It can be easily checked that $A^T(v)\,A(v) = I$. This shows that the special Lorentz transformation may be considered as an orthogonal transformation which represents a rotation in a 4-dimensional space-time continuum. Moreover det A = +1, where det A denotes the determinant of the matrix A. Such a Lorentz transformation is said to be a **proper orthogonal transformation.**

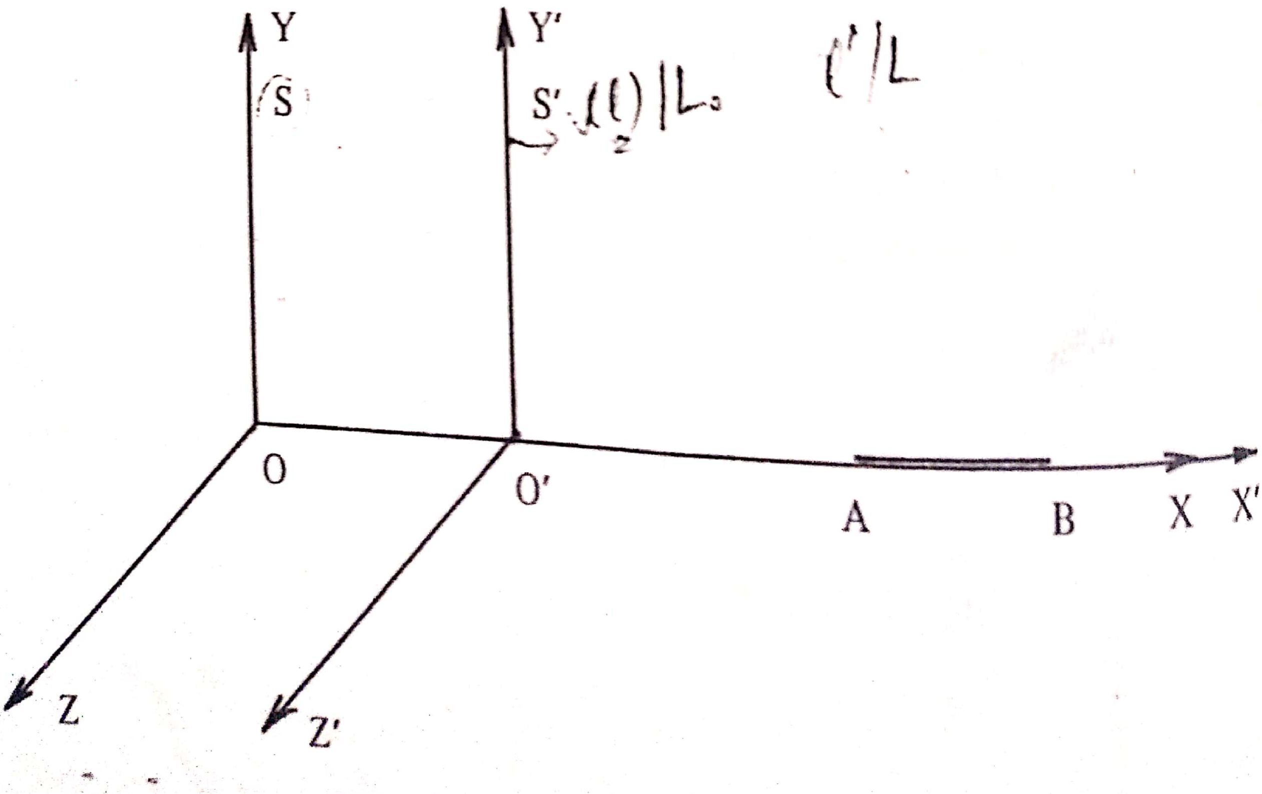
Consequences of the Lorentz Transformation

We shall now consider some of the consequences of the Lorentz transformation:

1. The Lorentz Contraction

Let us first study in detail the operation of measuring the length of an object in different frames of reference. Consider two standard inertial frames of reference S and S′, and a rod which is at rest along the x′-axis in the frame S′ (Fig. 2.2). Let ℓ be the length of this rod as measured by an observer in S′. Since the observer in S′ is at rest with respect to the rod, the length of the rod can be measured by noting the positions of the two ends of the rod, not necessarily at the same time. Let us see what would be the length of this rod as measured by an observer in S, i.e., by an observer who is at rest

$\ell' = L_0$



at same time.

in S. Since the rod is in motion relative to the observer in S, the length of the rod can be measured correctly only by noting simultaneously the end positions of the rod. This direct matching of a swiftly moving rod and measuring stick is difficult in practice. However, we will assume that this can always be done. Let x_1, x_2 be the end positions of the rod measured simultaneously by an observer in S and x_1', x_2' be the positions of the end points of the same rod as measured by an observer in S$'$. Then the length ℓ' of this rod as measured by the observer in S$'$ is given by (by S 's)

$$\ell' = x_2' - x_1' . \quad \ell = x_2 - x_1 \tag{2.21}$$

Now, by the Lorentz transformation, x_2' and x_1' are related to the space and time coordinates of the end points of the rod as measured in S by the equations

$x' = \gamma(x - vt)$.

