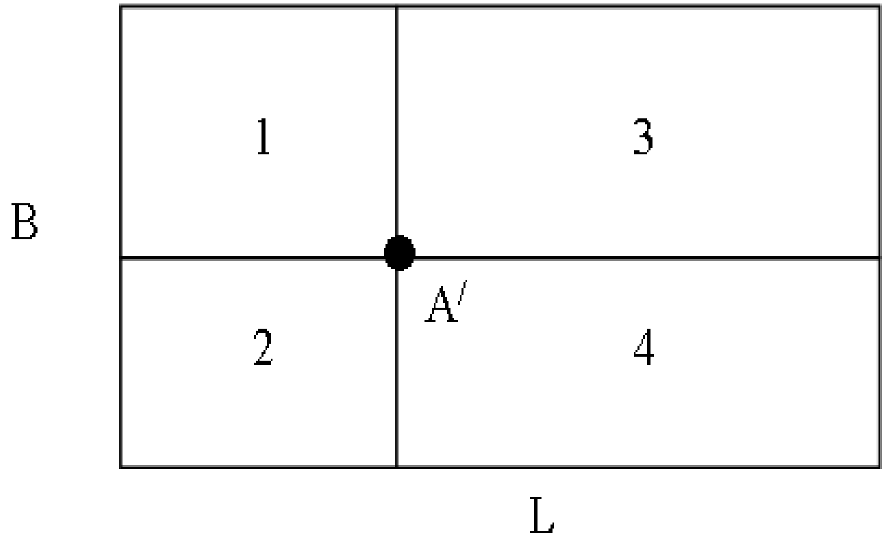
	n											
m	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4
0.1	0.00470	0.00917	0.01323	0.01678	0.01978	0.02223	0.02420	0.02576	0.02698	0.02794	0.02926	0.03007
0.2	0.00917	0.01790	0.02585	0.03280	0.03866	0.04348	0.04735	0.05042	0.05283	0.05471	0.05733	0.05894
0.3	0.01323	0.02585	0.03735	0.04742	0.05593	0.06294	0.06858	0.07308	0.07661	0.07938	0.08323	0.08561
0.4	0.01678	0.03280	0.04742	0.06024	0.07111	0.08009	0.08734	0.09314	0.09770	0.10129	0.10631	0.10941
0.5	0.01978	0.03866	0.05593	0.07111	0.08403	0.09473	0.10340	0.11035	0.11584	0.12018	0.12626	0.13003
0.6	0.02223	0.04348	0.06294	0.08009	0.09473	0.10688	0.11679	0.12474	0.13105	0.13605	0.14309	0.14749
0.7	0.02420	0.04735	0.06858	0.08734	0.10340	0.11679	0.12772	0.13653	0.14356	0.14914	0.15703	0.16199
0.8	0.02576	0.05042	0.07308	0.09314	0.11035	0.12474	0.13653	0.14607	0.15371	0.15978	0.16843	0.17389
0.9	0.02698	0.05283	0.07661	0.09770	0.11584	0.13105	0.14356	0.15371	0.16185	0.16835	0.17766	0.18357
1.0	0.02794	0.05471	0.07938	0.10129	0.12018	0.13605	0.14914	0.15978	0.16835	0.17522	0.18508	0.19139
1.2	0.02926	0.05733	0.08323	0.10631	0.12626	0.14309	0.15703	0.16843	0.17766	0.18508	0.19584	0.20278
1.4	0.03007	0.05894	0.08561	0.10941	0.13003	0.14749	0.16199	0.17389	0.18357	0.19139	0.20278	0.21020
1.6	0.03058	0.05994	0.08709	0.11135	0.13241	0.15028	0.16515	0.17739	0.18737	0.19546	0.20731	0.21510
1.8	0.03090	0.06058	0.08804	0.11260	0.13395	0.15207	0.16720	0.17967	0.18986	0.19814	0.21032	0.21836
2.0	0.03111	0.06100	0.08867	0.11342	0.13496	0.15326	0.16856	0.18119	0.19152	0.19994	0.21235	0.22058
2.5	0.03138	0.06155	0.08948	0.11450	0.13628	0.15483	0.17036	0.18321	0.19375	0.20236	0.21512	0.22364
3.0	0.03150	0.06178	0.08982	0.11495	0.13684	0.15550	0.17113	0.18407	0.19470	0.20341	0.21633	0.22499
4.0	0.03158	0.06194	0.09007	0.11527	0.13724	0.15598	0.17168	0.18469	0.19540	0.20417	0.21722	0.22600
5.0	0.03160	0.06199	0.09014	0.11537	0.13737	0.15612	0.17185	0.18488	0.19561	0.20440	0.21749	0.22632
6.0	0.03161	0.06201	0.09017	0.11541	0.13741	0.15617	0.17191	0.18496	0.19569	0.20449	0.21760	0.22644
8.0	0.03162	0.06202	0.09018	0.11543	0.13744	0.15621	0.17195	0.18500	0.19574	0.20455	0.21767	0.22652
10.0	0.03162	0.06202	0.09019	0.11544	0.13745	0.15622	0.17196	0.18502	0.19576	0.20457	0.21769	0.22654
∞	0.03162	0.06202	0.09019	0.11544	0.13745	0.15623	0.17197	0.18502	0.19577	0.20458	0.21770	0.22656

The variation of I_3 with m and n is shown in figure-5.
The stress at any point below a rectangular loaded area can be found by Eq-6 or the figure.

Application of the method

$$p = q[I_{3(1)} + I_{3(2)} + I_{3(3)} + I_{3(4)}] \quad (8)$$

Where $I_{3(1)}$, $I_{3(2)}$, $I_{3(3)}$, and $I_{3(4)}$ are the values of I_3 for rectangles 1, 2, 3 and 4, respectively.



