STRESS DISTRIBUTION

9.1 INTRODUCTION

Stresses within a soil mass are produced due to the following two types of loading:

(i) Stresses due to self weight of a soil mass are usually known as overburden stresses or geostatic stresses or gravitational stresses.

$$\sigma_z = \gamma z \qquad 9.1$$

Where,

σ_z = total vertical stress (overburden pressure) at depth, z below the surface, assuming pore pressure is zero

γ = unit weight of the soil mass;

If pore pressure is present,

$$\sigma_z = \bar{\sigma}_z + u \quad \text{or}$$

$$\sigma_z = \gamma' Z + u \qquad 9.2$$

Where,

$\bar{\sigma}_z$ = intergranular stress or effective stress

u = pore pressure or neutral pressure i.e. the stress shared by pore fluid.

γ' = buoyant weight or submerged weight of the soil.

(ii) Stresses caused by external loading such as due to structural loads.

These stresses may be shear stresses, lateral stresses or vertical compressive stresses. In this chapter, we shall consider only the vertical compressive stresses induced due to external load, σ_v. The knowledge of σ_v is essential for computation of settlement of structures.

9.2 METHODS OF ESTIMATING σ_v IN SOILS

There are basically two methods:

(a) Approximate Methods, and

(b) Methods based on the theory of elasticity.

Approximate Method

A simple empirical method is to assume load spread along a slope of 2:1 (V:H). This is based on the assumption that the area over which the load acts increases in a systematic way with depth. As the same vertical force is spread over a wider area, the intensity of load decreases with depth.

Fig. 9.1 (a,b) represent stresses under strip footing and a rectangular footing respectively, by this method.

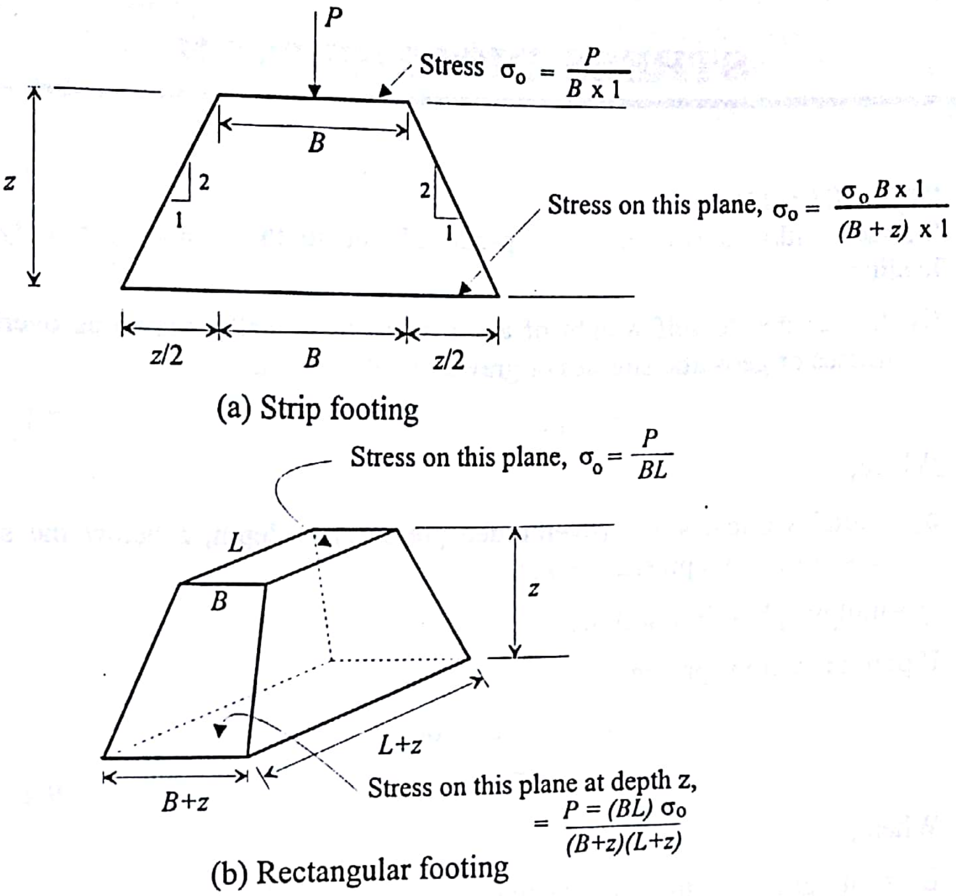


Figure 9.1 The 2:1 approximation for the distribution of vertical stress with depth.

For strip footing, $\sigma_z = \dfrac{\sigma_o(B \times 1)}{(B+z)\times 1}$ 9.3

For rectangular footing, $\sigma_z = \dfrac{\sigma_o(BL)}{(B+z)(L+z)}$ 9.4

Where σ_o = the contact pressure immediately underneath the footing.

Methods Based on Theory of Elasticity
It is a very tedious matter to obtain the elastic solution for a given loading and set of boundary conditions. In this chapter we are not concerned with how to obtain solutions but rather with how to use these solutions and as such several graphical solutions are included.

(i) Boussinesq (1885) Method

Assumption

(a) The soil is elastic, homogeneous material

(b) The soil is isotropic

(c) Load is a single point load applied to a point on the horizontal boundary.

(d) The soil mass is a semi-infinite medium which extends infinitely in all directions from a level surface.

(e) The soil is weightless

According to Boussinesq (1885), the value of the vertical stress, at any depth z below the surface is given by:

$$\sigma_z = \left(\frac{3Q}{2\pi z^2}\right)\left(\frac{1}{\left[1+(r/z)^2\right]^{5/2}}\right)$$

or

$$\sigma_z = \left(\frac{Q}{z^2}\right)\left(\frac{3/2\pi}{\left[1+(r/z)^2\right]^{5/2}}\right) \qquad 9.5$$

$$= \frac{Q}{z^2} K_B \qquad 9.6$$

$$\left\{\text{using } K_B = \left(\frac{3/2\pi}{\left[1+(r/z)^2\right]^{5/2}}\right)\right\}$$

Where,

Q = point load

z = depth below ground surface to the point where σ_z is desired.

r = horizontal distance from point load to the point where σ_z is desired (see Fig. 9.2a)

K_B = Boussinesq's co-efficient of stress. (see Fig. 9.2b)

Note: Equations 9.5 and 9.6 are independent of the material's properties.

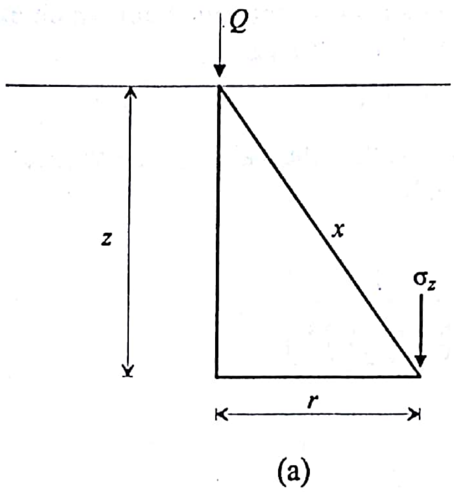
For line load (force per unit length), equation 9.6 may be integrated and the value of stress in this case is given by:

$$\sigma_z = \frac{2Q}{\pi} \cdot \frac{z^3}{x^4} \qquad 9.7$$

Where,

Q = line load i.e. load /unit length.

$x = (r^2 + z^2)^{1/2}$ (see Fig. 9.2a).



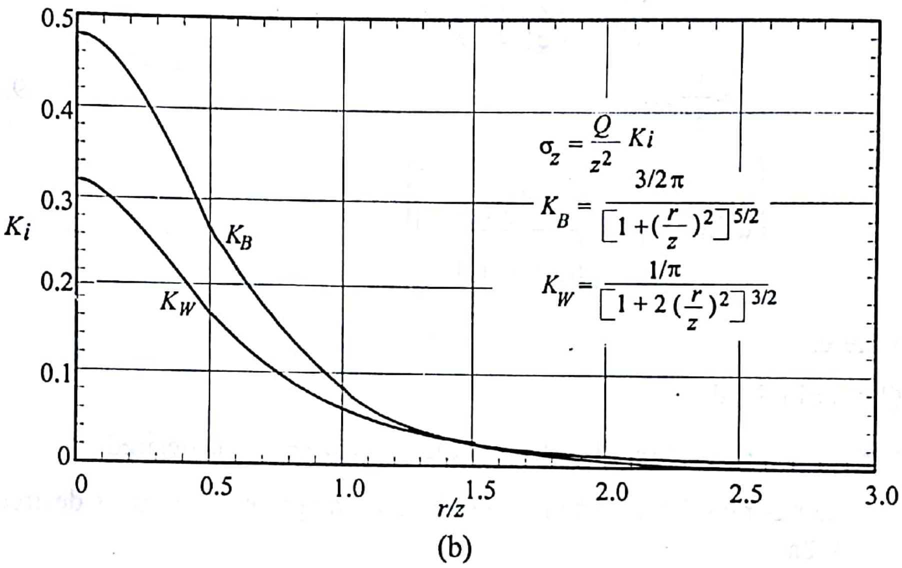


Figure 9.2 (a) Definition of terms used in Eq. 9.6 and 9.7; (b) relationship between K_B, K_W, and r/z for a point load (after Taylor, 1948)

In 1935, Newmark integrated the Boussinesq's point load equation and presented following equation for the vertical stress under the corner of a uniformly loaded rectangular area:

$$\sigma_z = \sigma_o \frac{1}{4\pi}\left[\frac{2mn(m^2+n^2)^{1/2}}{m^2+n^2+1+m^2n^2} \times \frac{(m^2+n^2+2)}{(m^2+n^2+1)} + \tan^{-1}\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+1-m^2n^2}\right] \quad 9.8$$

Where,

σ_o = contact stress underneath the loaded area.

