We have shown earlier the pair of view factors $F_{i \to j}$ and $F_{j \to i}$ are related to each other by

$$A_i F_{i \to j} = A_j F_{j \to i} \qquad \text{(12-11)}$$

This relation is referred to as the **reciprocity relation** or the **reciprocity rule,** and it enables us to determine the counterpart of a view factor from a knowledge of the view factor itself and the areas of the two surfaces. When determining the pair of view factors $F_{i \to j}$ and $F_{j \to i}$, it makes sense to evaluate first the easier one directly and then the more difficult one by applying the reciprocity relation.

2 The Summation Rule

The radiation analysis of a surface normally requires the consideration of the radiation coming in or going out in all directions. Therefore, most radiation problems encountered in practice involve enclosed spaces. When formulating a radiation problem, we usually form an *enclosure* consisting of the surfaces interacting radiatively. Even openings are treated as imaginary surfaces with radiation properties equivalent to those of the opening.

The conservation of energy principle requires that the entire radiation leaving any surface i of an enclosure be intercepted by the surfaces of the enclosure. Therefore, *the sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself, must equal unity.* This is known as the **summation rule** for an enclosure and is expressed as (Fig. 12–9)

$$\sum_{j=1}^{N} F_{i \to j} = 1 \qquad \text{(12-12)}$$

where N is the number of surfaces of the enclosure. For example, applying the summation rule to surface 1 of a three-surface enclosure yields

$$\sum_{j=1}^{3} F_{1 \to j} = F_{1 \to 1} + F_{1 \to 2} + F_{1 \to 3} = 1$$

The summation rule can be applied to each surface of an enclosure by varying i from 1 to N. Therefore, the summation rule applied to each of the N surfaces of an enclosure gives N relations for the determination of the view factors. Also, the reciprocity rule gives $\frac{1}{2} N(N-1)$ additional relations. Then the total number of view factors that need to be evaluated directly for an N-surface enclosure becomes

$$N^2 - [N + \tfrac{1}{2}N(N-1)] = \tfrac{1}{2}N(N-1)$$

For example, for a six-surface enclosure, we need to determine only $\frac{1}{2} \times 6(6-1) = 15$ of the $6^2 = 36$ view factors directly. The remaining 21 view factors can be determined from the 21 equations that are obtained by applying the reciprocity and the summation rules.

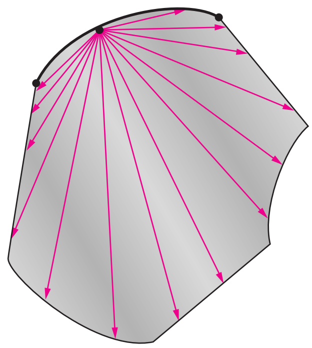


FIGURE 12–9
Radiation leaving any surface i of an enclosure must be intercepted completely by the surfaces of the enclosure. Therefore, the sum of the view factors from surface i to each one of the surfaces of the enclosure must be unity.

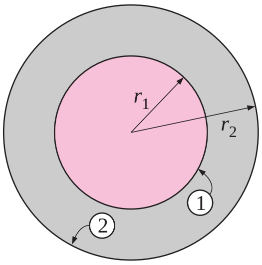


FIGURE 12–10
The geometry considered in Example 12–1.

EXAMPLE 12–1 View Factors Associated with Two Concentric Spheres

Determine the view factors associated with an enclosure formed by two spheres, shown in Figure 12–10.

SOLUTION The view factors associated with two concentric spheres are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis The outer surface of the smaller sphere (surface 1) and inner surface of the larger sphere (surface 2) form a two-surface enclosure. Therefore, $N = 2$ and this enclosure involves $N^2 = 2^2 = 4$ view factors, which are F_{11}, F_{12}, F_{21}, and F_{22}. In this two-surface enclosure, we need to determine only

$$\tfrac{1}{2}N(N-1) = \tfrac{1}{2} \times 2(2-1) = 1$$

view factor directly. The remaining three view factors can be determined by the application of the summation and reciprocity rules. But it turns out that we can determine not only one but *two* view factors directly in this case by a simple *inspection:*

$$F_{11} = 0, \qquad \text{since no radiation leaving surface 1 strikes itself}$$
$$F_{12} = 1, \qquad \text{since all radiation leaving surface 1 strikes surface 2}$$

Actually it would be sufficient to determine only one of these view factors by inspection, since we could always determine the other one from the summation rule applied to surface 1 as $F_{11} + F_{12} = 1$.

The view factor F_{21} is determined by applying the reciprocity relation to surfaces 1 and 2:

$$A_1F_{12} = A_2F_{21}$$

which yields

$$F_{21} = \frac{A_1}{A_2}F_{12} = \frac{4\pi r_1^2}{4\pi r_2^2} \times 1 = \left(\frac{r_1}{r_2}\right)^2$$

Finally, the view factor F_{22} is determined by applying the summation rule to surface 2:

$$F_{21} + F_{22} = 1$$

and thus

$$F_{22} = 1 - F_{21} = 1 - \left(\frac{r_1}{r_2}\right)^2$$

Discussion Note that when the outer sphere is much larger than the inner sphere ($r_2 \gg r_1$), F_{22} approaches one. This is expected, since the fraction of radiation leaving the outer sphere that is intercepted by the inner sphere will be negligible in that case. Also note that the two spheres considered above do not need to be concentric. However, the radiation analysis will be most accurate for the case of concentric spheres, since the radiation is most likely to be uniform on the surfaces in that case.

3 The Superposition Rule

Sometimes the view factor associated with a given geometry is not available in standard tables and charts. In such cases, it is desirable to express the given geometry as the sum or difference of some geometries with known view factors, and then to apply the **superposition rule,** which can be expressed as *the view factor from a surface i to a surface j is equal to the sum of the view factors from surface i to the parts of surface j.* Note that the reverse of this is not true. That is, the view factor from a surface *j* to a surface *i* is *not* equal to the sum of the view factors from the parts of surface *j* to surface *i*.

Consider the geometry in Figure 12–11, which is infinitely long in the direction perpendicular to the plane of the paper. The radiation that leaves surface 1 and strikes the combined surfaces 2 and 3 is equal to the sum of the radiation that strikes surfaces 2 and 3. Therefore, the view factor from surface 1 to the combined surfaces of 2 and 3 is

$$F_{1 \to (2,3)} = F_{1 \to 2} + F_{1 \to 3} \tag{12-13}$$

Suppose we need to find the view factor $F_{1 \to 3}$. A quick check of the view factor expressions and charts in this section will reveal that such a view factor cannot be evaluated directly. However, the view factor $F_{1 \to 3}$ can be determined from Eq. 12–13 after determining both $F_{1 \to 2}$ and $F_{1 \to (2,3)}$ from the chart in Figure 12–12. Therefore, it may be possible to determine some difficult view factors with relative ease by expressing one or both of the areas as the sum or differences of areas and then applying the superposition rule.

To obtain a relation for the view factor $F_{(2,3) \to 1}$, we multiply Eq. 12–13 by A_1,

$$A_1 F_{1 \to (2,3)} = A_1 F_{1 \to 2} + A_1 F_{1 \to 3}$$

and apply the reciprocity relation to each term to get

$$(A_2 + A_3) F_{(2,3) \to 1} = A_2 F_{2 \to 1} + A_3 F_{3 \to 1}$$

or

$$F_{(2,3) \to 1} = \frac{A_2 F_{2 \to 1} + A_3 F_{3 \to 1}}{A_2 + A_3} \tag{12-14}$$

Areas that are expressed as the sum of more than two parts can be handled in a similar manner.

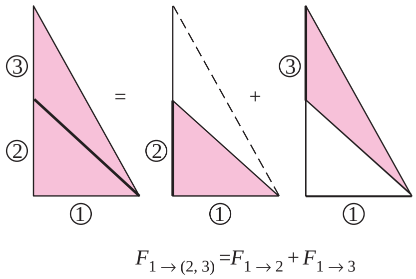


FIGURE 12–11
The view factor from a surface to a composite surface is equal to the sum of the view factors from the surface to the parts of the composite surface.