Increase of stress at any point below a rectangular loaded flexible area

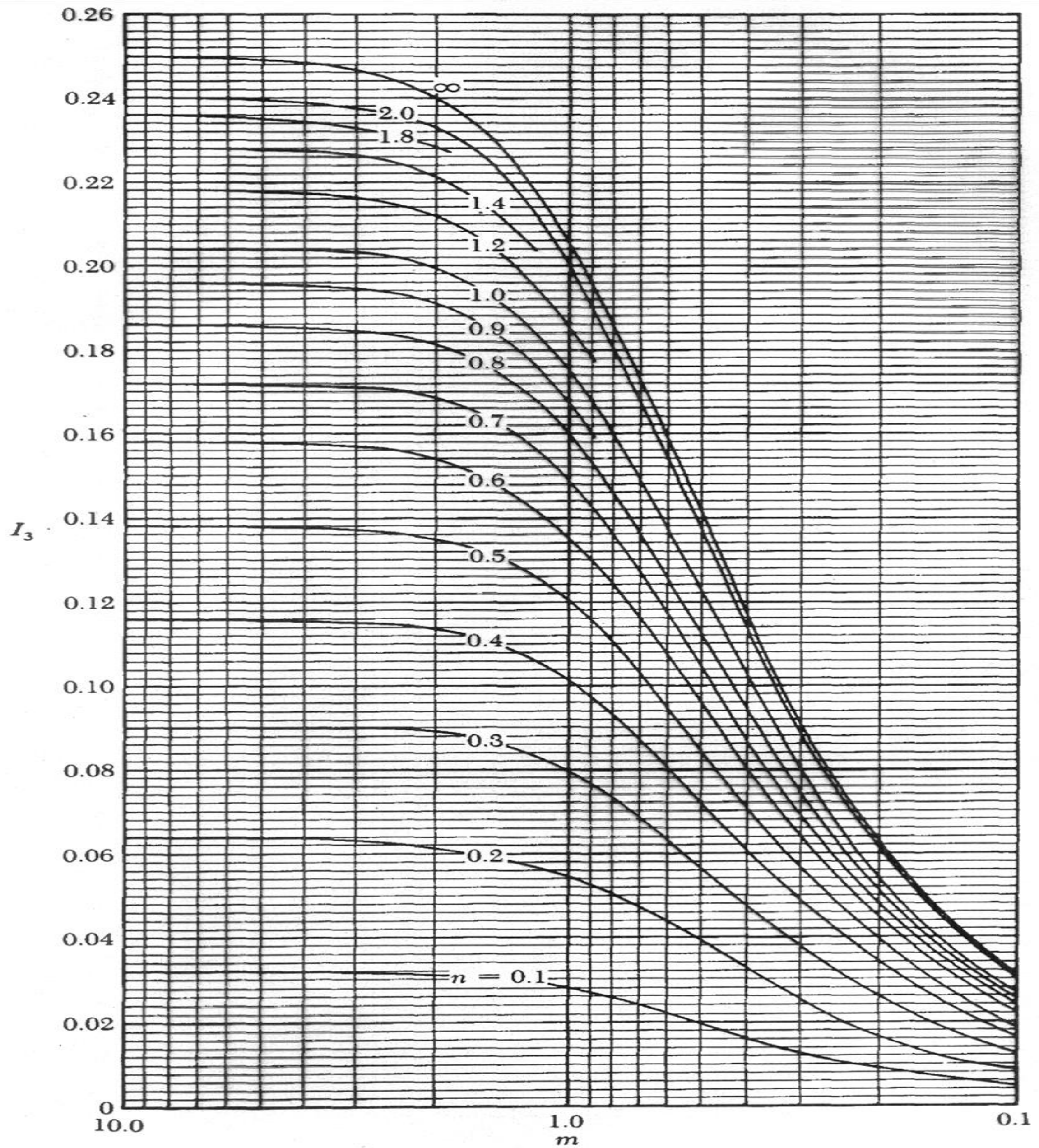


Fig: 5. Variation of I_3 with m and n

- **Fig: 7. shows the variation of σ_z/q below the CENTER of a rectangular loaded area with L/B = 1, 1.5, 2, and ∞, which has been calculated by using Eq-6.**

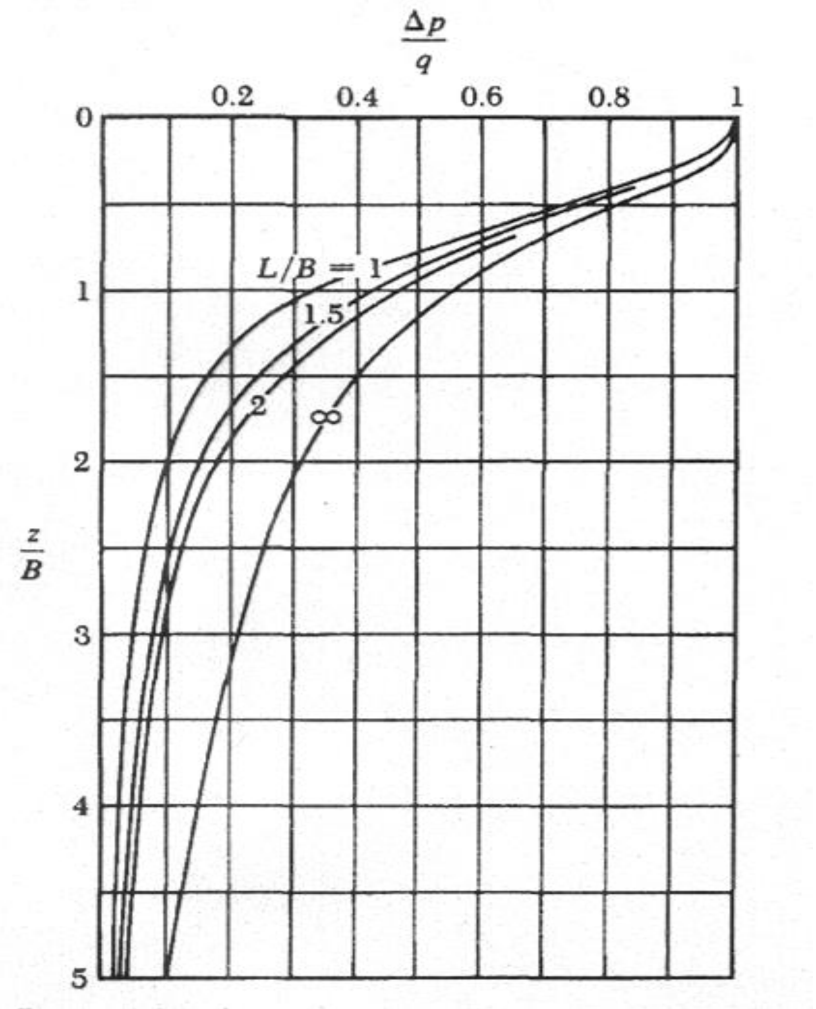


Figure-7: Stress increase under CENTER of a uniformly loaded rectangular flexible area

Average Vertical Stress Increase Due to a Rectangular Loaded Area

- The variation of vertical stress below the corner of a uniformly loaded rectangular area (using Eq., $\sigma_z = qI_3$) is shown in Fig: 8.
- For the settlement calculation of a compressible layer, it is always required to determine the average stress increase, σz (av), below the corner of a uniformly loaded area within the influence zone, most commonly with limits of z = 0 (from footing level) to z = H (generally = 2B) as shown in Fig-8. This can be calculated as follows:

