**Example No 1:-** Find the solution of differe-
-ntial equation. (A/2007 B.Z.U)

$$y''(t) + 2y'(t) + 17y = 34 \qquad y(0) = 3,\ y'(0) = 11$$

**Solution:-** $y''(t) + 2y'(t) + 17y = 34$

where

$$b = 34 \qquad y(0) = 3$$

$$a_1 = 2 \qquad y'(0) = 11$$

$$a_2 = 17$$

The particular integral is

$Y_p = \frac{b}{a_2} = \frac{34}{17} = 2$

$Y_c = e^{ht}\left[A_5 \cos vt + A_6 \sin vt\right]$

$h = \frac{-a_1}{2} = \frac{-2}{2} = -1$

$v = \sqrt{4a_2 - (a_1)^2}/2$

$v = \sqrt{4(17) - (2)^2}/2$

$v = \frac{\sqrt{68-4}}{2} = \frac{\sqrt{64}}{2} = 8/2 \Rightarrow \boxed{v = 4}$

**check :-**

$(a_1)^2 \quad 4a_2$

$(2)^2 \quad 4(17)$

$4 < 68$

So it is complex roots.

$Y_c = e^{-t}(A_5 \cos 4t + A_6 \sin 4t)$

The general solution is

$Y(t) = Y_c + Y_p$

$Y(t) = e^{-t}(A_5 \cos 4t + A_6 \sin 4t) + 2$

putting $t = 0$

$Y(0) = e^{0}(A_5 \cos 4(0) + A_6 \sin 4(0)) + 2$

$Y(0) = (A_5 \cos 0 + A_6 \sin 0) + 2$

$Y(0) = A_5(1) + 0 + 2$

$Y(0) = A_5 + 2$

$\left(\begin{array}{l} \because \sin 0 = 0 \\ \because \cos 0 = 1 \\ \because e^{0} = 1 \end{array}\right)$

$3 = A_5 + 2 \Rightarrow A_5 = 3 - 2$

$$\boxed{A_5 = 1}$$

Differentiating w.r.to "t" we have by (u.v).

$$y(t) = e^{-t}(A_5 \cos 4t + A_6 \sin 4t)$$

$$y'(t) = (e^{-t})'(A_5 \cos 4t + A_6 \sin 4t) + e^{-t}(A_5 \cos 4t + A_6 \sin 4t)'$$

$$= (-1)e^{-t}(A_5 \cos 4t + A_6 \sin 4t) + e^{-t}(A_5(-4\sin 4t + 4A_6 \cos 4t)$$

$$= -e^{-t}(A_5 \cos 4t + A_6 \sin 4t) + e^{-t}(-A_5 4\sin 4t + 4A_6 \cos 4t) \quad \left(\begin{array}{l} \because \frac{d}{dt}(\cos 4t) = -4\sin 4t \\ \because \frac{d}{dt}(\sin 4t) = 4\cos 4t \end{array}\right).$$

Put $t = 0$

$$y'(0) = -e^{0}(4A_5 \cos 4(0) + A_6 4\sin 4(0)) + e^{0}(-A_5 4\sin 4(0) + 4A_6 \cos 4(0))$$

$$y'(0) = -A_5 \cos 0 + 4A_6 \cos 0 \qquad \left(\begin{array}{l} \because \cos 0 = 1 \\ \sin 0 = 0 \\ e^{0} = 1 \end{array}\right)$$

$$y'(0) = -A_5(1) + 4A_6(1)$$

$$y'(0) = -A_5 + 4A_6$$

$$11 = -1 + 4A_6 \Rightarrow 4A_6 = 11 + 1 \Rightarrow 4A_6 = 12$$

$$\boxed{A_6 = 3}$$

Therfore, the definite solution is

$$\boxed{\therefore y(t) = e^{-t}(\cos 4t + 3\sin 4t) + 2}$$

**<u>Verification:-</u>**

$$y(0) = e^{-t}(\cos 4t + 3\sin 4t) + 2$$

Put $t = 0$

$$y(0) = e^{0}(\cos 0 + 3\sin 0) + 2$$

$$y(0) = 1(1+0) + 2$$

$$y(0) = 1 + 2 \qquad (\because y(0) = 3)$$

$$3 = 3$$

Ans