$m = x/z$; $n = y/z$, and

x, y = dimensions of the rectangular area (length and width)

The parameters n, and m are interchangeable. Equation 9.7 may be reduced to:

$$\sigma_z = \sigma_o I \quad 9.9$$

Where,

I = stress influence factor which depends on n and m.

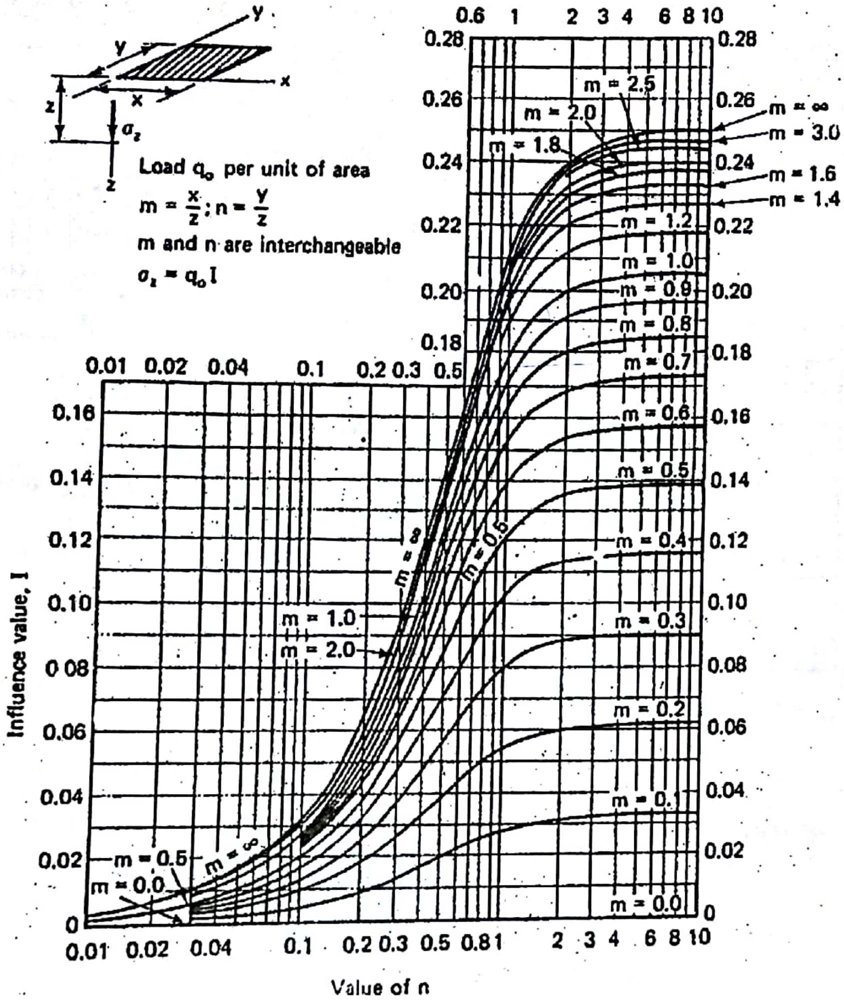


Figure 9.3 Influence value for vertical stress under corner of a uniformly loaded rectangular area (after US Navy, 1971)

Values of I are shown in Fig. 9.3 for various values of n and m and may be used for stress computation. US Navy (1971) prepared graphical charts for various types of loading by integrating the basic equation of Boussinesq. These charts are:

(i) Fig. 9.4 for stress variation underneath a uniformly loading circular area.

(ii) Fig. 9.5 represents the stress variations underneath a long embankment.

(iii) Fig. 9.6 for stresses underneath the corner of a triangular load.

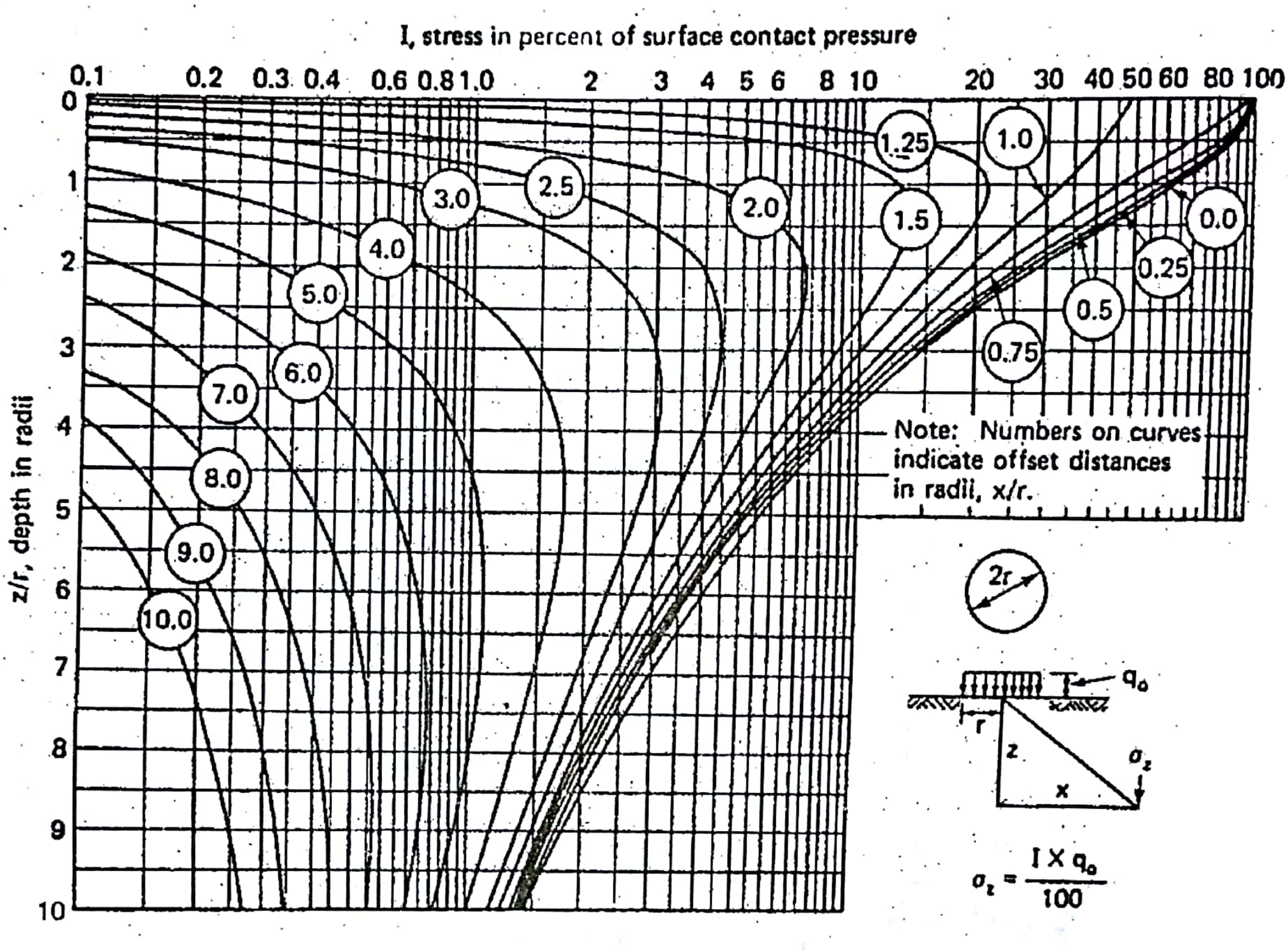


Figure 9.4 Influence values, expressed in percentage of surface contact pressure, q_o, for vertical stress under uniformly loaded circular area (after Foster and Ahlvin, 1954, as cited by US Navy, 1971