$$\sigma_{z(av)} = \frac{1}{H}\int_{o}^{H} (qI_3)dz = qI_4$$

$$I_4 = f(m', n')$$

$$m' = \frac{B}{H}$$

$$n' = \frac{L}{H}$$

$$m' = \frac{L}{B}$$

$$n' = \frac{z}{b}$$

$$b = \frac{B}{2}$$

Table 5.3 Variation of I_c with m_1 and n_1

n_1	m_1 = 1	2	3	4	5	6	7	8	9	10
0.20	0.994	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
0.40	0.960	0.976	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977
0.60	0.892	0.932	0.936	0.936	0.937	0.937	0.937	0.937	0.937	0.937
0.80	0.800	0.870	0.878	0.880	0.881	0.881	0.881	0.881	0.881	0.881
1.00	0.701	0.800	0.814	0.817	0.818	0.818	0.818	0.818	0.818	0.818
1.20	0.606	0.727	0.748	0.753	0.754	0.755	0.755	0.755	0.755	0.755
1.40	0.522	0.658	0.685	0.692	0.694	0.695	0.695	0.696	0.696	0.696
1.60	0.449	0.593	0.627	0.636	0.639	0.640	0.641	0.641	0.641	0.642
1.80	0.388	0.534	0.573	0.585	0.590	0.591	0.592	0.592	0.593	0.593
2.00	0.336	0.481	0.525	0.540	0.545	0.547	0.548	0.549	0.549	0.549
3.00	0.179	0.293	0.348	0.373	0.384	0.389	0.392	0.393	0.394	0.395
4.00	0.108	0.190	0.241	0.269	0.285	0.293	0.298	0.301	0.302	0.303
5.00	0.072	0.131	0.174	0.202	0.219	0.229	0.236	0.240	0.242	0.244
6.00	0.051	0.095	0.130	0.155	0.172	0.184	0.192	0.197	0.200	0.202
7.00	0.038	0.072	0.100	0.122	0.139	0.150	0.158	0.164	0.168	0.171
8.00	0.029	0.056	0.079	0.098	0.113	0.125	0.133	0.139	0.144	0.147
9.00	0.023	0.045	0.064	0.081	0.094	0.105	0.113	0.119	0.124	0.128
10.00	0.019	0.037	0.053	0.067	0.079	0.089	0.097	0.103	0.108	0.112

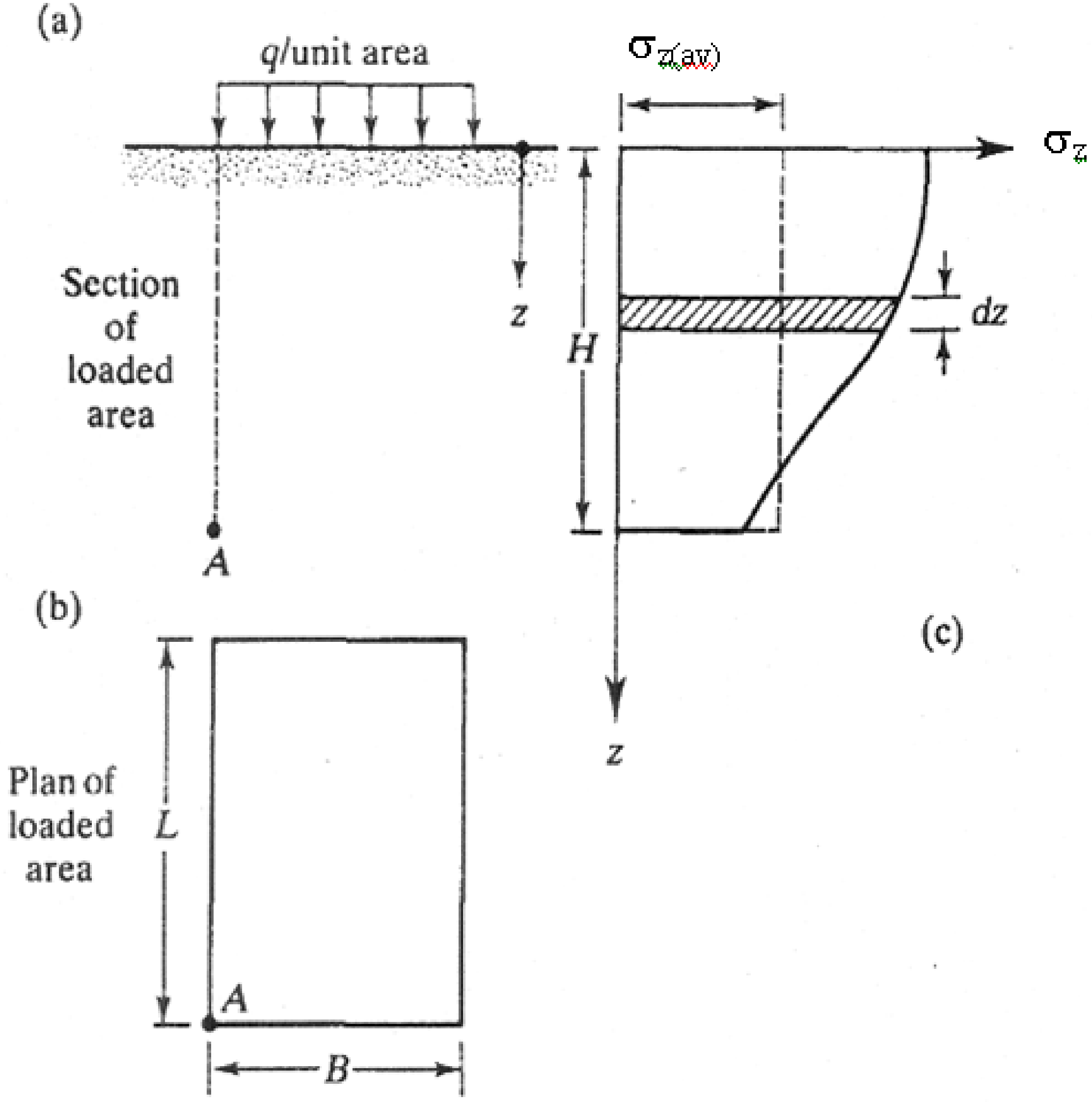


Figure 8- Average vertical stress increase due to a rectangular loaded flexible area

Westergaard's Method:

- Boussinesq's equation has been developed for homogenous soil.
- Sedimentary soil deposits are generally stratified and anisotropic.
- They commonly have alternating layers of sandy and silt/clay soil. For such conditions Westergaard's (1938) presented an equation for subsurface stresses.

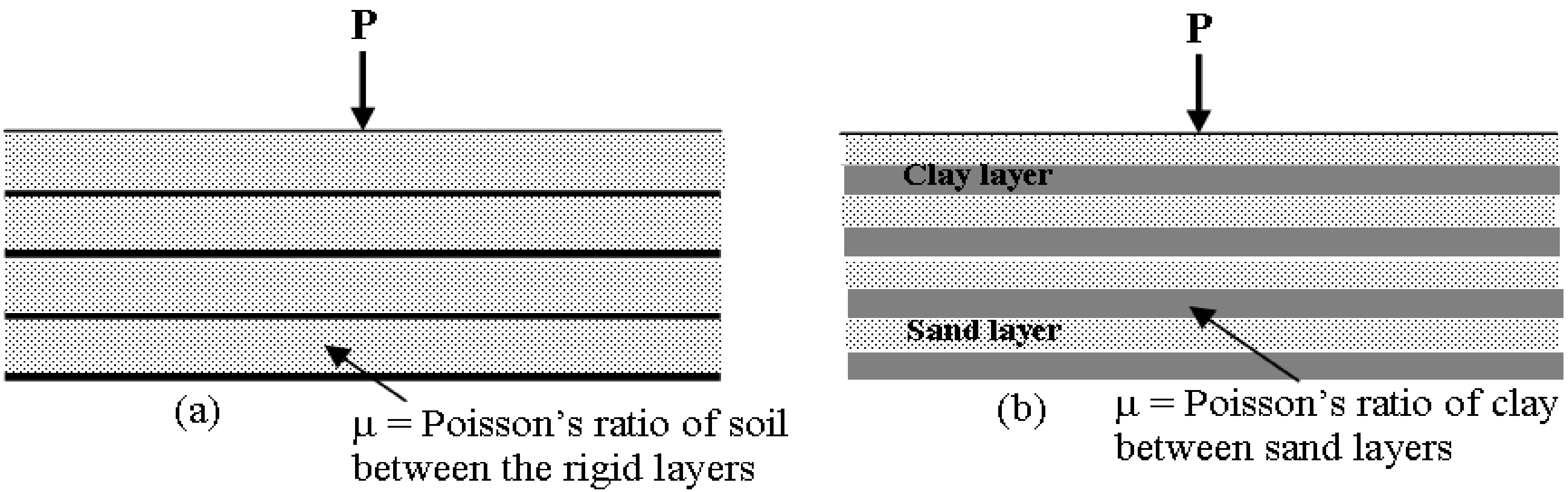


Fig: (a) Assumed soil model for development of equation
(b) Real soil model for application of equation