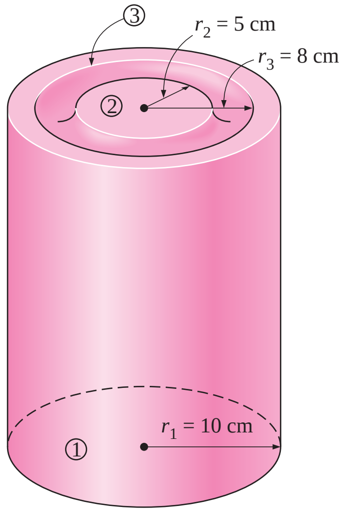


FIGURE 12–12
The cylindrical enclosure considered in Example 12–2.

EXAMPLE 12–2 Fraction of Radiation Leaving through an Opening

Determine the fraction of the radiation leaving the base of the cylindrical enclosure shown in Figure 12–12 that escapes through a coaxial ring opening at its top surface. The radius and the length of the enclosure are $r_1 = 10$ cm and $L = 10$ cm, while the inner and outer radii of the ring are $r_2 = 5$ cm and $r_3 = 8$ cm, respectively.

SOLUTION The fraction of radiation leaving the base of a cylindrical enclosure through a coaxial ring opening at its top surface is to be determined.

Assumptions The base surface is a diffuse emitter and reflector.

Analysis We are asked to determine the fraction of the radiation leaving the base of the enclosure that escapes through an opening at the top surface. Actually, what we are asked to determine is simply the *view factor* $F_{1 \to \text{ring}}$ from the base of the enclosure to the ring-shaped surface at the top.

We do not have an analytical expression or chart for view factors between a circular area and a coaxial ring, and so we cannot determine $F_{1 \to \text{ring}}$ directly. However, we do have a chart for view factors between two coaxial parallel disks, and we can always express a ring in terms of disks.

Let the base surface of radius $r_1 = 10$ cm be surface 1, the circular area of $r_2 = 5$ cm at the top be surface 2, and the circular area of $r_3 = 8$ cm be surface 3. Using the superposition rule, the view factor from surface 1 to surface 3 can be expressed as

$$F_{1 \to 3} = F_{1 \to 2} + F_{1 \to \text{ring}}$$

since surface 3 is the sum of surface 2 and the ring area. The view factors $F_{1 \to 2}$ and $F_{1 \to 3}$ are determined from the chart in Figure 12–7.

$$\frac{L}{r_1} = \frac{10 \text{ cm}}{10 \text{ cm}} = 1 \quad \text{and} \quad \frac{r_2}{L} = \frac{5 \text{ cm}}{10 \text{ cm}} = 0.5 \xrightarrow{\text{(Fig. 12–7)}} F_{1 \to 2} = 0.11$$

$$\frac{L}{r_1} = \frac{10 \text{ cm}}{10 \text{ cm}} = 1 \quad \text{and} \quad \frac{r_3}{L} = \frac{8 \text{ cm}}{10 \text{ cm}} = 0.8 \xrightarrow{\text{(Fig. 12–7)}} F_{1 \to 3} = 0.28$$

Therefore,

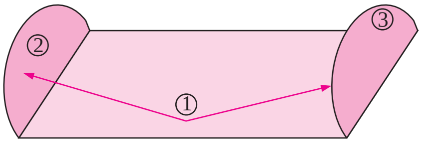
$$F_{1 \to \text{ring}} = F_{1 \to 3} - F_{1 \to 2} = 0.28 - 0.11 = \mathbf{0.17}$$

which is the desired result. Note that $F_{1 \to 2}$ and $F_{1 \to 3}$ represent the fractions of radiation leaving the base that strike the circular surfaces 2 and 3, respectively, and their difference gives the fraction that strikes the ring area.

4 The Symmetry Rule

The determination of the view factors in a problem can be simplified further if the geometry involved possesses some sort of symmetry. Therefore, it is good practice to check for the presence of any *symmetry* in a problem before attempting to determine the view factors directly. The presence of symmetry can be determined *by inspection*, keeping the definition of the view factor in mind. Identical surfaces that are oriented in an identical manner with respect to another surface will intercept identical amounts of radiation leaving that surface. Therefore, the **symmetry rule** can be expressed as *two (or more) surfaces that possess symmetry about a third surface will have identical view factors from that surface* (Fig. 12–13).

The symmetry rule can also be expressed as *if the surfaces j and k are symmetric about the surface* i *then* $F_{i \to j} = F_{i \to k}$. Using the reciprocity rule, we can show that the relation $F_{j \to i} = F_{k \to i}$ is also true in this case.



$F_{1 \to 2} = F_{1 \to 3}$

(Also, $F_{2 \to 1} = F_{3 \to 1}$)

FIGURE 12–13
Two surfaces that are symmetric about a third surface will have the same view factor from the third surface.

EXAMPLE 12–3 **View Factors Associated with a Tetragon**

Determine the view factors from the base of the pyramid shown in Figure 12–14 to each of its four side surfaces. The base of the pyramid is a square, and its side surfaces are isosceles triangles.

SOLUTION The view factors from the base of a pyramid to each of its four side surfaces for the case of a square base are to be determined.
Assumptions The surfaces are diffuse emitters and reflectors.
Analysis The base of the pyramid (surface 1) and its four side surfaces (surfaces 2, 3, 4, and 5) form a five-surface enclosure. The first thing we notice about this enclosure is its symmetry. The four side surfaces are symmetric about the base surface. Then, from the *symmetry rule,* we have

$$F_{12} = F_{13} = F_{14} = F_{15}$$

Also, the *summation rule* applied to surface 1 yields

$$\sum_{j=1}^{5} F_{1j} = F_{11} + F_{12} + F_{13} + F_{14} + F_{15} = 1$$

However, $F_{11} = 0$, since the base is a *flat* surface. Then the two relations above yield

$$F_{12} = F_{13} = F_{14} = F_{15} = \mathbf{0.25}$$

Discussion Note that each of the four side surfaces of the pyramid receive one-fourth of the entire radiation leaving the base surface, as expected. Also note that the presence of symmetry greatly simplified the determination of the view factors.

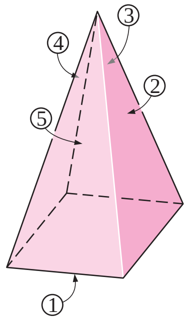


FIGURE 12–14
The pyramid considered in Example 12–3.

EXAMPLE 12–4 **View Factors Associated with a Triangular Duct**

Determine the view factor from any one side to any other side of the infinitely long triangular duct whose cross section is given in Figure 12–15.

SOLUTION The view factors associated with an infinitely long triangular duct are to be determined.
Assumptions The surfaces are diffuse emitters and reflectors.
Analysis The widths of the sides of the triangular cross section of the duct are L_1, L_2, and L_3, and the surface areas corresponding to them are A_1, A_2, and A_3, respectively. Since the duct is infinitely long, the fraction of radiation leaving any surface that escapes through the ends of the duct is negligible. Therefore, the infinitely long duct can be considered to be a three-surface enclosure, $N = 3$.

This enclosure involves $N^2 = 3^2 = 9$ view factors, and we need to determine

$$\tfrac{1}{2}N(N-1) = \tfrac{1}{2} \times 3(3-1) = 3$$

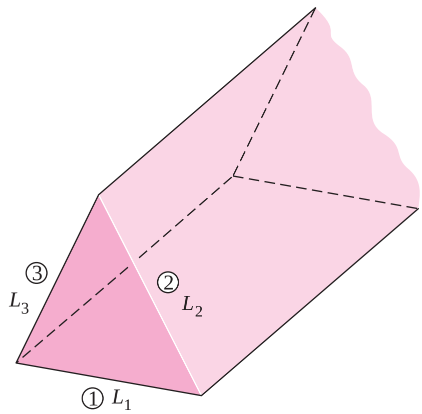


FIGURE 12–15
The infinitely long triangular duct considered in Example 12–4.

of these view factors directly. Fortunately, we can determine all three of them by inspection to be

$$F_{11} = F_{22} = F_{33} = 0$$

since all three surfaces are flat. The remaining six view factors can be determined by the application of the summation and reciprocity rules.