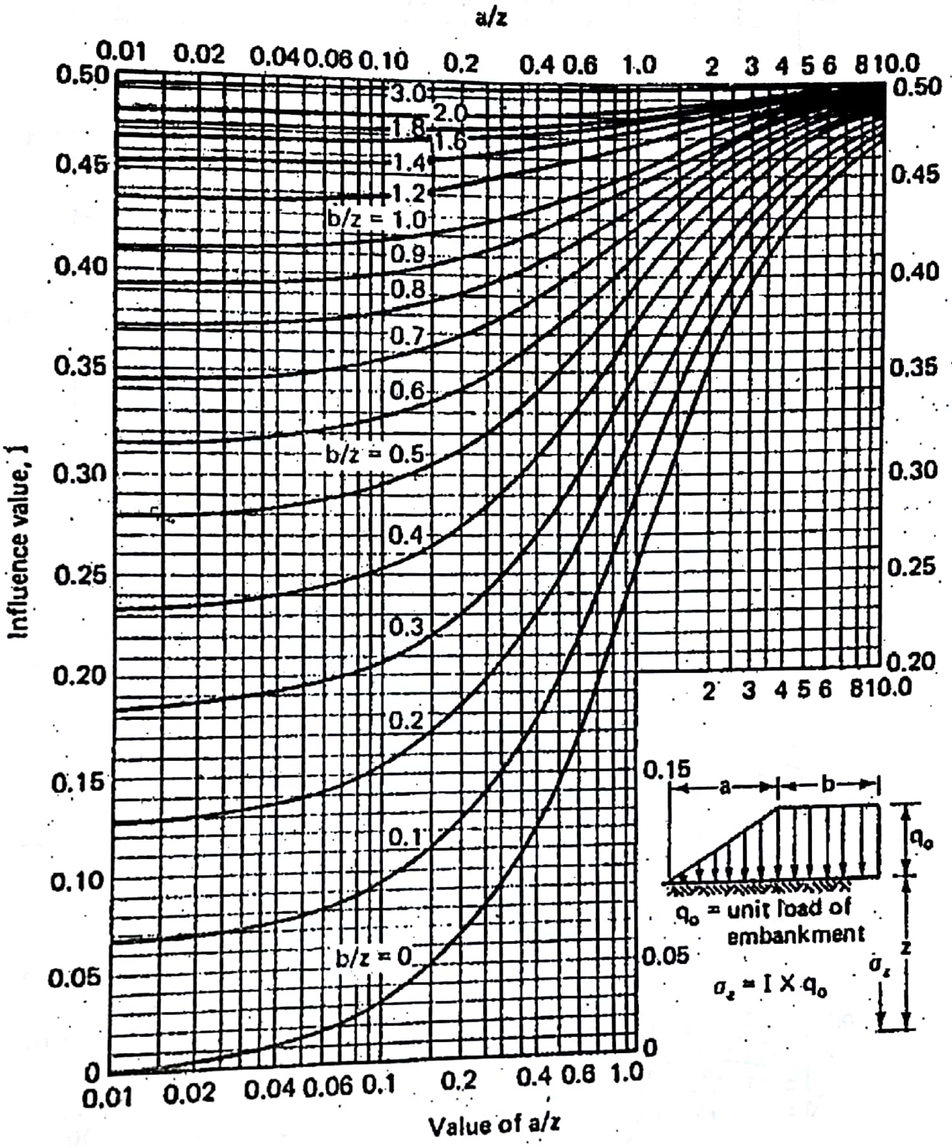


Figure 9.5 Influence values for vertical stress under a very long embankment; length = ∞ (after US Navy, 1971, after Osterberg, 1957).

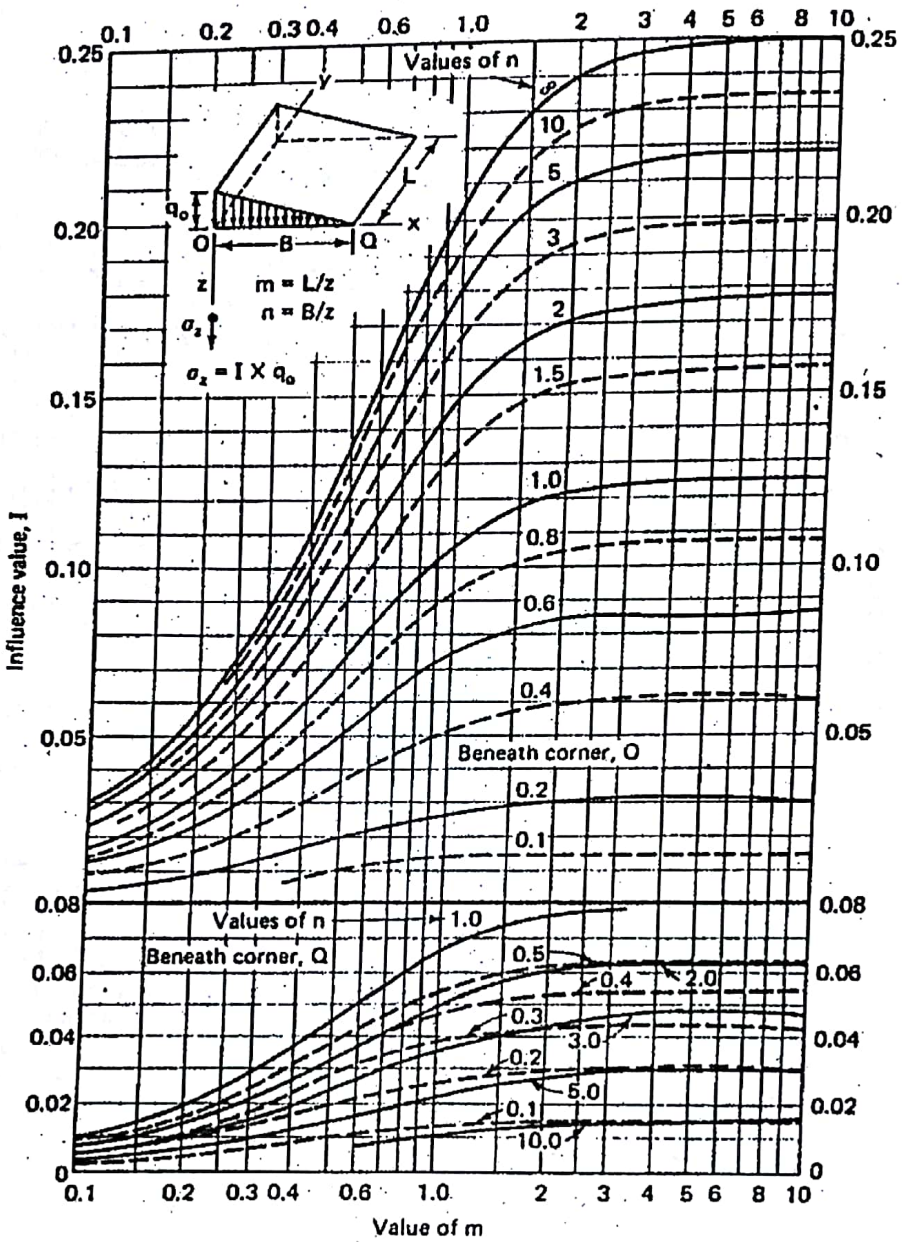


Figure 9.6 Influence values for vertical stress under the corners of a triangular load of limited length (after US Navy, 1971).

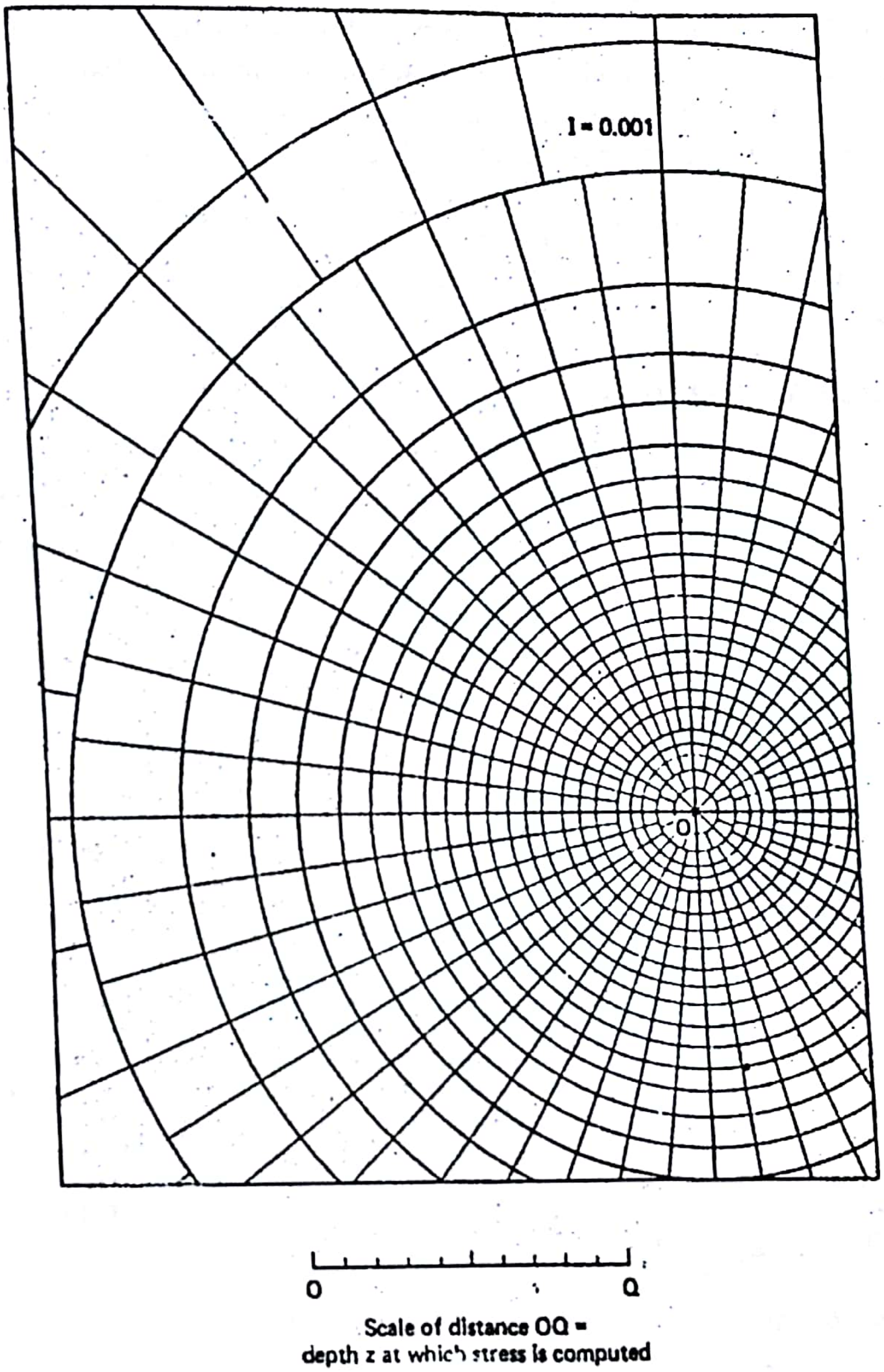


Figure 9.7 Influence chart for vertical stress on horizontal planes (after Newmark, 1942).