$$\sigma_z = \frac{p\sqrt{(1-2\mu)/(2-2\mu)}}{2\pi z^2[1-2\mu)/(2-2\mu)+(r/z)^2]^{3/2}} \qquad \textbf{(1)}$$

$$\sigma_z = \frac{p\sqrt{(a)}}{2\pi z^2[a+(r/z)^2]^{3/2}}$$

- Where
- σ_z = vertical stress at depth z
- p = concentrated load
- μ = Poisson's ratio (ratio of the strain in a direction normal to an applied stress to the strain parallel to the applied stress)
- z = depth
- r = horizontal distance from point of application of p to point at which q is desired
- a = (1-2μ)/(2-2μ)
- If Poisson's ratio is taken to be zero, Eq. (1) changes to

$$\sigma_z = \frac{p}{\pi z^2[(1+2(r/z)^2]^{3/2}}$$

Comparison Of Westergaard's And Boussinesq's Equations

Sr. No.	WESTERGAARD'S EQUATION	BOUSSINESQ'S EQUATION
1	The equation gives the stress as a function of depth (z) below the point of application of load and the horizontal distance (r) from the load axis.	The equation also gives the stress as a function of depth (z) below the point of application of load and the horizontal distance (r) from the load axis.

Comparison Of Westergaard's And Boussinesq's Equations

Sr. No.	WESTERGAARD'S EQUATION	BOUSSINESQ'S EQUATION
3	Westergaard's equation is based on alternating thin layers of elastic material between layers of inelastic material.	Boussinesq's equation is developed for homogenous material.
4	Westergaard's equation includes poisson's ratio (μ).	For Boussinesq's equation (μ) is taken as zero.
5	Westergaard's equation is not widely used.	For most practical problems in geotechnical engineering, Boussinesq's equation is preferred.

APPROXIMATE METHOD

- The method assumes that the area affected by applied load gradually increases with depth.
- Although the method is not accurate, yet it is used to estimate the stress for preliminary design.
- Some of the assumptions made for the distribution of load with depth are given below:
- The area of stress spreads with 2:1 slope as shown in the figure. 3
- The area of stress spreads within lines making an angle of 30^0 with the vertical

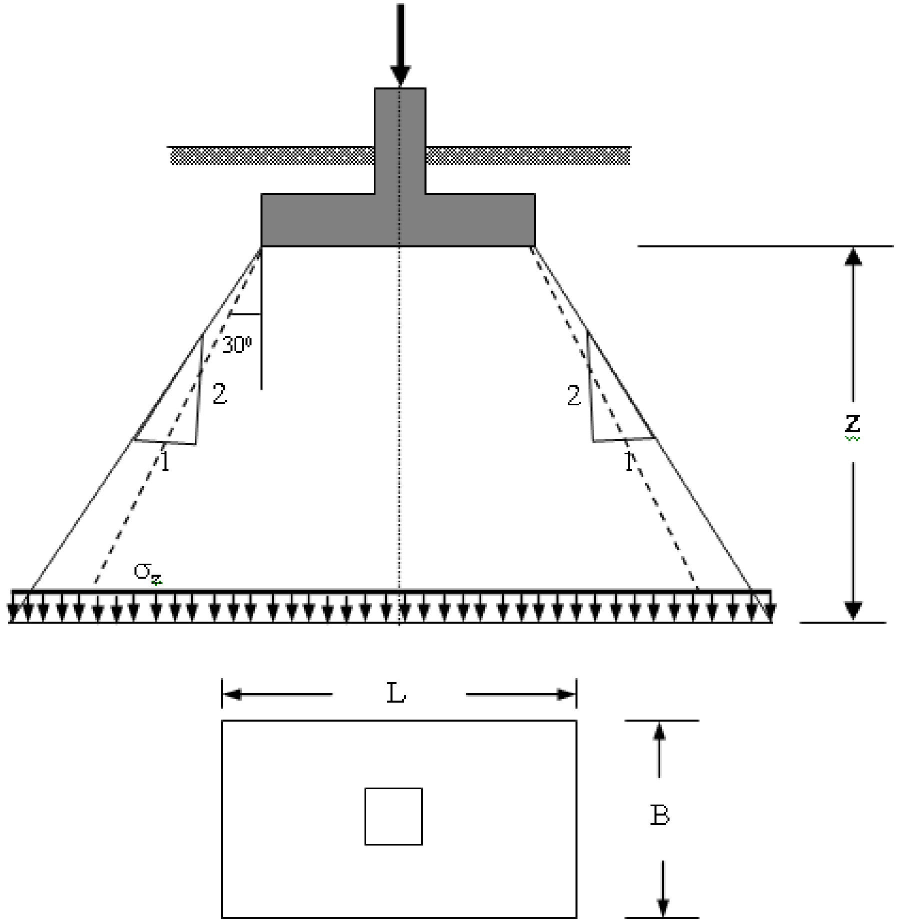
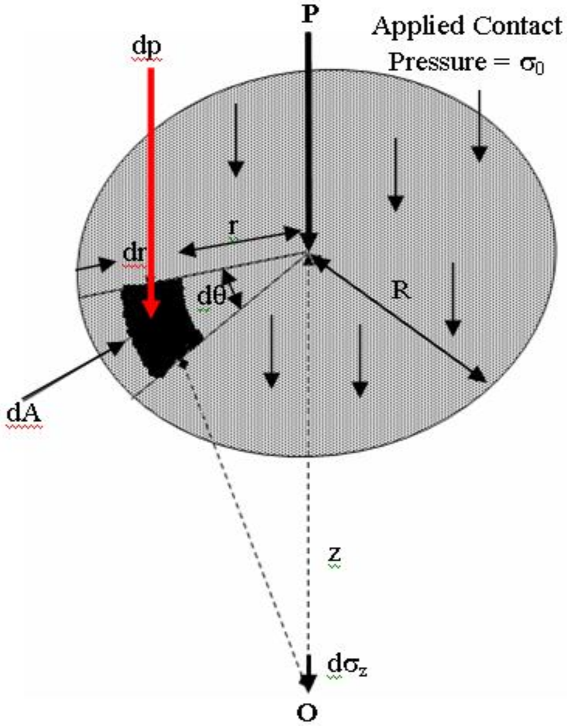


Fig: 11 Stress distribution with 2:1 slope,
(dotted lines indicate distribution of stress at 30^0 angle)

NEWMARK'S INFLUENCE CHART

- The Boussinesq's and Westergaard's theories have been developed for point loads.
- Newmark developed chart based on Boussinesq's equation for the determination of stress under the centre of a circular loaded area.
- Consider a circular foundation subjected to loading as in the figure.