Applying the summation rule to each of the three surfaces gives

$$\begin{aligned} F_{11} + F_{12} + F_{13} &= 1 \\ F_{21} + F_{22} + F_{23} &= 1 \\ F_{31} + F_{32} + F_{33} &= 1 \end{aligned}$$

Noting that $F_{11} = F_{22} = F_{33} = 0$ and multiplying the first equation by A_1, the second by A_2, and the third by A_3 gives

$$\begin{aligned} A_1F_{12} + A_1F_{13} &= A_1 \\ A_2F_{21} + A_2F_{23} &= A_2 \\ A_3F_{31} + A_3F_{32} &= A_3 \end{aligned}$$

Finally, applying the three reciprocity relations $A_1F_{12} = A_2F_{21}$, $A_1F_{13} = A_3F_{31}$, and $A_2F_{23} = A_3F_{32}$ gives

$$\begin{aligned} A_1F_{12} + A_1F_{13} &= A_1 \\ A_1F_{12} + A_2F_{23} &= A_2 \\ A_1F_{13} + A_2F_{23} &= A_3 \end{aligned}$$

This is a set of three algebraic equations with three unknowns, which can be solved to obtain

$$\begin{aligned} F_{12} &= \frac{A_1 + A_2 - A_3}{2A_1} = \frac{\boldsymbol{L_1 + L_2 - L_3}}{\boldsymbol{2L_1}} \\ F_{13} &= \frac{A_1 + A_3 - A_2}{2A_1} = \frac{\boldsymbol{L_1 + L_3 - L_2}}{\boldsymbol{2L_1}} \\ F_{23} &= \frac{A_2 + A_3 - A_1}{2A_2} = \frac{\boldsymbol{L_2 + L_3 - L_1}}{\boldsymbol{2L_2}} \end{aligned} \qquad \textbf{(12-15)}$$

Discussion Note that we have replaced the areas of the side surfaces by their corresponding widths for simplicity, since $A = Ls$ and the length s can be factored out and canceled. We can generalize this result as *the view factor from a surface of a very long triangular duct to another surface is equal to the sum of the widths of these two surfaces minus the width of the third surface, divided by twice the width of the first surface.*

View Factors between Infinitely Long Surfaces: The Crossed-Strings Method

Many problems encountered in practice involve geometries of constant cross section such as channels and ducts that are *very long* in one direction relative to the other directions. Such geometries can conveniently be considered to be *two-dimensional,* since any radiation interaction through their end surfaces will be negligible. These geometries can subsequently be modeled as being *infinitely long,* and the view factor between their surfaces can be determined by the amazingly simple *crossed-strings method* developed by H. C. Hottel in the 1950s. The surfaces of the geometry do not need to be flat; they can be convex, concave, or any irregular shape.

To demonstrate this method, consider the geometry shown in Figure 12–16, and let us try to find the view factor $F_{1 \to 2}$ between surfaces 1 and 2. The first thing we do is identify the endpoints of the surfaces (the points A, B, C, and D) and connect them to each other with tightly stretched strings, which are indicated by dashed lines. Hottel has shown that the view factor $F_{1 \to 2}$ can be expressed in terms of the lengths of these stretched strings, which are straight lines, as

$$F_{1 \to 2} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1} \tag{12-16}$$

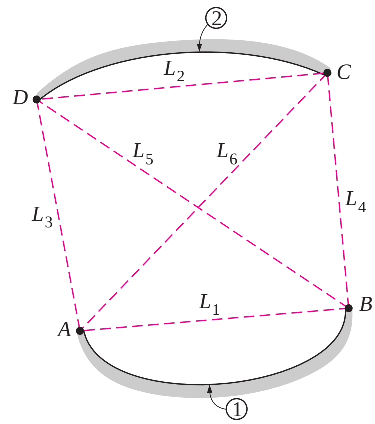


FIGURE 12–16
Determination of the view factor $F_{1 \to 2}$ by the application of the crossed-strings method.

Note that $L_5 + L_6$ is the sum of the lengths of the *crossed strings,* and $L_3 + L_4$ is the sum of the lengths of the *uncrossed strings* attached to the endpoints. Therefore, Hottel's crossed-strings method can be expressed verbally as

$$F_{i \to j} = \frac{\Sigma\,(\text{Crossed strings}) - \Sigma\,(\text{Uncrossed strings})}{2 \times (\text{String on surface } i)} \tag{12-17}$$

The crossed-strings method is applicable even when the two surfaces considered share a common edge, as in a triangle. In such cases, the common edge can be treated as an imaginary string of zero length. The method can also be applied to surfaces that are partially blocked by other surfaces by allowing the strings to bend around the blocking surfaces.

EXAMPLE 12–5 The Crossed-Strings Method for View Factors

Two infinitely long parallel plates of widths $a = 12$ cm and $b = 5$ cm are located a distance $c = 6$ cm apart, as shown in Figure 12–17. (*a*) Determine the view factor $F_{1 \to 2}$ from surface 1 to surface 2 by using the crossed-strings method. (*b*) Derive the crossed-strings formula by forming triangles on the given geometry and using Eq. 12–15 for view factors between the sides of triangles.

SOLUTION The view factors between two infinitely long parallel plates are to be determined using the crossed-strings method, and the formula for the view factor is to be derived.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis (*a*) First we label the endpoints of both surfaces and draw straight dashed lines between the endpoints, as shown in Figure 12–17. Then we identify the crossed and uncrossed strings and apply the crossed-strings method (Eq. 12–17) to determine the view factor $F_{1 \to 2}$:

$$F_{1 \to 2} = \frac{\Sigma\,(\text{Crossed strings}) - \Sigma\,(\text{Uncrossed strings})}{2 \times (\text{String on surface 1})} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$

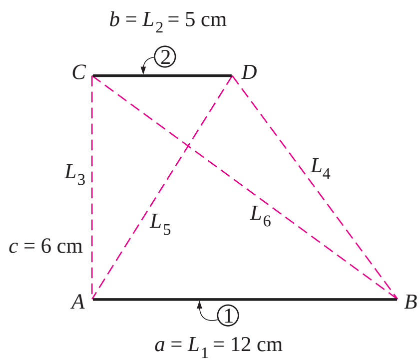


FIGURE 12–17
The two infinitely long parallel plates considered in Example 12–5.

where

$$L_1 = a = 12 \text{ cm} \qquad L_4 = \sqrt{7^2 + 6^2} = 9.22 \text{ cm}$$
$$L_2 = b = 5 \text{ cm} \qquad L_5 = \sqrt{5^2 + 6^2} = 7.81 \text{ cm}$$
$$L_3 = c = 6 \text{ cm} \qquad L_6 = \sqrt{12^2 + 6^2} = 13.42 \text{ cm}$$

Substituting,

$$F_{1 \to 2} = \frac{[(7.81 + 13.42) - (6 + 9.22)] \text{ cm}}{2 \times 12 \text{ cm}} = \mathbf{0.250}$$

(*b*) The geometry is infinitely long in the direction perpendicular to the plane of the paper, and thus the two plates (surfaces 1 and 2) and the two openings (imaginary surfaces 3 and 4) form a four-surface enclosure. Then applying the summation rule to surface 1 yields

$$F_{11} + F_{12} + F_{13} + F_{14} = 1$$

But $F_{11} = 0$ since it is a flat surface. Therefore,

$$F_{12} = 1 - F_{13} - F_{14}$$

where the view factors F_{13} and F_{14} can be determined by considering the triangles *ABC* and *ABD,* respectively, and applying Eq. 12–15 for view factors between the sides of triangles. We obtain

$$F_{13} = \frac{L_1 + L_3 - L_6}{2L_1}, \qquad F_{14} = \frac{L_1 + L_4 - L_5}{2L_1}$$

Substituting,

$$F_{12} = 1 - \frac{L_1 + L_3 - L_6}{2L_1} - \frac{L_1 + L_4 - L_5}{2L_1}$$
$$= \boldsymbol{\frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}}$$

which is the desired result. This is also a miniproof of the crossed-strings method for the case of two infinitely long plain parallel surfaces.

12–3 ▪ RADIATION HEAT TRANSFER: BLACK SURFACES

So far, we have considered the nature of radiation, the radiation properties of materials, and the view factors, and we are now in a position to consider the rate of heat transfer between surfaces by radiation. The analysis of radiation exchange between surfaces, in general, is complicated because of reflection: a radiation beam leaving a surface may be reflected several times, with partial reflection occurring at each surface, before it is completely absorbed. The analysis is simplified greatly when the surfaces involved can be approximated as blackbodies because of the absence of reflection. In this section, we consider radiation exchange between *black surfaces* only; we will extend the analysis to reflecting surfaces in the next section.