- **Newmark Chart for Irregular Shapes (1942)**

Newmark (1942) developed influence charts using the basic equation of Boussinesq from which the vertical stress may be computed underneath the loaded areas of irregular shapes. Fig 9.7 shows such a chart. On the chart line *OQ* represents the distance below the ground surface *z* for which the vertical stress, σ_v is desired, and this distance is used as the scale for a drawing of the loaded area. The vertical stress is computed by merely counting the number of areas or blocks (meshes) on the chart, within the boundary of the loaded area that is drawn to the proper scale and then placed upon the chart. The number of areas is multiplied by an influence value *I*, noted on the chart and by the contact stress to obtain the stress at the desired depth. The point at which the vertical stress is required is placed over the center of the chart. The use of the chart is explained in a worked example 9.3

- **Westergaard's Method (1938)**

Assumptions

Natural soils, in general, do not approach the ideal assumptions of Boussinesq. In fact many sedimentary soils are formed by aggregation of alternate horizontal layers of silt and clay (e.g. varved clays) or the material beneath a pavement. Westergaard (1938) presented a theory in which he assumed the soil interspersed with infinitely thin but perfectly rigid layers that allow only vertical movement but no lateral movement.

For point load, Westergaard's vertical stress equation is given below:

$$\sigma_z = \frac{Q}{2\pi z^2}\frac{\sqrt{a}}{[a+(r/z)^2]^{3/2}} \qquad 9.10$$

Where all terms are the same as that of Boussinesq (Fig. 9.2) except *a*

$a = (1-2\mu)/(2-2\mu)$

μ = poisson's ratio of the soil.

For $\mu = 0$, the maximum value of σ_z is given by:

$$\sigma_z = \frac{Q}{\pi z^2}\frac{1}{[1+2(r/z)^2]^{3/2}} \qquad 9.11$$

Which may be written as:

$$\sigma_z = \frac{Q}{z^2}K_w \qquad 9.12$$

Which is comparable to equation 9.6. (For values of K_W, see Fig. 9.2b).

Typical values of poisson's ratio, μ for silts, and sands range from 0.2 for loose materials to 0.4 for dense materials. Values for saturated clays vary from about 0.4 to 0.5. Also see Table 9.1 for general range of μ for different soils.

Table 9.1 Values or value ranges for Poisson's ratio μ (Bowles, 1996)

Type of soil	μ
Clay, saturated	0.4-0.5
Clay, unsaturated	0.1-0.3
Sandy clay	0.2-0.3
Silt	0.3-0.35
Sand. gravely sand commonly used	−0.1-1.00 0.3-0.4
Rock	0.1-0.4 (depends somewhat on type of rock)
Loess	0.1-0.3
Ice	0.36
Concrete	0.15

9.3 COMPARISON OF BOUSSINESQ AND WESTERGAARD THEORIES

Fig. 9.2(b) represents a comparison between Boussinesq and Westergaard theories. The following are the observations:

- For r/z <1.5, Boussinesq's values of stress influence factor K_B are larger than Westergaard influence factor K_w
- For r/z ≥1.5, both theories provide almost identical values of influence factors.

Many engineers, favour using Boussinesq theory, because of its simplicity but some engineers are of the opinion that for sedimentary soils or layered deposits, Westergaard's solution is preferable. Graphs and Charts similar to Boussinesq's theory are presented in Fig. 9.8.

Tables 9.2 through 9.4 present the influence values for vertical stress under a corner of a uniformly loaded rectangular area, under the center of a square and under the center of a strip footing.

9.4 STRESS DISTRIBUTION DIAGRAMS

Using Boussinesq's and Westergaard's equations, the following diagrams can be presented:

(i) The stress isobar diagrams (also known as pressure bulb or stress contour diagram).

(ii) The vertical stress distribution on a horizontal plane, Z units below the loaded surface or ground surface.

(iii) The vertical stress distribution, r units, away from the line of action of the single load.

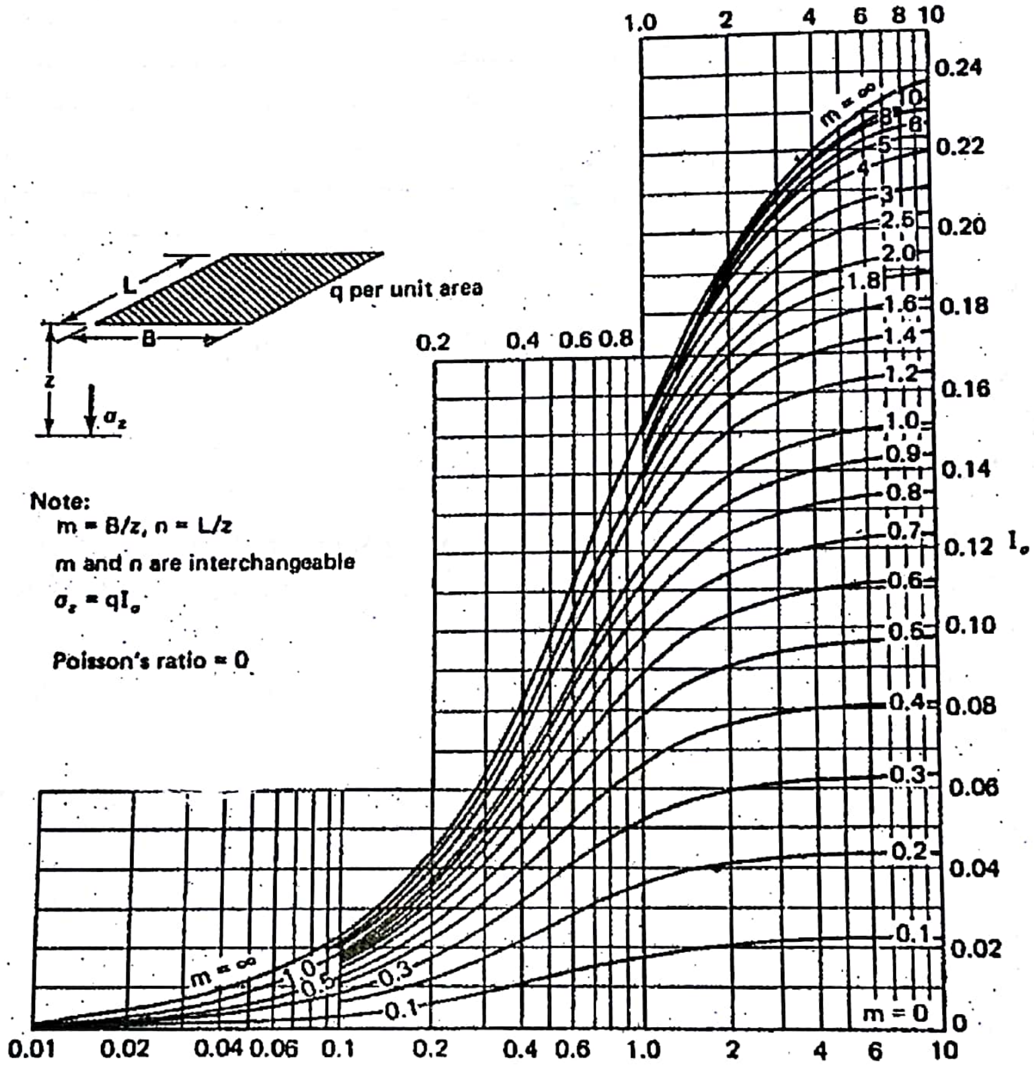
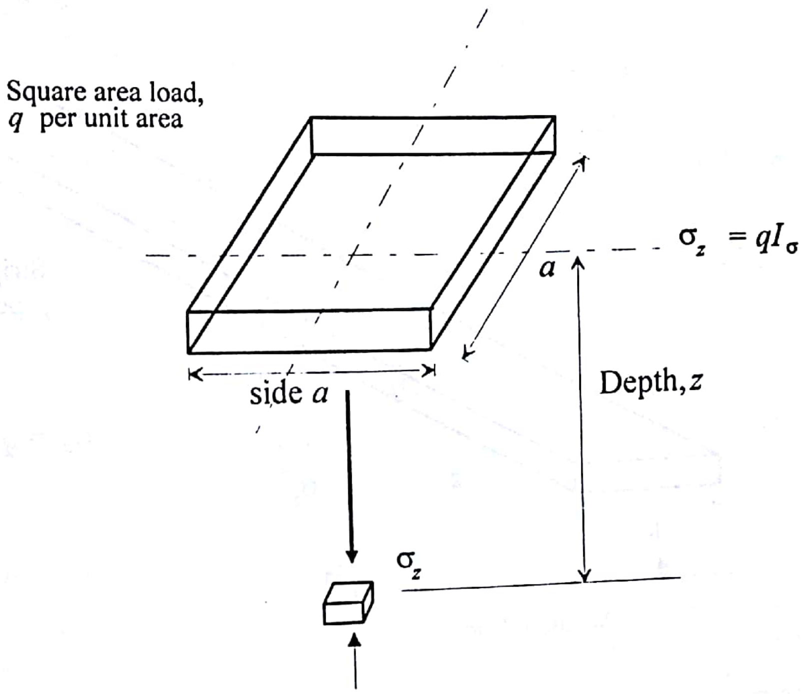


Figure 9.8 Influence values for vertical stress under corners of a uniformly loaded rectangular area for the Westergaard theory (after Duncan and Buchignani, 1976).