Foundation area = A,
Radius of the area = R,
Uniformly distributed applied load per unit area.= σ_0



The total load P can be written as,

$$P = \int_0^A \sigma_0 d_A$$

NEWMARK'S INFLUENCE CHART

Consider an elementary area d_A, within the loaded area as shown in figure.

The load d_p on this elementary area can be written as, $d_p = \sigma_0 \, d_A$

$= \sigma_0 \, (r \, d\theta \, d_r)$ Where $d_A = r \, d\theta \, d_r$

This load d_p can be considered as a point load. The stress at point O caused by this load, can be determined by using Boussinesq's eq.

$$d\sigma_z = \frac{3 \, d_p}{2\pi z^2 \left[1 + \left(\frac{r}{z} \right)^2 \right]^{5/2}}$$

Putting the value of d_p in the above equation

$$d\sigma_z = \frac{3(\sigma_0\, rd\theta\, dr)}{2\pi z^2\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}$$

Thus the total stress caused by the entire loaded area A can be obtained by integration of the above equation, or

$$d\sigma_z = \frac{3(\sigma_0\, rd\theta\, dr)}{2\pi z^2\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}$$

Where σ_0 = load per unit area on the foundation or the contact stress.

$$\sigma_z = \frac{3\sigma_0}{2\pi}\int_0^{2\pi}\int_0^{R} \frac{z^3}{(z^2+r^2)^{5/2}}\, r\, d\theta\, dr$$

After integration and simplifying we get as follows

$$\sigma_z = \sigma_0 \left\{ 1 - \left[\frac{1}{1+(R/z)^2} \right]^{3/2} \right\}$$

$$\frac{\sigma_z}{\sigma_0} = \left\{ 1 - \left[\frac{1}{1+(R/z)^2} \right]^{3/2} \right\}$$

The above equation can be rewritten as follows:

$$\frac{R}{z} = \left[\left(1 - \frac{\sigma_z}{\sigma_0} \right)^{-2/3} - 1 \right]^{1/2}$$

Table: Values of R/z for various values of σ_z / σ_0

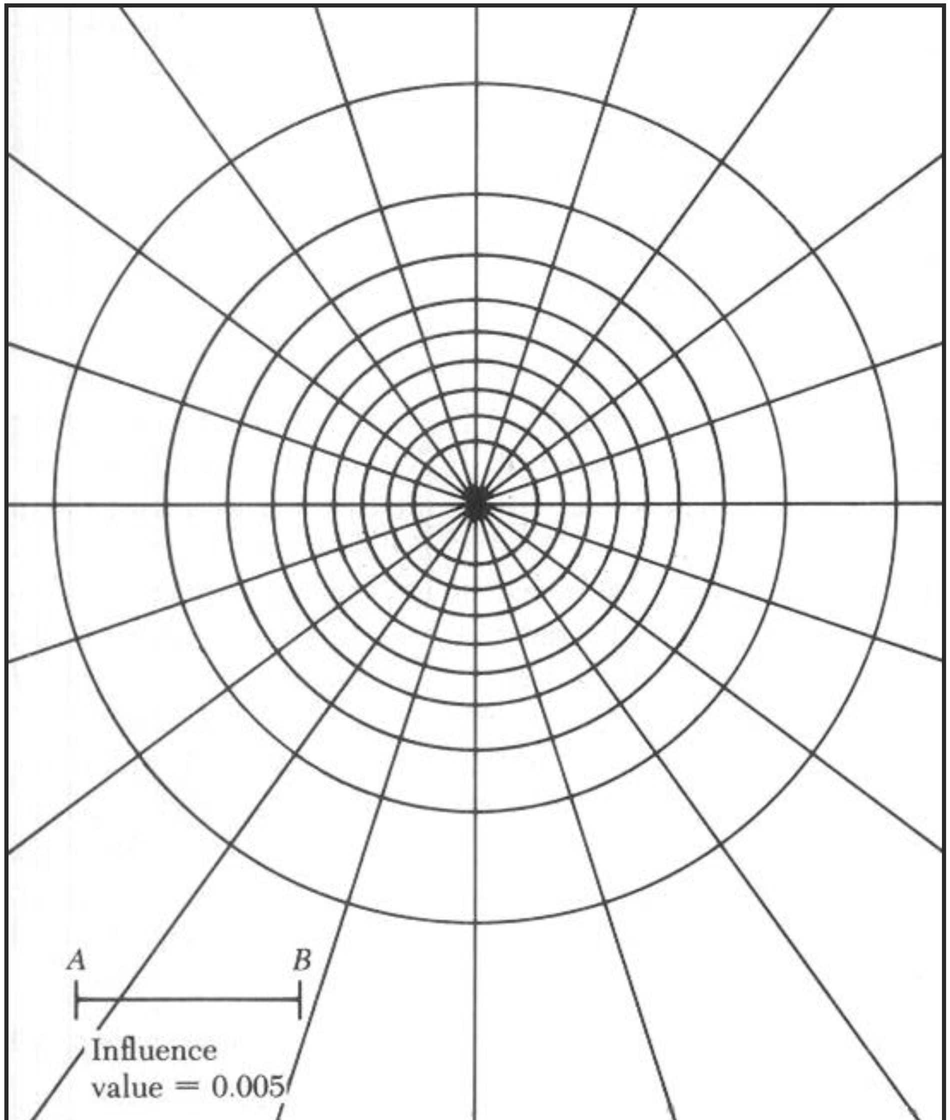
σ_z / σ_0	R/z
0	0
0.1	0.2698
0.2	0.4005
0.3	0.5181
0.4	0.6370
0.5	0.7664
0.6	0.9174
0.7	1.1097
0.8	1.3871
0.9	1.9084
1.0	∞

- Concentric circles can be drawn using non-dimensional values of R/z as shown in figure below.
- These circles have been divided by suitable number of equally spaced radial lines to form orthogonal mesh.
- This is referred as the Newmark's chart.

$$Influence\,value = IV = \frac{1}{\text{number of meshes on the chart}}$$

$$Influence\,value = IV = \frac{1}{\text{number of circles x number of rays}}$$

- For the Influence Chart shown in the figure,
- Iv = 1/10x20 = 1/200 = 0.005



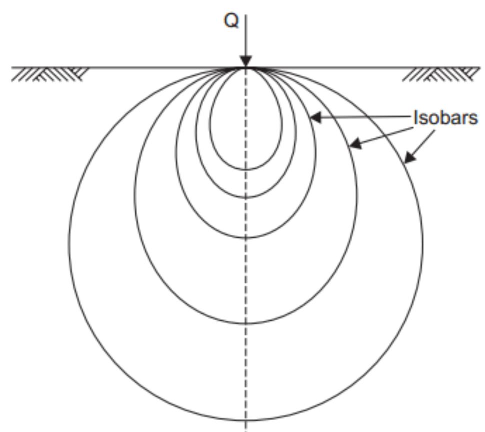
$$\sigma_z = (I_v)(n)(\sigma_0)$$

Figure: Newmark's influence chart for computing vertical pressure.