Consider two black surfaces of arbitrary shape maintained at uniform temperatures T_1 and T_2, as shown in Figure 12–18. Recognizing that radiation leaves a black surface at a rate of $E_b = \sigma T^4$ per unit surface area and that the view factor $F_{1\rightarrow 2}$ represents the fraction of radiation leaving surface 1 that strikes surface 2, the *net* rate of radiation heat transfer from surface 1 to surface 2 can be expressed as

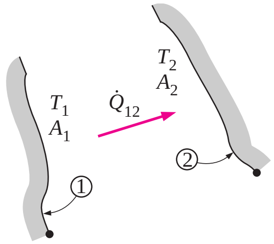


FIGURE 12–18
Two general black surfaces maintained at uniform temperatures T_1 and T_2.

$$\dot{Q}_{1\rightarrow 2} = \begin{pmatrix}\text{Radiation leaving}\\ \text{the entire surface 1}\\ \text{that strikes surface 2}\end{pmatrix} - \begin{pmatrix}\text{Radiation leaving}\\ \text{the entire surface 2}\\ \text{that strikes surface 1}\end{pmatrix}$$

$$= A_1 E_{b1} F_{1\rightarrow 2} - A_2 E_{b2} F_{2\rightarrow 1} \qquad \text{(W)} \tag{12-18}$$

Applying the reciprocity relation $A_1F_{1\rightarrow 2} = A_2F_{2\rightarrow 1}$ yields

$$\dot{Q}_{1\rightarrow 2} = A_1 F_{1\rightarrow 2}\, \sigma(T_1^4 - T_2^4) \qquad \text{(W)} \tag{12-19}$$

which is the desired relation. A negative value for $\dot{Q}_{1\rightarrow 2}$ indicates that net radiation heat transfer is from surface 2 to surface 1.

Now consider an *enclosure* consisting of *N black* surfaces maintained at specified temperatures. The *net* radiation heat transfer *from* any surface *i* of this enclosure is determined by adding up the net radiation heat transfers from surface *i* to each of the surfaces of the enclosure:

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i\rightarrow j} = \sum_{j=1}^{N} A_i F_{i\rightarrow j}\sigma(T_i^4 - T_j^4) \qquad \text{(W)} \tag{12-20}$$

Again a negative value for $\dot{Q}$ indicates that net radiation heat transfer is *to* surface *i* (i.e., surface *i gains* radiation energy instead of losing). Also, the net heat transfer from a surface to itself is zero, regardless of the shape of the surface.

EXAMPLE 12–6 Radiation Heat Transfer in a Black Furnace

Consider the 5-m × 5-m × 5-m cubical furnace shown in Figure 12–19, whose surfaces closely approximate black surfaces. The base, top, and side surfaces of the furnace are maintained at uniform temperatures of 800 K, 1500 K, and 500 K, respectively. Determine (*a*) the net rate of radiation heat transfer between the base and the side surfaces, (*b*) the net rate of radiation heat transfer between the base and the top surface, and (*c*) the net radiation heat transfer from the base surface.

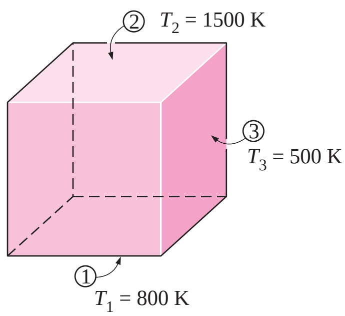


FIGURE 12–19
The cubical furnace of black surfaces considered in Example 12–6.

SOLUTION The surfaces of a cubical furnace are black and are maintained at uniform temperatures. The net rate of radiation heat transfer between the base and side surfaces, between the base and the top surface, from the base surface are to be determined.

Assumptions The surfaces are black and isothermal.

Analysis (*a*) Considering that the geometry involves six surfaces, we may be tempted at first to treat the furnace as a six-surface enclosure. However, the four side surfaces possess the same properties, and thus we can treat them as a single side surface in radiation analysis. We consider the base surface to be surface 1, the top surface to be surface 2, and the side surfaces to be surface 3. Then the problem reduces to determining $\dot{Q}_{1\to3}$, $\dot{Q}_{1\to2}$, and $\dot{Q}_1$.

The net rate of radiation heat transfer $\dot{Q}_{1\to3}$ from surface 1 to surface 3 can be determined from Eq. 12–19, since both surfaces involved are black, by replacing the subscript 2 by 3:

$$\dot{Q}_{1\to3} = A_1F_{1\to3}\sigma(T_1^4 - T_3^4)$$

But first we need to evaluate the view factor $F_{1\to3}$. After checking the view factor charts and tables, we realize that we cannot determine this view factor directly. However, we can determine the view factor $F_{1\to2}$ directly from Figure 12–5 to be $F_{1\to2} = 0.2$, and we know that $F_{1\to1} = 0$ since surface 1 is a plane. Then applying the summation rule to surface 1 yields

$$F_{1\to1} + F_{1\to2} + F_{1\to3} = 1$$

or

$$F_{1\to3} = 1 - F_{1\to1} - F_{1\to2} = 1 - 0 - 0.2 = 0.8$$

Substituting,

$$\begin{aligned}\dot{Q}_{1\to3} &= (25\ \text{m}^2)(0.8)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(800\ \text{K})^4 - (500\ \text{K})^4]\\ &= \mathbf{394 \times 10^3\ W = 394\ kW}\end{aligned}$$

(*b*) The net rate of radiation heat transfer $\dot{Q}_{1\to2}$ from surface 1 to surface 2 is determined in a similar manner from Eq. 12–19 to be

$$\begin{aligned}\dot{Q}_{1\to2} &= A_1F_{1\to2}\sigma(T_1^4 - T_2^4)\\ &= (25\ \text{m}^2)(0.2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(800\ \text{K})^4 - (1500\ \text{K})^4]\\ &= \mathbf{-1319 \times 10^3\ W = -1319\ kW}\end{aligned}$$

The negative sign indicates that net radiation heat transfer is from surface 2 to surface 1.

(*c*) The net radiation heat transfer from the base surface $\dot{Q}_1$ is determined from Eq. 12–20 by replacing the subscript *i* by 1 and taking $N = 3$:

$$\begin{aligned}\dot{Q}_1 &= \sum_{j=1}^{3}\dot{Q}_{1\to j} = \dot{Q}_{1\to1} + \dot{Q}_{1\to2} + \dot{Q}_{1\to3}\\ &= 0 + (-1319\ \text{kW}) + (394\ \text{kW})\\ &= \mathbf{-925\ kW}\end{aligned}$$

Again the negative sign indicates that net radiation heat transfer is *to* surface 1. That is, the base of the furnace is gaining net radiation at a rate of about 925 kW.

12–4 ▪ RADIATION HEAT TRANSFER: DIFFUSE, GRAY SURFACES

The analysis of radiation transfer in enclosures consisting of black surfaces is relatively easy, as we have seen above, but most enclosures encountered in practice involve nonblack surfaces, which allow multiple reflections to occur. Radiation analysis of such enclosures becomes very complicated unless some simplifying assumptions are made.

To make a simple radiation analysis possible, it is common to assume the surfaces of an enclosure to be *opaque*, *diffuse*, and *gray*. That is, the surfaces are nontransparent, they are diffuse emitters and diffuse reflectors, and their radiation properties are independent of wavelength. Also, each surface of the enclosure is *isothermal*, and both the incoming and outgoing radiation are *uniform* over each surface. But first we review the concept of radiosity discussed in Chap. 11.

Radiosity

Surfaces emit radiation as well as reflect it, and thus the radiation leaving a surface consists of emitted and reflected parts. The calculation of radiation heat transfer between surfaces involves the *total* radiation energy streaming away from a surface, with no regard for its origin. The *total radiation energy leaving a surface per unit time and per unit area* is the **radiosity** and is denoted by J (Fig. 12–20).

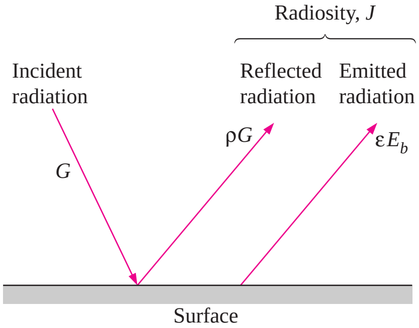


FIGURE 12–20
Radiosity represents the sum of the radiation energy emitted and reflected by a surface.