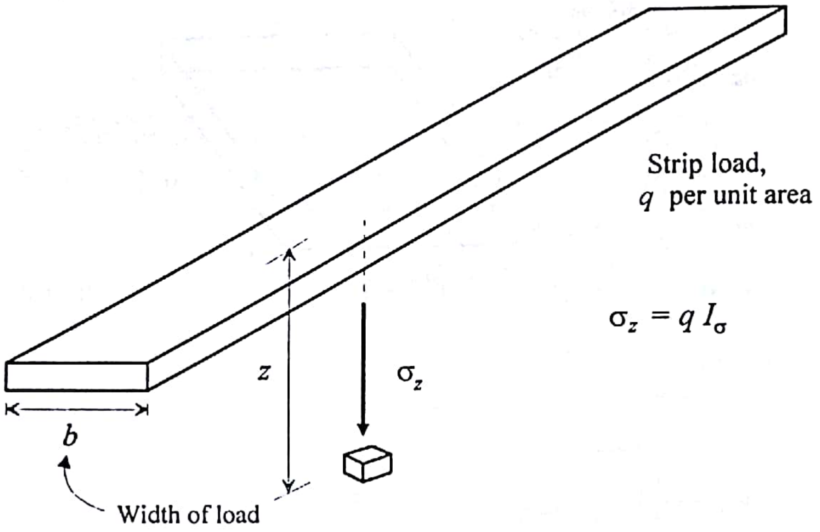
Table 9.2 Influence values for vertical stress under the center of a square uniformly loaded area*



a/z	I_σ Boussinesq	I_σ Westergaard
∞	1.0000	1.0000
20	0.9992	0.9365
16	0.9984	0.9199
12	0.9968	0.8944
10	0.9944	0.8734
8	0.9892	0.8435
6	0.9756	0.7926
5	0.9604	0.7525
4	0.9300	0.6971
3.6	0.9096	0.6659
3.2	0.8812	0.6309
2.8	0.8408	0.5863
2.4	0.7832	0.5328
2.0	0.7008	0.4647
1.8	0.6476	0.4246
1.6	0.5844	0.3794
1.4	0.5108	0.3291
1.2	0.4276	0.2858
1.0	0.3360	0.2165
0.8	0.2410	0.1560
0.6	0.1494	0.0999
0.4	0.0716	0.0477
0.2	0.0188	0.0127
00	0.0000	0.0000

* After Duncan and Buchignani (1976).

Table 9.3 Influence values for vertical stress under the center of an infinitely long strip load*

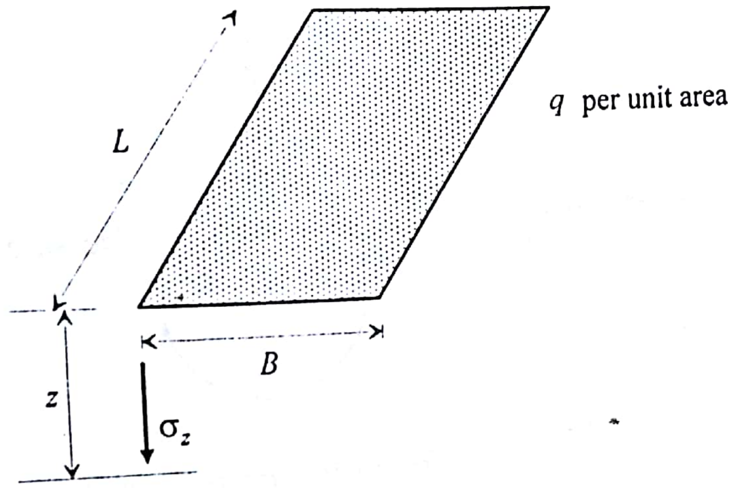


b/z	I_{σ} Boussinesq	I_{σ} Westergaard
∞	1.000	1.000
100	1.000	0.990
10	0.997	0.910
9	0.996	0.901
8	0.994	0.888
7	0.991	0.874
6.5	0.989	0.864
6.0	0.986	0.853
5.5	0.983	0.835
5.0	0.977	0.824
4.5	0.970	0.807
4.0	0.960	0.784
3.5	0.943	0.756
3.0	0.920	0.719
2.5	0.889	0.672
2.0	0.817	0.608
1.5	0.716	0.519
1.2	0.624	0.448
1.0	0.550	0.392
0.8	0.462	0.328
0.5	0.306	0.216
0.2	0.127	0.089
0.1	0.064	0.045
0	0.000	0.000

* After Duncan and Buchignani (1976)

Table 9.4 Influence values for vertical stress under corner of a uniformly loaded rectangular area*

$$\sigma_z = q\, I_\sigma$$



Boussinesq Case

B/z	L/z = 0.1	0.2	0.4	0.6	0.8	1.0	2.0	∞
0.1	0.005	0.009	0.017	0.022	0.026	0.028	0.031	0.032
0.2	0.009	0.018	0.033	0.043	0.050	0.055	0.061	0.062
0.4	0.017	0.033	0.060	0.080	0.093	0.101	0.113	0.115
0.6	0.022	0.043	0.080	0.107	0.125	0.136	0.153	0.156
0.8	0.026	0.050	0.093	0.125	0.146	0.160	0.181	0.185
1.0	0.028	0.055	0.101	0.136	0.160	0.175	0.200	0.205
2.0	0.031	0.061	0.113	0.153	0.181	0.200	0.232	0.240
∞	0.032	0.062	0.115	0.156	0.185	0.205	0.240	0.250

Westergaard Case

B/z	L/z = 0.1	0.2	0.4	0.6	0.8	1.0	2.0	∞
0.1	0.003	0.006	0.011	0.014	0.017	0.018	0.021	0.022
0.2	0.006	0.012	0.021	0.028	0.033	0.036	0.041	0.044
0.4	0.011	0.021	0.039	0.052	0.060	0.066	0.077	0.082
0.6	0.014	0.028	0.052	0.069	0.081	0.089	0.104	0.112
0.8	0.017	0.033	0.060	0.081	0.095	0.105	0.125	0.135
1.0	0.018	0.036	0.066	0.089	0.105	0.116	0.140	0.152
2.0	0.021	0.041	0.077	0.104	0.125	0.140	0.174	0.196
∞	0.022	0.044	0.082	0.112	0.135	0.152	0.196	0.250

* After Duncan and Buchignani (1976)

- **Pressure Bulb**

A pressure bulb is a stress isobar that is a line connecting all the points of equal stress (Fig. 9.9). For a given load, an infinite number of isobars can be drawn.

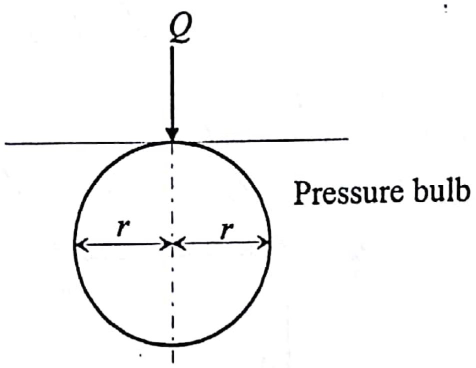


Figure 9.9 *Pressure bulb*

- **Seat of Settlement (Active Zone)**

Terzaghi (1936) recommended that the vertical stress is considered of negligible magnitude when smaller than 20% of the applied contact pressure, σ_0, and that about 80% of the total settlement takes place due to the compression of soil within this pressure bulb (i.e. an isobar of $\sigma_z = 0.2\sigma_0$) and the region within the $0.2\sigma_0$ isobar is known as a seat of settlement or active zone.

For an elastic isotropic, semi-infinite solid media, the depth of the $0.2\sigma_0$ isobar is about 1.5*B* (where *B* is the width of the loaded area or footing). Thus the wider the loaded area, the deeper is its effect.

The depth within which soil compression contributes significantly to surface settlement is called as critical depth (*CD*). For fine-grained soils, the *CD* extends to the point where the applied stress decreases to 10% of effective overburden pressure. For coarse-gained soils, the *CD* extends to a point where the applied stress decreases to 20% of the effective overburden pressure.

- **Vertical Stress Distribution Diagram on a Horizontal Plane**

$$\sigma_z = \frac{Q}{z^2} k_I \qquad 9.13$$

To plot the vertical stress distribution diagram on a horizontal plane, the value of *z* is kept constant in equation 9.13 and for various values of *r*, σ_z computed. The shape of such a diagram is shown in Fig.9.10 below:

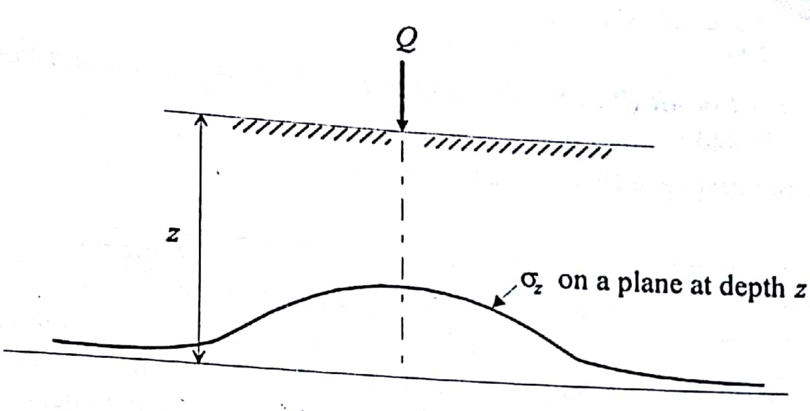


Figure 9.10 *Stress on a plane at a depth z*

• **Vertical Stress Distribution Along a Vertical Plane**
Such a diagram shows the variation of σ_z with depth at a constant distance of r from the line of action of the point load. Fig.9.11 represents such diagrams.