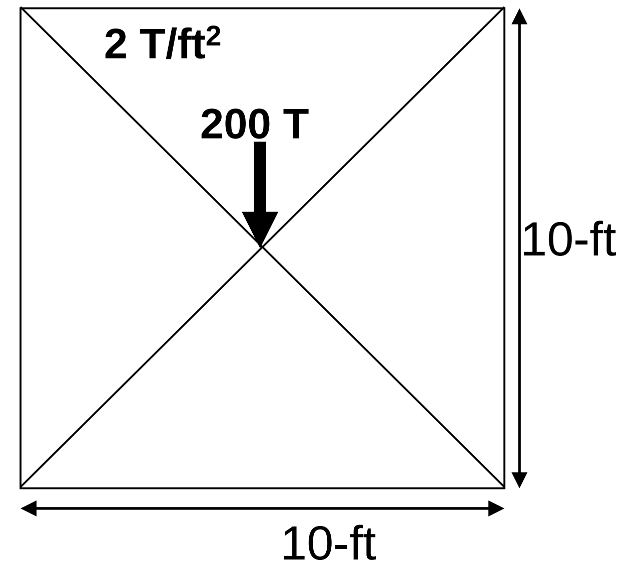
Pressure Bulb

- A pressure bulb is stress isobar that is line connecting all the points of equal Vertical stress..
- For a given load infinite number of isobar can be drawn.
- Seat of settlement (Active Zone)
- Terzaghi recommended that the vertical stress is considered negligible when smaller than 20% of the applied contact pressure.
- 80% of settlement takes place within the pressure bulb.
- Region between the pressure bulb is known as Seat of settlement.

$$\sigma z = \frac{Q}{Z^2} k_b$$



Find the intensity of vertical pressure at 10’ of a square footing having dimensions 10’x10’ and a load of 2 T/ft^2 act on the footing. Find vertical pressure at centre of footing?



Solution.

1. Boussinesq Equation

We have to find vertical pressure at center of footing of dimensions 10’x10’ at 10’ depth.
i.e z = 10 ft
we consider the footing as a whole i.e.

$$P = \sigma_{app} \times A_{footing} \quad = 2 \times 100 = 200 \text{ T}$$

$$r = 0$$

according to Boussinesq Equation

$$\sigma_z = \frac{P}{Z^2} \times \frac{3}{2\pi\left(1 + \frac{r^2}{2^2}\right)^{5/2}}$$

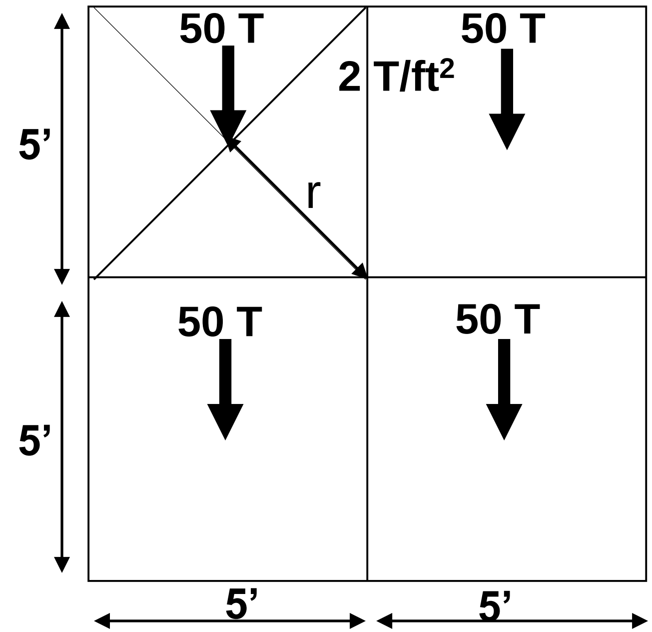
By putting values

$$\sigma_Z = \frac{200}{10^2} \times \frac{3}{2\pi(1)^{5/2}} = 0.955 \quad \text{T/ft}^2$$

2. Distributing the load on 4-areas (each 5’ x 5’)

1.a. Boussinesq’s Equation

Now we have to find vertical stress at center of footing of dimensions 10’x10’ at 10’ depth.
i.e z = 10 ft
but considering the 4 loads as shown, it is more close to realistic situation



$$P_1 = \sigma_{app} \times A_{footing} \quad = 2 \times 5 \times 5 = 50\ \mathrm{T}$$

$$r = \sqrt{2.5^2 \times 2.5^2} = 3.5355$$

according to the Equation stress due to one load $\sigma_z = \frac{P}{Z^2} \times \frac{3}{2\pi\left(1 + \frac{r^2}{Z^2}\right)^{5/2}}$

By putting values

$$\sigma_Z = \frac{50}{10^2} \times \frac{3}{2\pi(1.125)^{5/2}} = 0.178 \quad \mathrm{T/ft}^2$$

Stress due to four loads $= 0.178 \times 4 = 0.712\ \ \mathrm{T/ft}^2$

The true stress will be determined by considering 100 loads each of 2-tons applied on 1-square foot area.

3. Westergaard's Equation

$$\sigma_z = \frac{P}{\pi Z^2 \left[1 + 2(r/z)^2\right]^{3/2}}$$

where

$$P = \sigma \times A = 2 \times 10^2 = 200\ \text{T} \qquad Z = 10\ ft \qquad r = 0$$

By putting values

$$\sigma_Z = \frac{200}{\pi 10^2 \left[1 + 2(0)^2\right]^{3/2}} = 0.637\ \text{T/ft}^2$$

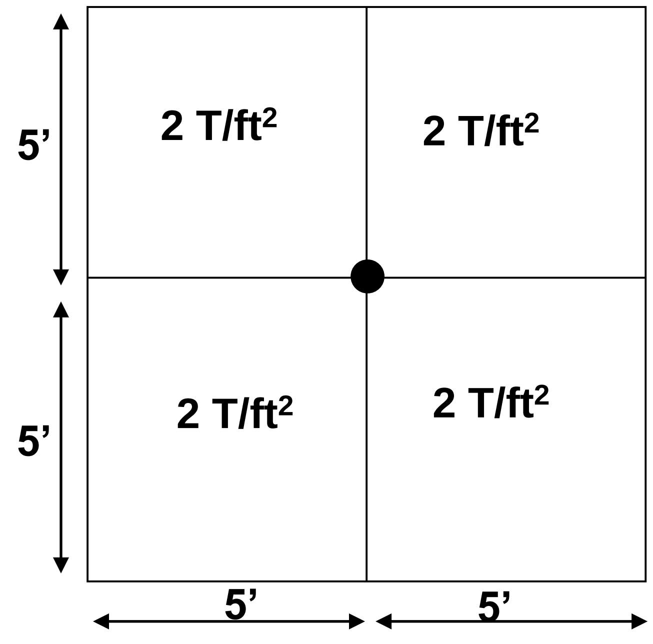
4. Stress due to uniformly loaded rectangular area.

The method is applicable to find stress under the corner of a loaded area.

