**Regression &correlation**

**Regression:**

The term regression was introduced by the English biometrician, Sir Francis Galton. Regression is relationship between independent and dependent variable. It investigates the dependence of one variable called dependent variable, one or more other variables, called independent variable. The dependent variable is assumed to be a random variable whereas the independent variables are assumed to have fixed values. Regression model is

Y=$α+βx+ε$

Where y is dependent variable and x is independent model

“$α$” is called intercept and “$β$” is called slope and $ε is $error term

**Types of regression:**

Simple linear regression

Multiple linear regression

**Simple linear regression**

When there is one dependent and one independent variable that is called simple linear regression.

Y=$α+βx+ε$

**Multiple linear regression**

Whenthere is one dependent and more than one independent variables is called multiple linear

Regression.

 Y=$α+β\_{x\_{1}}+β\_{x\_{2}}+β\_{x\_{3}}+ε$

According to the nature of relationship there are two types of models

* **Exact model / deterministics model.**
* **Probalastic model /statistical model.**
1. **Exact model / Deterministic model.**

If a regression or model contain exact relationship between variables then model is called exact model

Foe example:

F=ma

F=32+C (9/5)

1. **Probalastic model /statistical model.**

When we involve all other factors in model or consider uncertain factors is called probalastic model.

For example

Y=a+bx+e where “e” is (error term) means other factors.

**The principle of least squares:**

The principle of least squares (LS) consists of determining the values of the unknown parameters that will minimize the sum of squares of errors (residuals).

**Error**

Error are defined as the differences between observed values and the estimated value by the fitted Model equation.

**Regression Equation:**

$\hat{Y}$**=a+b**$X\_{i}$**+**$ ε$

$$to find a ?$$

$a=\overbar{Y}$**-b**$\overbar{X}$

$to find b?$ **(y on x)**

$$ $$

$b\_{yx}$**=**$\frac{n\sum\_{}^{}xy-\left(\sum\_{}^{}x\right)\left(\sum\_{}^{}y\right)}{n\sum\_{}^{}x^{2}-\left(\sum\_{}^{}x\right)^{2}}$

$to find b\_{xy}$**(x on y)**

**bxy=**$ \frac{n\sum\_{}^{}xy-\left(\sum\_{}^{}x\right)\left(\sum\_{}^{}y\right)}{n\sum\_{}^{}y^{2}-\left(\sum\_{}^{}y\right)^{2}}$

**Example NO.1**

**Compute the least squares regression equation of Y on X for the following .what is the regression co efficient and what does it mean?**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **x** | **5** | **6** | **8** | **10** | **12** | **13** | **15** | **16** | **17** |
| **y** | **16** | **19** | **23** | **28** | **36** | **41** | **44** | **45** | **50** |

**Solution**

The estimated regression line is Y on X is

$\hat{Y}$**=a+bX**

**Necessary calculations are:**

|  |  |  |  |
| --- | --- | --- | --- |
| **x** | **Y** | **Xy** | $$x^{2}$$ |
| 5 | 16 | 80 | 25 |
| 6 | 19 | 114 | 36 |
| 8 | 23 | 184 | 64 |
| 10 | 28 | 280 | 100 |
| 12 | 36 | 432 | 144 |
| 13 | 41 | 533 | 169 |
| 15 | 44 | 660 | 225 |
| 16 | 45 | 720 | 256 |
| 17 | 50 | 850 | 289 |
| $\sum\_{}^{}x$**=102** | $\sum\_{}^{}y$**=302** | $\sum\_{}^{}xy$**=3853** | **1308** |

Here y on x so

**byx =**$\frac{n\sum\_{}^{}xy-\left(\sum\_{}^{}x\right)\left(\sum\_{}^{}y\right)}{n\sum\_{}^{}x^{2}-\left(\sum\_{}^{}x\right)^{2}}$

$$Type equation here.$$

n=9

$b\_{yx}$=$\frac{9\left(3853\right)-(102)(302)}{9\left(1308\right)-\left(102\right)^{2}}$

b= 2.831

now we have to find a so

**a**$=\overbar{Y}$**-b**$\overbar{X}$

$\overbar{Y}=\frac{\sum\_{}^{}y}{n}$ **=**$\frac{302}{9}$**=33.56**

$\overbar{X}=\frac{\sum\_{}^{}x}{n}$**=**$\frac{102}{9}$**=11.33**

**a**=33.56-2.831(11.33)

**a=1.47**

**Hence the desire estimated regression equation is**

$\hat{Y}$**=a+b**$X\_{i}$**+**$ ε$

$$\hat{Y}=1.47+2.831X$$

**Example no.2**

in an experiment **to measure** the stiffness of a spring ,the length of the spring under different loads was measured as follows

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **X=loads**  | **3** | **5** | **6** | **9** | **10** | **12** | **15** | **20** | **22** | **28** |
| **Y=length** | **10** | **12** | **15** | **18** | **20** | **22** | **27** | **30** | **32** | **34** |

Find the regression equation

1. **The length,given the weight on the spring (y on x)**
2. **The weight ,given the length of the spring (x on y)**

**Solution:**

**Second part x on y**

$\hat{X}$**=a+bY**

|  |  |  |  |
| --- | --- | --- | --- |
| **X** | **Y** | $$y^{2}$$ | **Xy** |
| 3 | 10 | 100 | 30 |
| 5 | 12 | 144 | 60 |
| 6 | 15 | 225 | 90 |
| 9 | 18 | 324 | 162 |
| 10 | 20 | 400 | 200 |
| 12 | 22 | 484 | 264 |
| 15 | 27 | 729 | 405 |
| 20 | 30 | 900 | 600 |
| 22 | 32 | 1024 | 704 |
| 28 | 34 | 1156 | 932 |
| $\sum\_{}^{}x$**=130** | $\sum\_{}^{}y$**=220** | **5486** | **3467** |

**bxy=**$\frac{n\sum\_{}^{}xy-\left(\sum\_{}^{}x\right)\left(\sum\_{}^{}y\right)}{n\sum\_{}^{}y^{2}-\left(\sum\_{}^{}y\right)^{2}}$

$b\_{xy}$=$\frac{10\left(3467\right)-(130)(220)}{10\left(5486\right)-\left(220\right)^{2}}$=0.94

now we have to find a so

**a**$=\overbar{X}$**-b**$\overbar{Y}$

$\overbar{Y}=\frac{\sum\_{}^{}y}{n}$ **=**$\frac{220}{10}$**=22**

$\overbar{X}=\frac{\sum\_{}^{}x}{n}$**=**$\frac{130}{10}$**=13**

**a=xbar-b(ybar)**

**a**=13-0.94(22)

**a=-7.68**

**Hence the desire estimated regression equation is**

$\hat{X}$**=a+ bY+**$ ε$

$$\hat{Y}=-7.68+0.94X+ε$$

 **Part (i) is for home assignment.**

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