For a surface i that is *gray* and *opaque* ($\varepsilon_i = \alpha_i$ and $\alpha_i + \rho_i = 1$), the radiosity can be expressed as

$$\begin{aligned} J_i &= \begin{pmatrix}\text{Radiation emitted}\\ \text{by surface } i\end{pmatrix} + \begin{pmatrix}\text{Radiation reflected}\\ \text{by surface } i\end{pmatrix} \\ &= \varepsilon_i E_{bi} + \rho_i G_i \\ &= \varepsilon_i E_{bi} + (1 - \varepsilon_i)G_i \qquad (\text{W/m}^2) \end{aligned} \tag{12-21}$$

where $E_{bi} = \sigma T_i^4$ is the blackbody emissive power of surface i and G_i is irradiation (i.e., the radiation energy incident on surface i per unit time per unit area).

For a surface that can be approximated as a *blackbody* ($\varepsilon_i = 1$), the radiosity relation reduces to

$$J_i = E_{bi} = \sigma T_i^4 \qquad \text{(blackbody)} \tag{12-22}$$

That is, *the radiosity of a blackbody is equal to its emissive power.* This is expected, since a blackbody does not reflect any radiation, and thus radiation coming from a blackbody is due to emission only.

Net Radiation Heat Transfer to or from a Surface

During a radiation interaction, a surface *loses* energy by emitting radiation and *gains* energy by absorbing radiation emitted by other surfaces. A surface experiences a net gain or a net loss of energy, depending on which quantity is larger. The *net* rate of radiation heat transfer from a surface i of surface area A_i is denoted by $\dot{Q}_i$ and is expressed as

$$\dot{Q}_i = \begin{pmatrix}\text{Radiation leaving} \\ \text{entire surface } i\end{pmatrix} - \begin{pmatrix}\text{Radiation incident} \\ \text{on entire surface } i\end{pmatrix}$$
$$= A_i(J_i - G_i) \qquad \text{(W)} \tag{12-23}$$

Solving for G_i from Eq. 12–21 and substituting into Eq. 12–23 yields

$$\dot{Q}_i = A_i\left(J_i - \frac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i}\right) = \frac{A_i \varepsilon_i}{1 - \varepsilon_i}(E_{bi} - J_i) \qquad \text{(W)} \tag{12-24}$$

In an electrical analogy to Ohm's law, this equation can be rearranged as

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i} \qquad \text{(W)} \tag{12-25}$$

where

$$R_i = \frac{1 - \varepsilon_i}{A_i \varepsilon_i} \tag{12-26}$$

is the **surface resistance** to radiation. The quantity $E_{bi} - J_i$ corresponds to a *potential difference* and the net rate of radiation heat transfer corresponds to *current* in the electrical analogy, as illustrated in Figure 12–21.

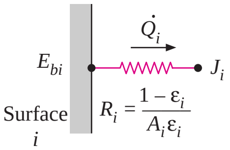


FIGURE 12–21
Electrical analogy of surface resistance to radiation.

The direction of the net radiation heat transfer depends on the relative magnitudes of J_i (the radiosity) and E_{bi} (the emissive power of a blackbody at the temperature of the surface). It will be *from* the surface if $E_{bi} > J_i$ and *to* the surface if $J_i > E_{bi}$. A negative value for $\dot{Q}_i$ indicates that heat transfer is *to* the surface. All of this radiation energy gained must be removed from the other side of the surface through some mechanism if the surface temperature is to remain constant.

The surface resistance to radiation for a *blackbody* is *zero* since $\varepsilon_i = 1$ and $J_i = E_{bi}$. The net rate of radiation heat transfer in this case is determined directly from Eq. 12–23.

Some surfaces encountered in numerous practical heat transfer applications are modeled as being *adiabatic* since their back sides are well insulated and the net heat transfer through them is zero. When the convection effects on the front (heat transfer) side of such a surface is negligible and steady-state conditions are reached, the surface must lose as much radiation energy as it gains, and thus $\dot{Q}_i = 0$. In such cases, the surface is said to *reradiate* all the radiation energy it receives, and such a surface is called a **reradiating surface.** Setting $\dot{Q}_i = 0$ in Eq. 12–25 yields

$$J_i = E_{bi} = \sigma T_i^4 \qquad \text{(W/m}^2\text{)} \tag{12-27}$$

Therefore, the *temperature* of a reradiating surface under steady conditions can easily be determined from the equation above once its radiosity is known. Note that the temperature of a reradiating surface is *independent of its emissivity.* In radiation analysis, the surface resistance of a reradiating surface is disregarded since there is no net heat transfer through it. (This is like the fact that there is no need to consider a resistance in an electrical network if no current is flowing through it.)

Net Radiation Heat Transfer between Any Two Surfaces

Consider two diffuse, gray, and opaque surfaces of arbitrary shape maintained at uniform temperatures, as shown in Figure 12–22. Recognizing that the radiosity J represents the rate of radiation leaving a surface per unit surface area and that the view factor $F_{i\to j}$ represents the fraction of radiation leaving surface i that strikes surface j, the *net* rate of radiation heat transfer from surface i to surface j can be expressed as

$$\dot{Q}_{i\to j} = \left(\begin{array}{c}\text{Radiation leaving}\\ \text{the entire surface } i\\ \text{that strikes surface } j\end{array}\right) - \left(\begin{array}{c}\text{Radiation leaving}\\ \text{the entire surface } j\\ \text{that strikes surface } i\end{array}\right) \quad \textbf{(12-28)}$$

$$= A_i J_i F_{i\to j} - A_j J_j F_{j\to i} \qquad \text{(W)}$$

Applying the reciprocity relation $A_i F_{i\to j} = A_j F_{j\to i}$ yields

$$\dot{Q}_{i\to j} = A_i F_{i\to j}(J_i - J_j) \qquad \text{(W)} \quad \textbf{(12-29)}$$

Again in analogy to Ohm's law, this equation can be rearranged as

$$\dot{Q}_{i\to j} = \frac{J_i - J_j}{R_{i\to j}} \qquad \text{(W)} \quad \textbf{(12-30)}$$

where

$$R_{i\to j} = \frac{1}{A_i F_{i\to j}} \quad \textbf{(12-31)}$$

is the **space resistance** to radiation. Again the quantity $J_i - J_j$ corresponds to a *potential difference*, and the net rate of heat transfer between two surfaces corresponds to *current* in the electrical analogy, as illustrated in Figure 12–22.

The direction of the net radiation heat transfer between two surfaces depends on the relative magnitudes of J_i and J_j. A positive value for $\dot{Q}_{i\to j}$ indicates that net heat transfer is *from* surface i *to* surface j. A negative value indicates the opposite.

In an N-surface enclosure, the conservation of energy principle requires that the net heat transfer from surface i be equal to the sum of the net heat transfers from surface i to each of the N surfaces of the enclosure. That is,

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i\to j} = \sum_{j=1}^{N} A_i F_{i\to j}(J_i - J_j) = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{i\to j}} \qquad \text{(W)} \quad \textbf{(12-32)}$$

The network representation of net radiation heat transfer from surface i to the remaining surfaces of an N-surface enclosure is given in Figure 12–23. Note that $\dot{Q}_{i\to i}$ (the net rate of heat transfer from a surface to itself) is zero regardless of the shape of the surface. Combining Eqs. 12–25 and 12–32 gives

$$\frac{E_{bi} - J_i}{R_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{i\to j}} \qquad \text{(W)} \quad \textbf{(12-33)}$$

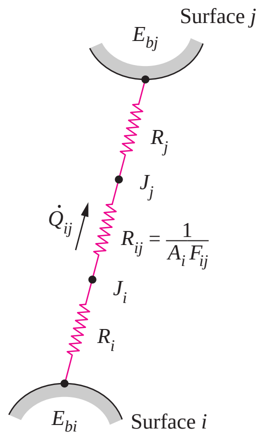


FIGURE 12–22
Electrical analogy of space resistance to radiation.