Footing is divided into 4 small square footings of 5’ x 5’.

The centre of the actual footing under which the stress is required becomes the corner of the area of 5 x5.

The relation for finding stress at centre of given footing is given by



$$\sigma_z = \int d\sigma_2 = \int_{r=0}^{B}\int_{x=0}^{L} \frac{3qz^3(dx.dy)}{2\pi(x^2+y^2+z^2)^{5/2}} = qI_3$$

where

$$I_3 = \frac{1}{4x}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left[\frac{m^2+n^2+1}{m^2+n^2+1}\right] + \tan^{-1}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\right]\right]$$

$$m = \frac{B}{Z} \quad \text{and} \quad \mathrm{n} = \frac{\mathrm{L}}{\mathrm{Z}} \quad \text{or} \quad \sigma_Z = q\left[I_{3(1)} + I_{3(2)} + I_{3(3)} + I_{3(4)}\right] = 4qI_3$$

As given footing has same length L and B. So $m = n = \frac{5}{10} = 0.5$

I_3 can be determined from the above equation, but it is very laborious,
It can be easily found by the graph or table on next pages

Therefore from graph of I_3 with m and n, we have $I_3 = 0.084$

$$\Rightarrow \quad \sigma_{10'} = 4\times 2\times 0.084 = 0.672 \text{ T/ft}^2$$