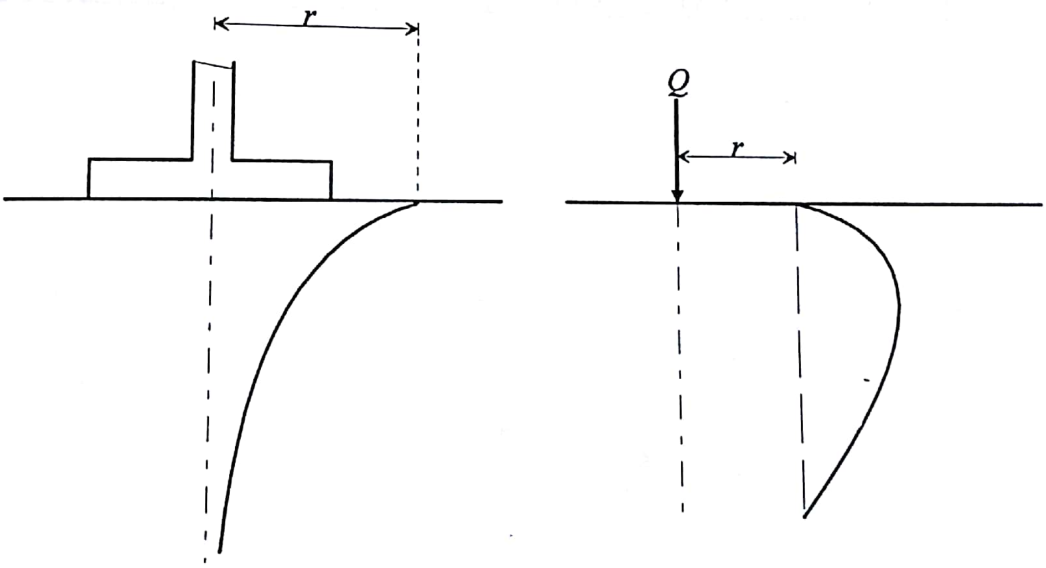


Figure 9.11 *Vertical stress variation on a plane at a distance r from the line of action of the load.*

Summary of Stresses in Soils

(1) Approximate method of 2:1 (V:H) slope may be used between depth $z = B$ and $z = 4B$. This method is not suitable for depths $z = 0 \rightarrow B$. For depths B to $4B$, the results are comparable with the theoretical methods.

(2) Point load is used for uniform loads when:

(a) $z \geq 3B$ or $6R$, where B is the width and R is the radius (Ref. NAVFAC DM 7.1)

(b) $z \geq 2B$ or $4R$ (Ref. Sowers, Introductory Soil Mechanics and Foundation Engg.)

(3) Westergaard and Boussinesq Comparison.

r/z	K_B	K_w	**Remarks**
0.0	0.4780	0.3180	• K_w=2/3K_B at r/z=0 i.e. Westergaard stress is about 67% of Boussinesq when r/z=0 (immediately beneath the load • For r/z =1.5, stress by both the methods are equal • For stress beyond r/z > 1.5, Westergaard is greater
0.2	0.4330	0.2840	
0.4	0.3290	0.2100	
0.6			
0.8	(–)0.1390	0.0925	
1.0	0.0840	0.0612	
1.2	0.0510	0.0416	
1.4	0.0320	0.0292	
1.5	0.0251	0.0247	
1.6	0.0200	0.0210	
1.8	0.0130	0.0156	
2.0	0.0085	0.0118	

WORKED EXAMPLES

Ex. 9.1

2 meters of fill (γ =20 kN/m³) is placed over a large area. On the top of the fill, a 3×4 m spread footing loaded with 1500 kN is placed. Assuming that the average density of the soil prior to placement of the fill is 16.5 kN/m³. The water table is very deep.

(a) Compute and plot the effective vertical stress profile with depth prior to fill placement.

(b) Compute and plot the added stress, $\Delta\sigma_z$, due to the fill.

(c) Compute the additional stress with depth due to the 3x4 m footing when the footing base is placed 1m below the top of the filled ground surface. Use 2:1 method. (Assume weight of footing plus backfill equals weight of soil removed).

Solution

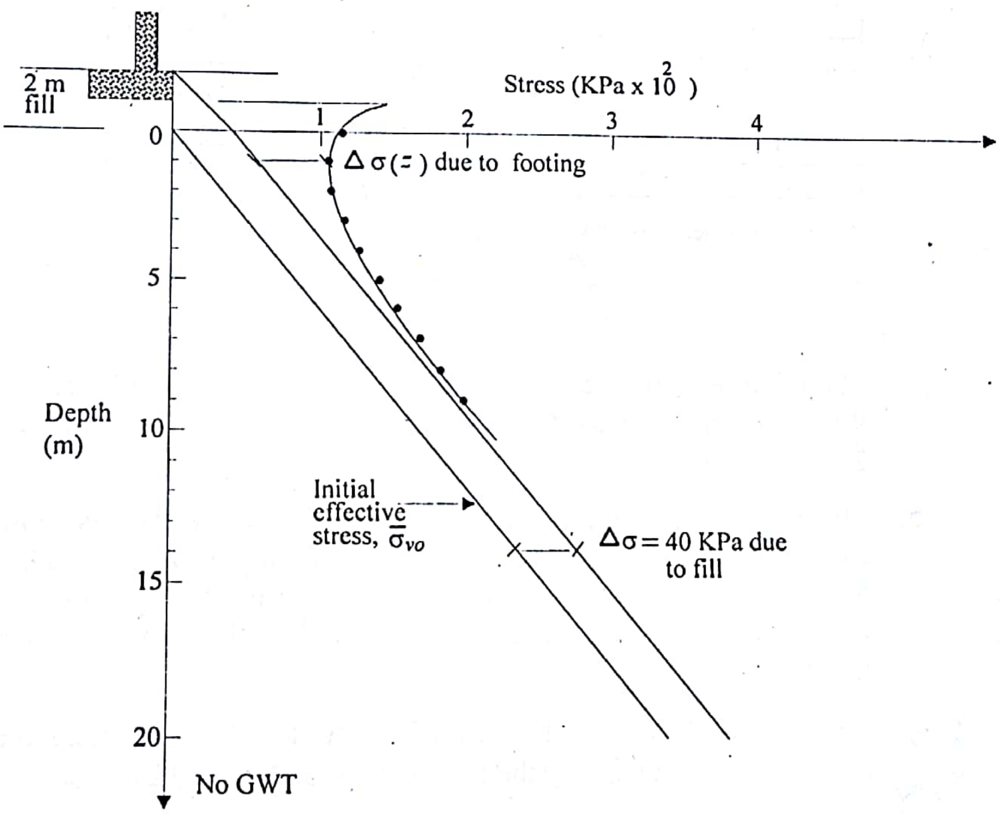
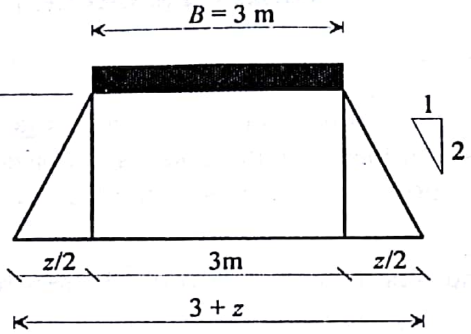


Figure Ex. 9.1 a



(1) z (m)	(2) (B+z)	(3) (L+z)	(4) Area (m^2)	(5) $\Delta\sigma(z)$ (KPa)
0	3	4	12	125
1	4	5	20	75
2	5	6	30	50
3	6	7	42	35.7
4	7	8	56	26.8
5	8	9	72	20.8
6	9	10	90	16.7
7	10	11	110	13.6
8	11	12	132	11.4
9	12	13	156	9.6
10	13	14	182	8.3

Note: z taken below bottom of footing.

Fig. Ex. 9.1b

(a) The initial effective stress distribution is calculated and plotted in Fig. Ex. 9.1a. The stress is zero at zero depth and 330 kPa at a depth of 20 m (16.5× 20 = 330 kPa).

(b) The added stress due to the 2 m fill is 2×20 = 40 kPa. This is shown in Fig. Ex. 9.1a by the line parallel to the in situ vertical effective stress line. Notice that at any depth, the additional stress due to the fill is a constant 40 kPa because the fill is large in aerial extent and thus 100% of its influence is felt throughout.

(c) The contact stress σ_o between the footing and the soil equals the column load, 1500 kN, divided by the footing area, 3×4 m, or 12 m², or

$$\sigma_o = \frac{\text{load}}{\text{area}} = \frac{1500\,\text{kN}}{12\,\text{m}^2} = 125\,\text{kN/m}^2$$

Using the 2:1 method, a tabulation of how the stress changes with depth z is shown in Fig. Ex. 9.1b. The change in stress $\Delta\sigma(z)$, in column 5 is added to the change in stress due to the fill in Fig. Ex. 9.1a. It can be seen that the stress due to the footing diminishes quite rapidly with depth.

footing

Ex. 9.2

For the footing of Ex. 9.1 loaded uniformly by 125 kPa, Compute:

(a) Vertical stress under the corner of the footing at a depth of 2m.

(b) Vertical stress under the center of the footing, and

(c) Compare the results with Ex. 9.1

Solution

(a) $x = 3$ m

$y = 4$ m

$z = 2$ m

$m = \frac{x}{2} = \frac{3}{2} = 1.5$ and $n = \frac{y}{2} = \frac{4}{2} = 2$

From Fig. 9.3, $I = 0.223$, From Eq. 9.9

$\sigma_z = \sigma_o I = 125 \times 0.223 = 27.9$ kPa

(b) to compute the stress under the center, it is necessary to divide the 3×4 m rectangular footing into four sections of 1.5×2 m in size. Find the stress under one corner and multiply this value by 4 to take into account the four quadrants of the uniformly loaded area. We can do this because, for an elastic material, superposition is valid.