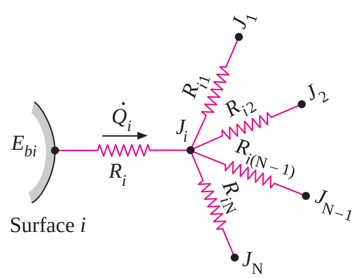


FIGURE 12–23
Network representation of net radiation heat transfer from surface i to the remaining surfaces of an N-surface enclosure.

which has the electrical analogy interpretation that *the net radiation flow from a surface through its surface resistance is equal to the sum of the radiation flows from that surface to all other surfaces through the corresponding space resistances.*

Methods of Solving Radiation Problems

In the radiation analysis of an enclosure, either the temperature or the net rate of heat transfer must be given for each of the surfaces to obtain a unique solution for the unknown surface temperatures and heat transfer rates. There are two methods commonly used to solve radiation problems. In the first method, Eqs. 12–32 (for surfaces with specified heat transfer rates) and 12–33 (for surfaces with specified temperatures) are simplified and rearranged as

Surfaces with specified net heat transfer rate $\dot{Q}_i$

$$\dot{Q}_i = A_i \sum_{j=1}^{N} F_{i \to j}(J_i - J_j) \tag{12-34}$$

Surfaces with specified temperature T_i

$$\sigma T_i^4 = J_i + \frac{1 - \varepsilon_i}{\varepsilon_i} \sum_{j=1}^{N} F_{i \to j}(J_i - J_j) \tag{12-35}$$

Note that $\dot{Q}_i = 0$ for insulated (or reradiating) surfaces, and $\sigma T_i^4 = J_i$ for black surfaces since $\varepsilon_i = 1$ in that case. Also, the term corresponding to $j = i$ will drop out from either relation since $J_i - J_j = J_i - J_i = 0$ in that case.

The equations above give N linear algebraic equations for the determination of the N unknown radiosities for an N-surface enclosure. Once the radiosities $J_1, J_2, \ldots, J_N$ are available, the unknown heat transfer rates can be determined from Eq. 12–34 while the unknown surface temperatures can be determined from Eq. 12–35. The temperatures of insulated or reradiating surfaces can be determined from $\sigma T_i^4 = J_i$. A positive value for $\dot{Q}_i$ indicates net radiation heat transfer *from* surface i to other surfaces in the enclosure while a negative value indicates net radiation heat transfer *to* the surface.

The systematic approach described above for solving radiation heat transfer problems is very suitable for use with today's popular equation solvers such as EES, Mathcad, and Matlab, especially when there are a large number of surfaces, and is known as the **direct method** (formerly, the *matrix method*, since it resulted in matrices and the solution required a knowledge of linear algebra). The second method described below, called the **network method,** is based on the electrical network analogy.

The network method was first introduced by A. K. Oppenheim in the 1950s and found widespread acceptance because of its simplicity and emphasis on the physics of the problem. The application of the method is straightforward: draw a surface resistance associated with each surface of an enclosure and connect them with space resistances. Then solve the radiation problem by treating it as an electrical network problem where the radiation heat transfer replaces the current and radiosity replaces the potential.

The network method is not practical for enclosures with more than three or four surfaces, however, because of the increased complexity of the network. Next we apply the method to solve radiation problems in two- and three-surface enclosures.

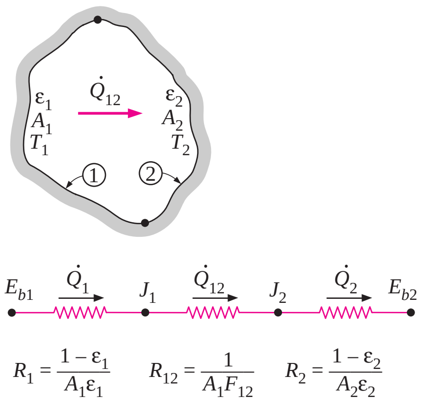


FIGURE 12–24
Schematic of a two-surface enclosure and the radiation network associated with it.

Radiation Heat Transfer in Two-Surface Enclosures

Consider an enclosure consisting of two opaque surfaces at specified temperatures T_1 and T_2, as shown in Fig. 12–24, and try to determine the net rate of radiation heat transfer between the two surfaces with the network method. Surfaces 1 and 2 have emissivities ε_1 and ε_2 and surface areas A_1 and A_2 and are maintained at uniform temperatures T_1 and T_2, respectively. There are only two surfaces in the enclosure, and thus we can write

$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$

That is, the net rate of radiation heat transfer from surface 1 to surface 2 must equal the net rate of radiation heat transfer *from* surface 1 and the net rate of radiation heat transfer *to* surface 2.

The radiation network of this two-surface enclosure consists of two surface resistances and one space resistance, as shown in Figure 12–24. In an electrical network, the electric current flowing through these resistances connected in series would be determined by dividing the potential difference between points A and B by the total resistance between the same two points. The net rate of radiation transfer is determined in the same manner and is expressed as

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2} = \dot{Q}_1 = -\dot{Q}_2$$

or

$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\,\varepsilon_1} + \dfrac{1}{A_1\,F_{12}} + \dfrac{1-\varepsilon_2}{A_2\,\varepsilon_2}} \qquad \text{(W)} \qquad \textbf{(12-36)}$$

This important result is applicable to any two gray, diffuse, opaque surfaces that form an enclosure. The view factor F_{12} depends on the geometry and must be determined first. Simplified forms of Eq. 12–36 for some familiar arrangements that form a two-surface enclosure are given in Table 12–3. Note that $F_{12} = 1$ for all of these special cases.

EXAMPLE 12–7 Radiation Heat Transfer between Parallel Plates

Two very large parallel plates are maintained at uniform temperatures $T_1 = 800$ K and $T_2 = 500$ K and have emissivities $\varepsilon_1 = 0.2$ and $\varepsilon_2 = 0.7$, respectively, as shown in Figure 12–25. Determine the net rate of radiation heat transfer between the two surfaces per unit surface area of the plates.

SOLUTION Two large parallel plates are maintained at uniform temperatures. The net rate of radiation heat transfer between the plates is to be determined.

Assumptions Both surfaces are opaque, diffuse, and gray.

Analysis The net rate of radiation heat transfer between the two plates per unit area is readily determined from Eq. 12–38 to be

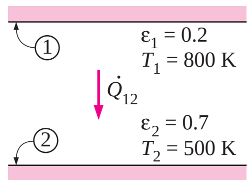
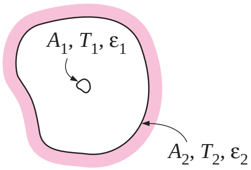


FIGURE 12–25
The two parallel plates considered in Example 12–7.

TABLE 12–3

Small object in a large cavity

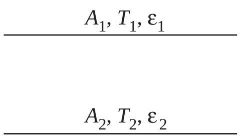


$$\frac{A_1}{A_2} \approx 0$$

$$F_{12} = 1$$

$$\dot{Q}_{12} = A_1\sigma\varepsilon_1(T_1^4 - T_2^4) \quad \textbf{(12-37)}$$

Infinitely large parallel plates

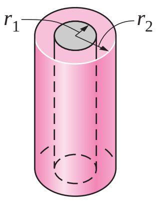


$$A_1 = A_2 = A$$

$$F_{12} = 1$$

$$\dot{Q}_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} \quad \textbf{(12-38)}$$

Infinitely long concentric cylinders

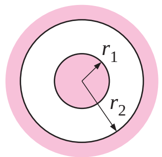


$$\frac{A_1}{A_2} = \frac{r_1}{r_2}$$

$$F_{12} = 1$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)} \quad \textbf{(12-39)}$$

Concentric spheres



$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$$

$$F_{12} = 1$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2} \quad \textbf{(12-40)}$$

$$\dot{q}_{12} = \frac{\dot{Q}_{12}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} = \frac{(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[(800\ \text{K})^4 - (500\ \text{K})^4]}{\dfrac{1}{0.2} + \dfrac{1}{0.7} - 1}$$

$$= \mathbf{3625\ W/m^2}$$

Discussion Note that heat at a net rate of 3625 W is transferred from plate 1 to plate 2 by radiation per unit surface area of either plate.

Radiation Heat Transfer in Three-Surface Enclosures

We now consider an enclosure consisting of three opaque, diffuse, gray surfaces, as shown in Figure 12–26. Surfaces 1, 2, and 3 have surface areas A_1, A_2, and A_3; emissivities ε_1, ε_2, and ε_3; and uniform temperatures T_1, T_2, and T_3, respectively. The radiation network of this geometry is constructed by following the standard procedure: draw a surface resistance associated with each of the three surfaces and connect these surface resistances with space resistances, as shown in the figure. Relations for the surface and space resistances are given by Eqs. 12–26 and 12–31. The three endpoint potentials E_{b1}, E_{b2}, and E_{b3} are considered known, since the surface temperatures are specified. Then all we need to find are the radiosities J_1, J_2, and J_3. The three equations for the determination of these three unknowns are obtained from the requirement that *the algebraic sum of the currents (net radiation heat transfer) at each node must equal zero.* That is,

$$\frac{E_{b1} - J_1}{R_1} + \frac{J_2 - J_1}{R_{12}} + \frac{J_3 - J_1}{R_{13}} = 0$$
$$\frac{J_1 - J_2}{R_{12}} + \frac{E_{b2} - J_2}{R_2} + \frac{J_3 - J_2}{R_{23}} = 0$$
$$\frac{J_1 - J_3}{R_{13}} + \frac{J_2 - J_3}{R_{23}} + \frac{E_{b3} - J_3}{R_3} = 0 \qquad \textbf{(12-41)}$$

Once the radiosities J_1, J_2, and J_3 are available, the net rate of radiation heat transfers at each surface can be determined from Eq. 12–32.

