The Dynamic Stability and Second order Differential Equations.

Case 1 (Distinct Real Roots)

We know that here $y(t) = y_c + y_p$ while $y_c = A_1 e^{r_1 t} + A_2 e^{r_2 t}$.

The dynamic stability will take place if $y(t) \to y_p$ as $t \to \infty$. This can happen if $y_c \to 0$ as $t \to \infty$ or $(A_1 e^{r_1 t} + A_2 e^{r_2 t}) \to 0$ as $t \to \infty$.

This is possible if and only if both the characteristic roots are negative, i.e., $r_1, r_2 < 0$.

This condition does not even permit one of the roots to be positive or zero.

Case 2 (Repeated Real Roots)

As here $y(t) = y_c + y_p$, while $y_c = A_1 e^{rt} + A_2 t e^{rt}$.

The dynamic stability will take place if $y(t) \longrightarrow y_p$ as $t \longrightarrow \infty$. This can happen if $y_c \longrightarrow 0$ as $t \longrightarrow \infty$ or $(A_1 e^{rt} + A_2 t e^{rt}) \longrightarrow 0$ as $t \longrightarrow \infty$.

This is possible if and if the only characteristic root is negative i.e., $r < 0$.

The second term of y_c, i.e., $A_2 \, t \, e^{rt}$ keeps a multiplicative 't'. If $r > 0$, the e^{rt} part will be explosive and t part will simply serve to intensify explosiveness. If $r < 0$, the e^{rt} part and the t part will move in opposite direction—e^{rt} will ever be decreasing and t ever increasing with the passage of time. Ultimately, the dampening force of e^{rt} part will over the exploding force of t part and as such $t \, e^{rt} \longrightarrow 0$ as $\longrightarrow \infty$

Case 3 (Complex Roots)

As here $y(t) = y_c + y_p$ while $y_c = e^{ht}(\alpha \cos vt + \beta \sin vt)$

The dynamic stability will take place if $y(t) \longrightarrow y_p$ as $\longrightarrow \infty$.

This can happen if $y_c \longrightarrow 0$ as $t \longrightarrow \infty$.

or $[e^{ht}(\alpha \cos vt + \beta \sin vt)] \longrightarrow 0$ as $t \longrightarrow \infty$.

Since the expression $(\alpha \cos vt + \beta \sin vt)$ can never converge to zero whatever value t may assume. Therefore, the key factor is the exponential term e^{ht} which can converge to zero if and only if $h < 0$.