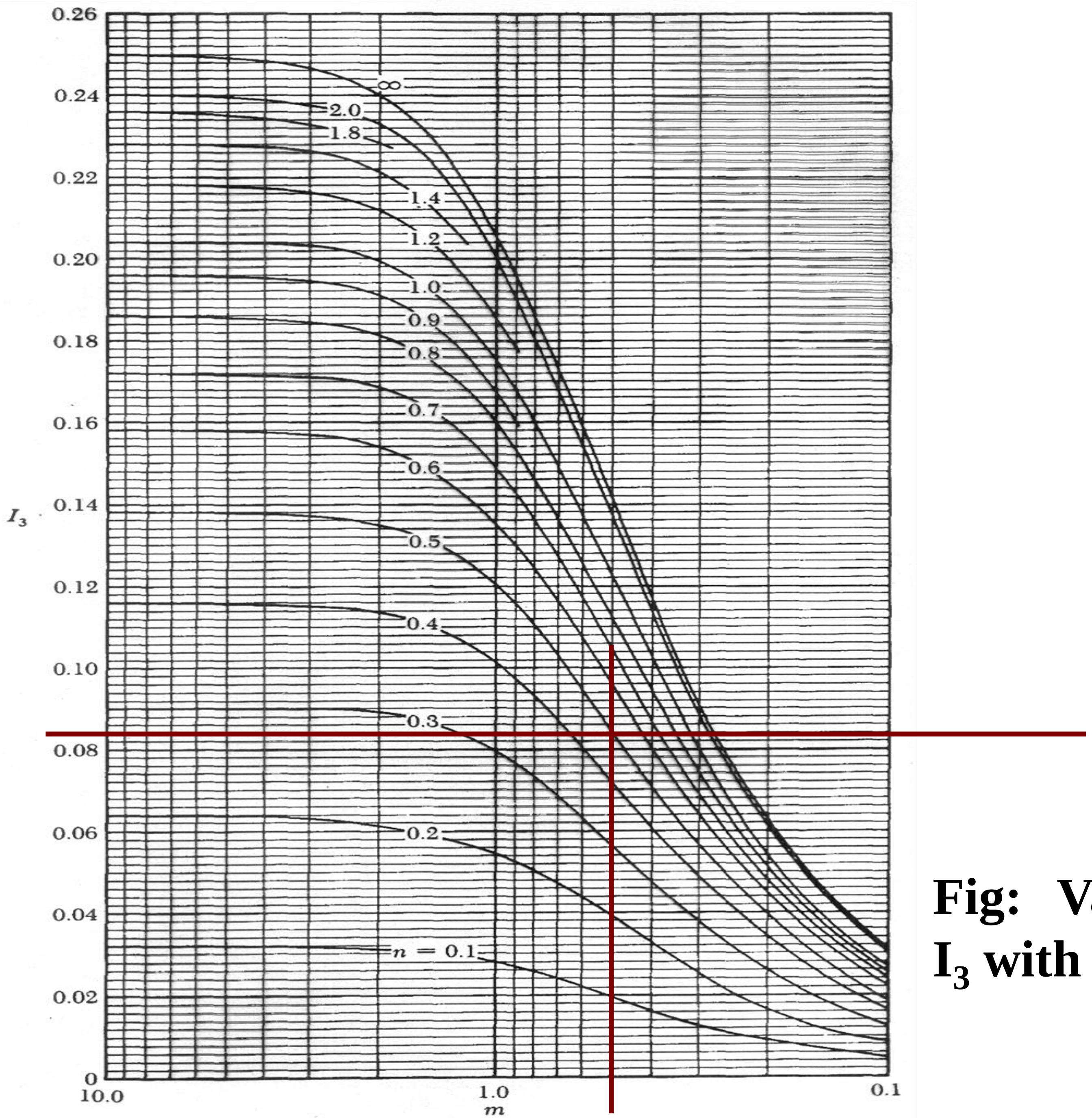


Fig: Variation of I_3 with m and n

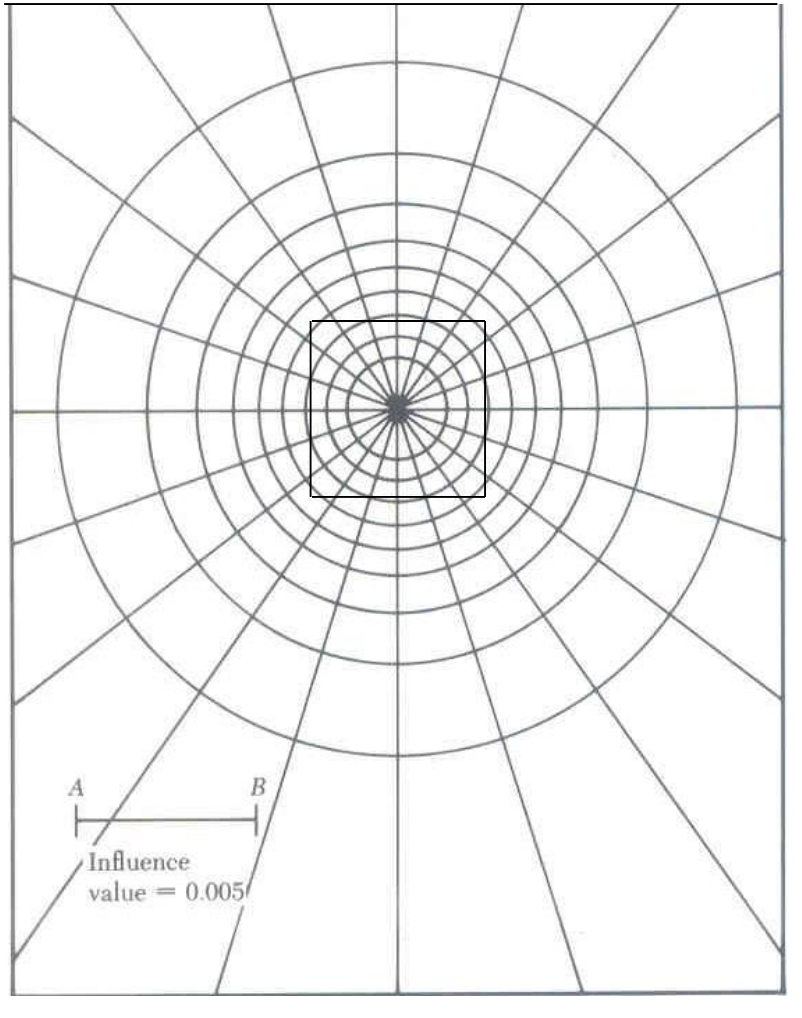
n	m = 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0
0.1	0.0047	0.0092	0.0132	0.0168	0.0198	0.0222	0.0242	0.0258	0.0270	0.0279	0.0293	0.0301	0.0306	0.0309	0.0311	0.0314	0.0315	0.0316	0.0316	0.0316
0.2	0.0092	0.0179	0.0259	0.0328	0.0387	0.0435	0.0474	0.0504	0.0528	0.0547	0.0573	0.0589	0.0599	0.0606	0.0610	0.0616	0.0618	0.0619	0.0620	0.0620
0.3	0.0132	0.0259	0.0374	0.0474	0.0559	0.0629	0.0686	0.0731	0.0766	0.0794	0.0832	0.0856	0.0871	0.0880	0.0887	0.0895	0.0898	0.0901	0.0901	0.0902
0.4	0.0168	0.0328	0.0474	0.0602	0.0711	0.0801	0.0873	0.0931	0.0977	0.1013	0.1063	0.1094	0.1114	0.1126	0.1134	0.1145	0.1150	0.1153	0.1154	0.1154
0.5	0.0198	0.0387	0.0559	0.0711	0.0840	0.0947	0.1034	0.1104	0.1158	0.1202	0.1263	0.1300	0.1324	0.1340	0.1350	0.1363	0.1368	0.1372	0.1374	0.1374
0.6	0.0222	0.0435	0.0629	0.0801	0.0947	0.1069	0.1168	0.1247	0.1311	0.1361	0.1431	0.1475	0.1503	0.1521	0.1533	0.1548	0.1555	0.1560	0.1561	0.1562
0.7	0.0242	0.0474	0.0686	0.0873	0.1034	0.1169	0.1277	0.1365	0.1436	0.1491	0.1570	0.1620	0.1652	0.1672	0.1686	0.1704	0.1711	0.1717	0.1719	0.1719
0.8	0.0258	0.0504	0.0731	0.0931	0.1104	0.1247	0.1365	0.1461	0.1537	0.1598	0.1684	0.1739	0.1774	0.1797	0.1812	0.1832	0.1841	0.1847	0.1849	0.1850
0.9	0.0270	0.0528	0.0766	0.0977	0.1158	0.1311	0.1436	0.1537	0.1619	0.1684	0.1777	0.1836	0.1874	0.1899	0.1915	0.1938	0.1947	0.1954	0.1956	0.1957
1.0	0.0279	0.0547	0.0794	0.1013	0.1202	0.1361	0.1491	0.1598	0.1684	0.1752	0.1851	0.1914	0.1955	0.1981	0.1999	0.2024	0.2034	0.2042	0.2044	0.2045
1.2	0.0293	0.0573	0.0832	0.1063	0.1263	0.1431	0.1570	0.1684	0.1777	0.1851	0.1958	0.2028	0.2073	0.2103	0.2124	0.2151	0.2163	0.2172	0.2175	0.2176
1.4	0.0301	0.0589	0.0856	0.1094	0.1300	0.1475	0.1620	0.1739	0.1836	0.1914	0.2028	0.2102	0.2151	0.2184	0.2206	0.2236	0.2250	0.2260	0.2263	0.2264
1.6	0.0306	0.0599	0.0871	0.1114	0.1324	0.1503	0.1652	0.1774	0.1874	0.1955	0.2073	0.2151	0.2203	0.2237	0.2261	0.2294	0.2309	0.2320	0.2323	0.2325
1.8	0.0309	0.0606	0.0880	0.1126	0.1340	0.1521	0.1672	0.1797	0.1899	0.1981	0.2103	0.2183	0.2237	0.2274	0.2299	0.2333	0.2350	0.2362	0.2366	0.2367
2.0	0.0311	0.0610	0.0887	0.1134	0.1350	0.1533	0.1686	0.1812	0.1915	0.1999	0.2124	0.2206	0.2261	0.2299	0.2325	0.2361	0.2378	0.2391	0.2395	0.2397
2.5	0.0314	0.0616	0.0895	0.1145	0.1363	0.1548	0.1704	0.1832	0.1938	0.2024	0.2151	0.2236	0.2294	0.2333	0.2361	0.2401	0.2420	0.2434	0.2439	0.2441
3.0	0.0315	0.0618	0.0898	0.1150	0.1368	0.1555	0.1711	0.1841	0.1947	0.2034	0.2163	0.2250	0.2309	0.2350	0.2378	0.2420	0.2439	0.2455	0.2461	0.2463
4.0	0.0316	0.0619	0.0901	0.1153	0.1372	0.1560	0.1717	0.1847	0.1954	0.2042	0.2172	0.2260	0.2320	0.2362	0.2391	0.2434	0.2455	0.2472	0.2479	0.2481
5.0	0.0316	0.0620	0.0901	0.1154	0.1374	0.1561	0.1719	0.1849	0.1956	0.2044	0.2175	0.2263	0.2324	0.2366	0.2395	0.2439	0.2460	0.2479	0.2486	0.2489
6.0	0.0316	0.0620	0.0902	0.1154	0.1374	0.1562	0.1719	0.1850	0.1957	0.2045	0.2176	0.2264	0.2325	0.2367	0.2397	0.2441	0.2463	0.2482	0.2489	0.2492

5. NEWMARK'S INFLUENCE CHART.

$$\text{Influence Value} = \text{IV} = \frac{1}{\text{No. of circles} \times \text{No. of Rays}} = \frac{1}{10 \times 20} = 0.005$$

$$No.\ \text{of meshes for given footing} = \text{n} = 66$$

$$\sigma_z = (1V)(n)(\sigma_0) = 0.005 \times 66 \times 2 = 0.66\ \text{T/ft}^2$$



6. APPROXIMATE METHOD

$$\sigma_z = \frac{p}{(B+z)(L+z)}$$

$$\sigma_z = \frac{200}{(10+10)(10+10)} = 0.5T/\ ft^2$$

Result:-

1- From Boussinesq Method $\sigma_z = 0.995\ \text{T/ft}^2$

2- From Boussinesq Method, using 4 loads $\sigma_z = 0.712\ \text{T/ft}^2$

3- From Westergaards Equation $\sigma_z = 0.637\ \text{T/ft}^2$

3- From influence factor I_3, $\sigma_z = 0.672\ \text{T/ft}^2$

4- From Approximate Analysis $\sigma_z = 0.50\ \text{T/ft}^2$

5- From Newmarks Influence Chart $\sigma_z = 0.66\ \text{T/ft}^2$

THANK YOU!