**4-Vectors**

A 4-vector $A_\mu$ in Minkowski's space-time continuum is defined as a quantity having four components $A_1$, $A_2$, $A_3$, $A_4$ which transform under a Lorentz transformation in the same way as space-time coordinates $x_1$, $x_2$, $x_3$, $x_4 = ict$, so that

$$A_1' = \gamma\left(A_1 + i\frac{v}{c}A_4\right)$$

$$A_2' = A_2$$

$$A_3' = A_3$$

$$A_4' = \gamma\left(A_4 - i\frac{v}{c}A_1\right).$$

For a given observer the first three components $x_1$, $x_2$, $x_3$ of $x_\mu$ behave like an ordinary 3-vector while the fourth component behaves as a scalar in 3-dimensional space. Since under a Lorentz transformation every 4-vector behaves the same way as $x_\mu$, this result is true for every 4-vector.

Squaring and adding the above four equations, we get

$$A_1'^2 + A_2'^2 + A_3'^2 + A_4'^2 = A_1^2 + A_2^2 + A_3^2 + A_4^2.$$

This result shows that the length of a 4-vector, viz., $(A_1^2 + A_2^2 + A_3^2 + A_4^2)^{1/2}$ does not change under a Lorentz transformation, i.e., the length of a 4-vector is an invariant quantity. Since $A_4^2$ is always negative, the length of a 4-vector can be zero even though it may have non-zero components.

A 4-vector is said to be **space-like**, **time-like** or **light-like** (also called null vector) according as the square of its length is positive, negative or zero. We shall use the subscripts 1, 2, 3, 4 to indicate the four components of a 4-vector while the components of a 3-vector would be specified by using Latin suffices x, y, z as subscripts. It should also be understood that the Greek suffices $\mu$, $\nu$, · · · will run from 1 to 4 while the Latin suffices i, j, · · · will run from 1 to 3.

It is customary to write a 4-vector $A_\mu$ as $A_\mu = (A_1, A_2, A_3, A_4)$ so that the square of its magnitude is given by

$$A_\mu A_\mu = A_1^2 + A_2^2 + A_3^2 + A_4^2.$$

Since, to a given observer, $A_i$'s behave as the component of a 3-
may write

$$A_\mu = (\mathbf{A}, A_4)$$

and
$$A_\mu A_\mu = A^2 + A_4^2 .$$

In particular, the position 4-vector $x_\mu$ is written as

$$x_\mu = (x_1, x_2, x_3, x_4) = (x, y, z, x_4) = (\mathbf{r}, x_4) = (\mathbf{r}, ict)$$

and its magnitude is given by the relation

$$x_\mu x_\mu = r^2 - c^2 t^2.$$