The set of equations above simplify further if one or more surfaces are "special" in some way. For example, $J_i = E_{bi} = \sigma T_i^4$ for a *black* or *reradiating* surface. Also, $\dot{Q}_i = 0$ for a reradiating surface. Finally, when the net rate of radiation heat transfer $\dot{Q}_i$ is specified at surface i instead of the temperature, the term $(E_{bi} - J_i)/R_i$ should be replaced by the specified $\dot{Q}_i$.

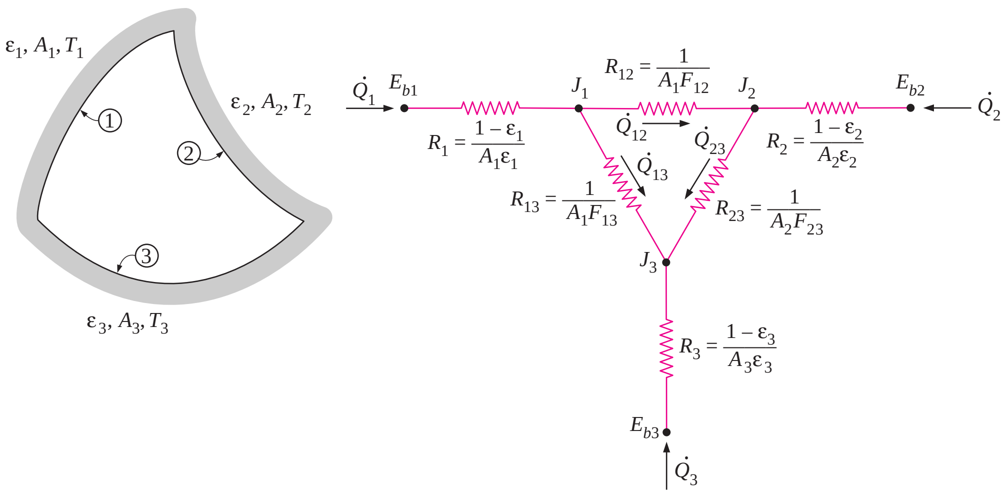


FIGURE 12–26
Schematic of a three-surface enclosure and the radiation network associated with it.

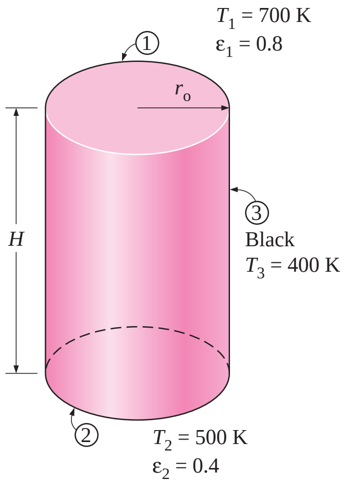


FIGURE 12–27
The cylindrical furnace considered in Example 12–8.

EXAMPLE 12–8 Radiation Heat Transfer in a Cylindrical Furnace

Consider a cylindrical furnace with $r_o = H = 1$ m, as shown in Figure 12–27. The top (surface 1) and the base (surface 2) of the furnace has emissivities $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.4$, respectively, and are maintained at uniform temperatures $T_1 = 700$ K and $T_2 = 500$ K. The side surface closely approximates a blackbody and is maintained at a temperature of $T_3 = 400$ K. Determine the net rate of radiation heat transfer at each surface during steady operation and explain how these surfaces can be maintained at specified temperatures.

SOLUTION The surfaces of a cylindrical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer at each surface during steady operation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Analysis We will solve this problem systematically using the direct method to demonstrate its use. The cylindrical furnace can be considered to be a three-surface enclosure with surface areas of

$$A_1 = A_2 = \pi r_o^2 = \pi(1\text{ m})^2 = 3.14\text{ m}^2$$
$$A_3 = 2\pi r_o H = 2\pi(1\text{ m})(1\text{ m}) = 6.28\text{ m}^2$$

The view factor from the base to the top surface is, from Figure 12–7, $F_{12} = 0.38$. Then the view factor from the base to the side surface is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \quad \rightarrow \quad F_{13} = 1 - F_{11} - F_{12} = 1 - 0 - 0.38 = 0.62$$

since the base surface is flat and thus $F_{11} = 0$. Noting that the top and bottom surfaces are symmetric about the side surface, $F_{21} = F_{12} = 0.38$ and $F_{23} = F_{13} = 0.62$. The view factor F_{31} is determined from the reciprocity relation,

$$A_1F_{13} = A_3F_{31} \quad \rightarrow \quad F_{31} = F_{13}(A_1/A_3) = (0.62)(0.314/0.628) = 0.31$$

Also, $F_{32} = F_{31} = 0.31$ because of symmetry. Now that all the view factors are available, we apply Eq. 12–35 to each surface to determine the radiosities:

$$\textit{Top surface } (i = 1): \quad \sigma T_1^4 = J_1 + \frac{1 - \varepsilon_1}{\varepsilon_1}[F_{1\rightarrow 2}(J_1 - J_2) + F_{1\rightarrow 3}(J_1 - J_3)]$$

$$\textit{Bottom surface } (i = 2): \quad \sigma T_2^4 = J_2 + \frac{1 - \varepsilon_2}{\varepsilon_2}[F_{2\rightarrow 1}(J_2 - J_1) + F_{2\rightarrow 3}(J_2 - J_3)]$$

$$\textit{Side surface } (i = 3): \quad \sigma T_3^4 = J_3 + 0 \text{ (since surface 3 is black and thus } \varepsilon_3 = 1)$$

Substituting the known quantities,

$$(5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)(700\text{ K})^4 = J_1 + \frac{1 - 0.8}{0.8}[0.38(J_1 - J_2) + 0.68(J_1 - J_3)]$$

$$(5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)(500\text{ K})^4 = J_2 + \frac{1 - 0.4}{0.4}[0.28(J_2 - J_1) + 0.68(J_2 - J_3)]$$

$$(5.67 \times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)(400\text{ K})^4 = J_3$$

Solving the equations above for J_1, J_2, and J_3 gives

$$J_1 = 11{,}418 \text{ W/m}^2,\ J_2 = 4562 \text{ W/m}^2, \quad \text{and} \quad J_3 = 1452 \text{ W/m}^2$$

Then the net rates of radiation heat transfer at the three surfaces are determined from Eq. 12–34 to be

$$\begin{aligned}
\dot{Q}_1 &= A_1[F_{1\to 2}(J_1 - J_2) + F_{1\to 3}(J_1 - J_3)]\\
&= (3.14 \text{ m}^2)[0.38(11{,}418 - 4562) + 0.62(11{,}418 - 1452)] \text{ W/m}^2\\
&= \mathbf{27.6 \times 10^3 \text{ W} = 27.6 \text{ kW}}\\
\dot{Q}_2 &= A_2[F_{2\to 1}(J_2 - J_1) + F_{2\to 3}(J_2 - J_3)]\\
&= (3.12 \text{ m}^2)[0.38(4562 - 11{,}418) + 0.62(4562 - 1452)] \text{ W/m}^2\\
&= \mathbf{-2.13 \times 10^3 \text{ W} = -2.13 \text{ kW}}\\
\dot{Q}_3 &= A_3[F_{3\to 1}(J_3 - J_1) + F_{3\to 2}(J_3 - J_2)]\\
&= (6.28 \text{ m}^2)[0.31(1452 - 11{,}418) + 0.31(1452 - 4562)] \text{ W/m}^2\\
&= \mathbf{-25.5 \times 10^3 \text{ W} = -25.5 \text{ kW}}
\end{aligned}$$

Note that the direction of net radiation heat transfer is *from* the top surface *to* the base and side surfaces, and the algebraic sum of these three quantities must be equal to zero. That is,

$$\dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 = 27.6 + (-2.13) + (-25.5) \cong 0$$

Discussion To maintain the surfaces at the specified temperatures, we must supply heat to the top surface continuously at a rate of 27.6 kW while removing 2.13 kW from the base and 25.5 kW from the side surfaces.