$$x_2' = \gamma\,(x_2 - v\,t_2)$$

$$x_1' = \gamma\,(x_1 - v\,t_1),$$

where t_1 and t_2 are the times at which the end positions x_1, x_2 of the rod are measured in the frame S. Since the coordinates x_1 and x_2 are measured simultaneously, we must have

$$t_2 = t_1 .$$

The above equations, therefore, reduce to

$$x_2' = \gamma\,(x_2 - v\,t_1)$$

$$x_1' = \gamma\,(x_1 - v\,t_1).$$

Substituting these expressions in equation (2.21), we obtain

$\ell' = \gamma x_2 - \gamma v t_1 - \gamma x_1 + \gamma v t_1$

$$\ell' = \gamma\,(x_2 - v\,t_1) - \gamma\,(x_1 - v\,t_1) = \gamma\,(x_2 - x_1) = \gamma\,\ell, \tag{2.22}$$

$\ell' = \gamma \ell$, $\ell = \frac{\ell'}{\gamma}$

where $\ell = x_2 - x_1$ is the length of the rod as measured in S. Since, the velocity v of a material object is always less than c, the factor $\gamma = 1/(1 - v^2/c^2)^{1/2}$ is always greater than unity. Thus from equation (2.22) we conclude that $\ell' > \ell$

Can a single clock in a frame of reference be sufficient to measure the time of occurrence of any event? It would be so, provided that some signal from the event reaches the clock, and we know the velocity v of the signal as well as the distance r of the clock from the point where the event has occurred. If the signal reaches the clock at time t, then the event must have occurred at time $t - r/v$. However, a much more convenient process would be to have a clock in the vicinity of the event, and note the time directly on that clock. In principle, this would be possible if synchronized clocks are distributed throughout the frame of reference, and an observer is stationed near each clock. Then whenever an event occurs, the time would be recorded on the clock placed in its vicinity, by the observer stationed nearby. In further discussion the words *time noted on a clock in a frame* would mean the *time noted on a clock, placed in the vicinity of the event, by an observer (which may be a camera) stationed nearby.*

2. Time Dilation

Let us now study the effect of the Lorentz transformation on the measurement of time. Consider two events, say the emission of two sparks, occurring at times t_1' and t_2' in the frame S', where $t_2' > t_1'$. Then in this frame the time interval between the sparks is $t_2' - t_1'$. Denote it by T'. Let us suppose that an observer in S observes the same sparks at times t_1 and t_2. Denote the time interval $t_2 - t_1$ by T. Then, by using equations (2.19), we obtain

$$T = t_2 - t_1$$

$$= \gamma\,(t_2' + \frac{v}{c^2}\,x_2') - \gamma\,(t_1' + \frac{v}{c^2}\,x_1')$$

$$= \gamma\,(t_2' - t_1') + \gamma\frac{v}{c^2}\,(x_2' - x_1').$$

Colocal Events
$x_1' = x_2'$
$t_1' \neq t_2'$

If in S' the two events take place at points for which x'-coordinates are equal, i.e., $x_2' = x_1'$, then the above equation reduces to

$$T = \gamma(t_2' - t_1') = \gamma\,T'. \qquad (2.23)$$

$T > T'$
(rest) Longer > slow (moving)

Equation (2.23) shows that for the two events having the same x'-coordinates the time interval T as measured by an observer in S would be longer than the interval T' between these very events as measured by an observer in the frame S'. If the two events are the ticks of a clock C' in the frame S', the

ind the speed of a space ship whose clock is 1 sec slow per hr relative to a clock at earth

$\Delta T = 1$ $\quad T - T' = 1$ $\quad T' = T - 1 = 3600 - 1 = 3599$

$v = 0.0707 \times 10^8$ m/sec

Find speed of spaceship if its clock slow by ... seen by the earth observer

$\frac{\Delta T}{T} = \frac{1}{100}$ $\frac{T - T'}{T} = \frac{1}{100}$ $1 - \frac{T'}{T} = \frac{1}{100}$ $\frac{T'}{T} = \frac{99}{100}$

$T = \gamma T'$ $V = 0.42 \times 10^8$ m/sec.

observer in S, after comparing the readings of this clock C′ with the readings of the synchronized clocks fixed in the frame S would find that the clock in S′ is going slow. Similarly, the observer in the frame S′ would find that the clock in S is going slow. This effect is called time-dilation and is usually expressed by saying that the moving clocks run slow as compared to the clocks at rest. If the moving clock is brought to rest, it regains its initial rate of flow of time. However, it will now lag behind in time when compared to the clocks which remained at rest. This delay in time was accumulated during the period of its motion, and unlike the Lorentz contraction, cannot be made good automatically.

Time dilation, like length contraction, is a real phenomenon. It may again be pointed out that the theory of relativity does not cast any new light on the nature of time; it only changes it from an absolute quantity to one possessing relative character.

Meson Decay

A very interesting example of time dilation is furnished by mu-mesons (μ-mesons), also called muons. These tiny particles are created at an altitude of more than 6000 m by cosmic rays coming from the outer space. These mesons have an average life of 2×10^{-6} s in their rest frame and travel towards the earth with a typical speed of 2.994×10^8 m s^{-1}. According to classical mechanics, before decaying, on an average they should travel a distance

d = velocity of light × mean life time of a μ-meson