$x = 1.5$ m

$y = 2$ m

$z = 2$ m; then

$m = \frac{x}{z} = \frac{1.5}{2} = 0.75$ and $n = \frac{y}{z} = \frac{2}{2} = 1$

The corresponding value of I from Fig. 9.3 is 0.159,

$\sigma_z = 4\sigma_o I = 4 \times 125 \times 0.159 = 79.5$ kPa

Thus the vertical stress under the center for this case is about three times that under the corner. This seems reasonable since the center is loaded from all sides but under the corner it is not.

(c) at a depth of 2 m below the 3×4 m footing, the vertical stress according to the 2:1 theory is 50 kPa (see Fig. Ex. 9.1b). This value represents the average stress beneath the footing at –2 m. The average of the corner and center stress by elastic theory is (27.9+79.9)/2 =53.7 kPa. Thus the 2:1 method underestimates the vertical stress at the center but overestimates σ_z at the corners.

Ex. 9.3

A uniform stress of 250 kPa is applied to the loaded area shown below:

Using Newmark chart, compute the stress at a depth of 80 m below the ground surface due to the loaded area under point O'.

Solution

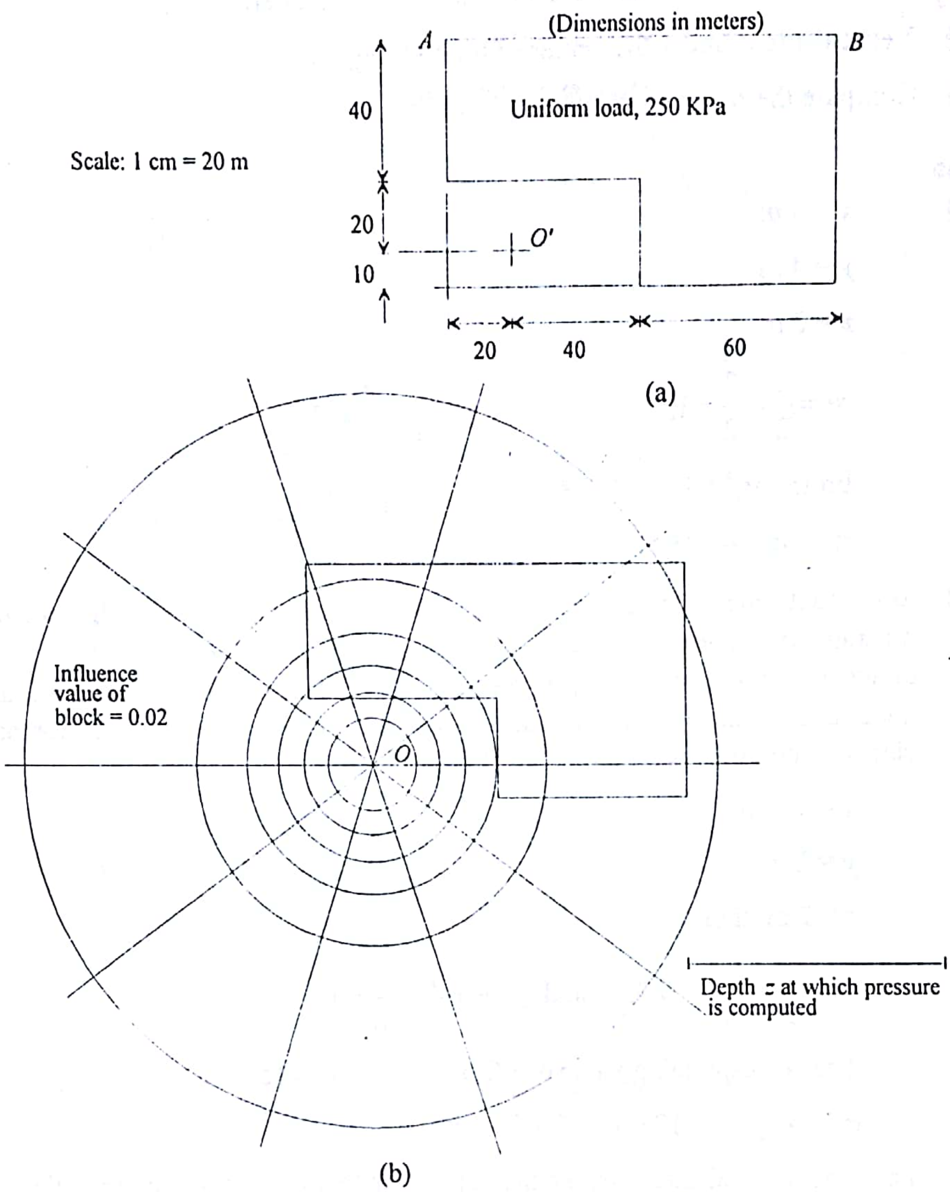


Figure Ex. 9.3 (*after Newmark,* 1942)

Draw the loaded area such that the length of the line $\overline{OQ}$ is scaled to 80m. For example, the distance $\overline{AB}$ in Fig. Ex. 9.3a is 1.5 times the distance $\overline{OQ}$.$\overline{OQ}$ = 80 m and $\overline{AB}$ = 120 m. Next, place point O', the point where the stress is required, over the center of the influence chart (as shown in Fig. Ex. 9.3b to a slightly smaller scale). The number of blocks (and partial blocks) are counted under the loaded area. In this case, about eight blocks are found. The vertical stress at 80 m is then indicated by:

$$\sigma_v = q_0 I \times \text{No. of blocks}$$

Where q_0 = surface or contact stress, and

I = influence value per block (0.02 in Fig. Ex. 9.3b)

Therefore,

$$\sigma_v = 250 \text{ kPa} \times 0.02 \times 8 \text{ blocks} = 40 \text{ kPa}$$

To compute the stress at other depths, the process is repeated by making other drawings for the different depths, changing the scale each time to correspond to the distance $\overline{OQ}$ on the influence chart.

Ex. 9.4

A rectangular area of 4×8 m is loaded uniformly with a load of 250 kPa

(a) Find the stress at a depth of z = 2 m below the corner of the area and underneath the middle point of the area.

(b) Compute the stress at 2 m depth below point O.

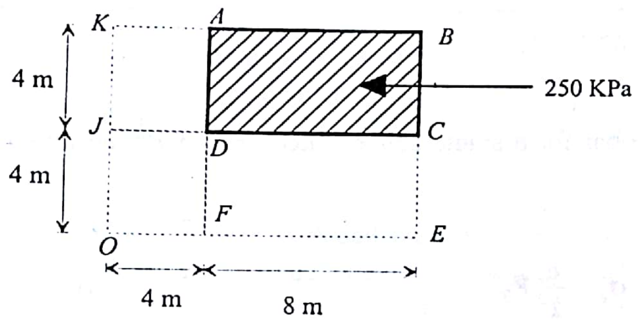


Figure Ex. 9.4

Solution

(a) Using stress under the corner method and Fig. 9.3

- **Stress at corner**

For $z = 2$ m

$$n = \frac{x}{z} = \frac{8}{2} = 4 \qquad m = \frac{y}{z} = \frac{4}{2} = 2$$

From Fig. 9.3 $\quad I = 0.24$

$\therefore\ \sigma_z = (0.24)(250) = 60$ kPa

- **Stress under mid point**

$$n = \frac{4}{2} = 2 \qquad m = \frac{2}{2} = 1$$

From Fig. 9.3 $\quad I = 0.2$

$\therefore\ \sigma_z = (0.2)(250)(4) = 200$ kPa

(b) Refer to Fig. Ex. 9.4 and the letter points as shown:

Add the rectangles in the following manner (+ve for loaded areas and –ve for unloaded areas):

+*OEBK* – *OECJ* – *OFAK* + *OFDJ* result in loaded rectangle we want *ABCD*.

Find four separate influence values from Fig. 9.3 for each rectangle at depth 2 m, then add and subtract the computed stresses. Note that it is necessary to add rectangle *OFDJ* because it was subtracted twice as part of rectangles *OFAK* and *OECJ*. The computations are shown in the following table:

Item	Area			
	+*OEBK*	−*OECJ*	−*OFAK*	+*OFDJ*
x	12	12	4	4
y	8	4	8	4
z	2	2	2	2
$m=x/z$	6	6	2	2
$n=y/z$	4	2	4	2
I	0.25	0.24	0.24	0.23
σ_z	+62.5	−60.0	−60.0	+57.5

Total $\sigma_z = 62.5 - 60.0 - 60.0 + 57.5 = 0.0$

Ex. 9.5

Plot an isobar for a single concentrated load of P=100 tons for a stress σ_z= 0.5 ton/sq. ft.

Solution

$$\sigma_z = \frac{P}{z^2} k_B \qquad \text{(i)}$$

Where,

$$k_B = \frac{0.478}{[1+(r/z)^2]^{2.5}} \qquad \text{(ii)}$$

From (i)

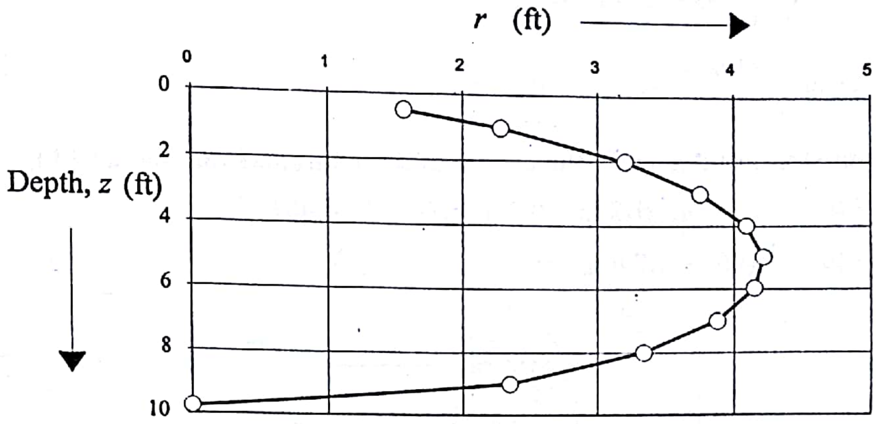
$$0.5 = \frac{100}{z^2} k_B \quad \Rightarrow \quad K_B = \frac{z^2}{200} \qquad \text{(iii)}$$

When r = 0, then K = 0.478, and the isobar crosses the line of action at a depth $z = \sqrt{200 \times 0.478} = 9.78$ ft.

From (ii) $$r/z = [(\frac{0.478}{k_B})^{1/2.5} - 1]^{1/2} \qquad \text{(iv)}$$

Thus for various values of z, compute corresponding values of K_B from equations (iii) and using these values of K_B , compute r/z from equation (iv) as shown below:

Depth, (z),ft	$K_B = \frac{z^2}{200}$	**(r/z)**	**(r) (ft)**	**σ_z (tsf)**
0.5	0.00125	3.129	1.564	0.5
1.0	0.005	2.280	2.280	0.5
2.0	0.020	1.600	3.200	0.5
3.0	0.045	1.284	3.760	0.5
4.0	0.080	1.022	4.088	0.5
5.0	0.125	0.843	4.213	0.5
6.0	0.180	0.691	4.148	0.5
7.0	0.245	0.554	3.875	0.5
8.0	0.320	0.417	3.338	0.5
9.0	0.405	0.262	2.356	0.5
9.78	0.478	0.000	0.000	0.5



Isobar of 0.5 tsf intensity (only one half is plotted as it is symmetrical

Ex. 9.6

(a) A column load of 100 tons is added on the surface of a homogeneous deposit on a 2m×2m area of sand extending to a great depth. Compute the increase in stress on a horizontal plane at a depth of 2 m below the base of the loaded area.

(b) Calculate also the stress at the center of the loaded area at 2m depth below the base.

(c) Compute the stresses if the load is assumed to be concentrated point load and point out the error in the assumption.

Solution

(a) Approximate Method of 2:1 (V:H) Slope

$$\sigma_z = \frac{100}{(B+z)^2} = \frac{100}{(2+2)^2} = \frac{100}{16} = 6.25 \text{ tons/sq. ft}$$

(b) Using Newmark Chart.

- Draw the plan of the footing on a scale such that AB on the chart = 2 m.
- Place the plan on the chart so that the center of the plan coincides with the center of the chart and count the No. of meshes = 64

$$\therefore \quad \sigma_z = (64)(0.005) \times \frac{100}{2 \times 2}$$

= (meshes)(influence value of the chart) (σ_o) = 8 ton/m²

(c) Boussinesq's Method.

$$\sigma_z = \frac{P^2}{z^2} k_B$$

For $r = 0$, along the line of action of the load

$K_B = 0.478$

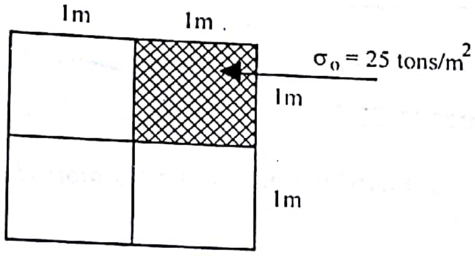
$$\therefore \sigma_z = 0.478\frac{100}{4} = 11.95 \text{ ton/m}^2$$

$$\text{Percent error} = \frac{11.95-8}{8}\times 100 = 49.375\%$$

(d) Using stress underneath the corner of a rectangular method and Fig. 9.3

For $z = 2$ m $\quad n = m = 1/2 \text{ m} = 0.5 \text{ m} = x/z \quad I = 0.087$

$\therefore \sigma_z = 4\,(0.087)(25) = 8.7 \text{ tons/m}^2$



Ex. 9.7

Given:

Column load = 600 kPa $\qquad$ Column footing size = 1m×1.5m

Required:

Stresses at 0.5, 1, 1.5, 2, 2.5, 3 and 4 m depths below base by different methods:

Solution

(i) Approximate Method

Depth, z (m)	$A = B\times L$ (m²)	$\sigma_z = P/A$ (kPa)
0.0	1×1.5	400
0.5	1.5×2.0	200
1.0	2.0×2.5	120
1.5	2.5×3.0	080
2.0	3.0×3.5	057
2.5	3.5×4.0	043
3.0	4.0×4.5	033
4.0	5.0×5.5	022

(ii) Boussinesq's Point Load Method

For $r = 0$, along the line of action of the load

$K_B = 0.478$

$$\therefore \sigma_z = \frac{P}{z^2}(0.478) = \frac{600\times 0.478}{z^2} = \frac{286.48}{z^2}$$

z (m)	0	0.5	1.0	1.5	2.0	2.5	3.0	4.0
σ_z	∞	1146	286	127	072	046	032	018

(iii) Using Fadum Method

$x = 0.75 \quad y = 0.5$

z (m)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	4.0
$m=x/z$	-	1.5	0.75	0.5	0.375	0.300	0.250	0.188
$n=y/z$	-	1.0	0.5	0.33	0.25	0.20	0.167	0.125
I_z	1.0	0.205	0.110	0.065	0.038	0.015	0.019	0.090
$\sigma_z=$	400	328	176	104	061	040	030	016

Ex. 9.8

Given:

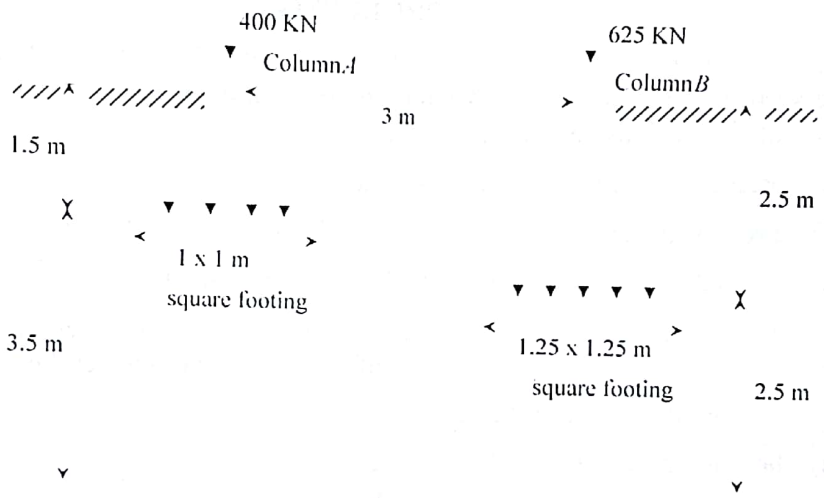


Figure Ex. 9.8

Required.

(a) Contact pressure below the footings

(b) Stress under the center of footings at a depth 5 m below GSL. using Boussingesq's method.

(c) Stress at 5 m depth midway between the columns

Solution

(a) Contact Pressure below the footings

- For footings of column A $\sigma_0 = 400/1 = 400$ kPa
- For footings of column B $\sigma_0 = 625/(1.25)^2 = 400$ kPa

(b) Pressure at 5 m below GSL.

- For column A:

$$\sigma_z = (0.478)\{\frac{400}{(3.5)^2}\} = 15.6 \text{ kPa}$$

- For column B:

$$\sigma_z = (0.478)\{\frac{400}{(3.5)^2}\} = 15.6 \text{ kPa}$$

(c) Pressure at middle point between the columns at 5m depth below GSL, i.e. r =1.5 m

$$\sigma_z = \frac{625}{6.25}\cdot\frac{0.478}{[1+(\frac{1.5}{2.5})^2]^{5/2}} + \frac{400}{12.25}\cdot\frac{0.478}{[1+(\frac{1.5}{3.5})^2]^{5/2}}$$

$$= (100)(0.2216) + (32.654) = 32.39 \text{ kPa}$$

PROBLEMS

9-1 A point load of 2000 kN is acting at the surface of a thick clay stratum. Compute the vertical stress at 1 m depth intervals to 10 m over a range of 5 m on either sides of the load, also at 1 m spacing and plot:

(i) pressure bulbs for 10 kPa, 20 kPa, and 40 kPa;

(ii) stress distribution diagram directly underneath the load; and

(iii) stress distribution on a horizontal plane at 5 m depth.

9-2 A flexible pad of 24m×12m carried a UDL (including its own weight) of 150 kPa. Determine the vertical stress at depths of 2, 4, 8, and 24 m under the points:

(i) beneath the center of the pad;

(ii) beneath the center of a 24 m edge;

(iii) beneath the center of a 12 m edge; and

(iv) beneath a corner.

9-3 The two columns (A and B) of a framed structure are 3 m centers apart. Column A is supported on a square pad 1.25m×1.25m. The base of which is 2.5 m below GSL. The footing of column B is 1m×1m and its base is at 1.5 m below GSL. The contact pressure under each column is 500 kPa.

Consider the bases as point loads to find the increase in stress at a depth 5 m below GSL:

(i) vertically below the columns centers

(ii) at a point midway between the two columns.

If the coefficient of compressibility of the soil is $m_v = 3\times10^{-3}$ m²/kN, compute the differential settlement between the columns, assume the thickness of the compressible soil to be 5 m below column A and 7 m below column B.