The direct method presented here is straightforward, and it does not require the evaluation of radiation resistances. Also, it can be applied to enclosures with any number of surfaces in the same manner.

EXAMPLE 12–9 Radiation Heat Transfer in a Triangular Furnace

A furnace is shaped like a long equilateral triangular duct, as shown in Figure 12–28. The width of each side is 1 m. The base surface has an emissivity of 0.7 and is maintained at a uniform temperature of 600 K. The heated left-side surface closely approximates a blackbody at 1000 K. The right-side surface is well insulated. Determine the rate at which heat must be supplied to the heated side externally per unit length of the duct in order to maintain these operating conditions.

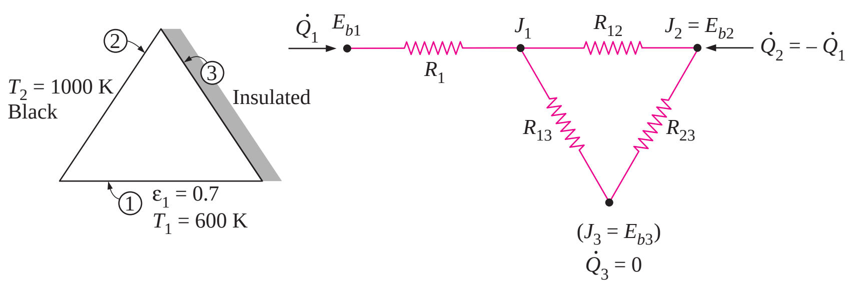


FIGURE 12–28
The triangular furnace considered in Example 12–9.

SOLUTION Two of the surfaces of a long equilateral triangular furnace are maintained at uniform temperatures while the third surface is insulated. The external rate of heat transfer to the heated side per unit length of the duct during steady operation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Analysis The furnace can be considered to be a three-surface enclosure with a radiation network as shown in the figure, since the duct is very long and thus the end effects are negligible. We observe that the view factor from any surface to any other surface in the enclosure is 0.5 because of symmetry. Surface 3 is a reradiating surface since the net rate of heat transfer at that surface is zero. Then we must have $\dot{Q}_1 = -\dot{Q}_2$, since the entire heat lost by surface 1 must be gained by surface 2. The radiation network in this case is a simple series–parallel connection, and we can determine $\dot{Q}_1$ directly from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{R_1 + \left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1}} = \frac{E_{b1} - E_{b2}}{\dfrac{1 - \varepsilon_1}{A_1\,\varepsilon_1} + \left(A_1 F_{12} + \dfrac{1}{1/A_1 F_{13} + 1/A_2 F_{23}}\right)^{-1}}$$

where

$$A_1 = A_2 = A_3 = wL = 1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2 \qquad \text{(per unit length of the duct)}$$
$$F_{12} = F_{13} = F_{23} = 0.5 \qquad \text{(symmetry)}$$
$$E_{b1} = \sigma T_1^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(600 \text{ K})^4 = 7348 \text{ W/m}^2$$
$$E_{b2} = \sigma T_2^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1000 \text{ K})^4 = 56{,}700 \text{ W/m}^2$$

Substituting,

$$\dot{Q}_1 = \frac{(56{,}700 - 7348) \text{ W/m}^2}{\dfrac{1 - 0.7}{0.7 \times 1 \text{ m}^2} + \left[(0.5 \times 1 \text{ m}^2) + \dfrac{1}{1/(0.5 \times 1 \text{ m}^2) + 1/(0.5 \times 1 \text{ m}^2)}\right]^{-1}}$$
$$= \mathbf{28.0 \times 10^3 = 28.0 \text{ kW}}$$

Therefore, heat at a rate of 28 kW must be supplied to the heated surface per unit length of the duct to maintain steady operation in the furnace.

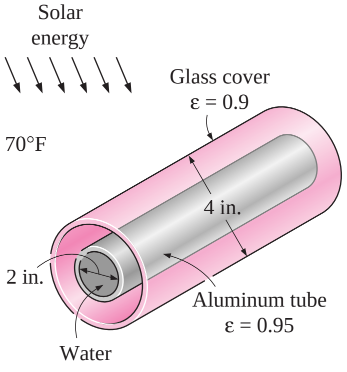


FIGURE 12–29
Schematic for Example 12–10.

EXAMPLE 12–10 Heat Transfer through a Tubular Solar Collector

A solar collector consists of a horizontal aluminum tube having an outer diameter of 2 in. enclosed in a concentric thin glass tube of 4-in. diameter, as shown in Figure 12–29. Water is heated as it flows through the tube, and the space between the aluminum and the glass tubes is filled with air at 1 atm pressure. The pump circulating the water fails during a clear day, and the water temperature in the tube starts rising. The aluminum tube absorbs solar radiation at a rate of 30 Btu/h per foot length, and the temperature of the ambient air outside is 70°F. The emissivities of the tube and the glass cover are 0.95 and 0.9, respectively. Taking the effective sky temperature to be 50°F, determine the temperature of the aluminum tube when steady operating conditions are established (i.e., when the rate of heat loss from the tube equals the amount of solar energy gained by the tube).

SOLUTION The circulating pump of a solar collector that consists of a horizontal tube and its glass cover fails. The equilibrium temperature of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube and its cover are isothermal. **3** Air is an ideal gas. **4** The surfaces are opaque, diffuse, and gray for infrared radiation. **5** The glass cover is transparent to solar radiation.

Properties The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and thus we cannot evaluate the average temperatures. Therefore, we will assume the glass temperature to be 110°F, and use properties at an anticipated average temperature of (70 + 110)/2 = 90°F (Table A-15E),

$$k = 0.01505 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F} \qquad \text{Pr} = 0.7275$$

$$\nu = 0.6310 \text{ ft}^2/\text{h} = 1.753 \times 10^{-4} \text{ ft}^2/\text{s} \qquad \beta = \frac{1}{T_{\text{ave}}} = \frac{1}{550 \text{ R}}$$

Analysis This problem was solved in Chapter 9 by disregarding radiation heat transfer. Now we will repeat the solution by considering natural convection and radiation occurring simultaneously.

We have a horizontal cylindrical enclosure filled with air at 1 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{\text{tube-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q}_{\text{solar gain}} = 30 \text{ Btu/h} \qquad \text{(per foot of tube)}$$

The heat transfer surface area of the glass cover is

$$A_o = A_{\text{glass}} = (\pi D_o L) = \pi(4/12 \text{ ft})(1 \text{ ft}) = 1.047 \text{ ft}^2 \qquad \text{(per foot of tube)}$$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, it is clear that the solution will require a trial-and-error approach. Assuming the glass cover temperature to be 110°F, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$\begin{aligned}
\text{Ra}_{D_o} &= \frac{g\beta(T_o - T_\infty) D_o^3}{\nu^2} \text{Pr} \\
&= \frac{(32.2 \text{ ft/s}^2)[1/(550 \text{ R})](110 - 70 \text{ R})(4/12 \text{ ft})^3}{(1.753 \times 10^{-4} \text{ ft}^2/\text{s})^2}(0.7275) = 2.054 \times 10^6 \\
\text{Nu} &= \left\{0.6 + \frac{0.387 \text{ Ra}_{D_o}^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.054 \times 10^6)^{1/6}}{[1 + (0.559/0.7275)^{9/16}]^{8/27}}\right\}^2 \\
&= 17.89
\end{aligned}$$

$$h_o = \frac{k}{D_o}\text{Nu} = \frac{0.01505\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}}{4/12\ \text{ft}}(17.89) = 0.8075\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

$$\dot{Q}_{o,\ \text{conv}} = h_o A_o(T_o - T_\infty) = (0.8075\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(1.047\ \text{ft}^2)(110 - 70){}^\circ\text{F}$$
$$= 33.8\ \text{Btu/h}$$

Also,

$$\dot{Q}_{o,\ \text{rad}} = \varepsilon_o \sigma A_o(T_o^4 - T_{\text{sky}}^4)$$
$$= (0.9)(0.1714 \times 10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(1.047\ \text{ft}^2)[(570\ \text{R})^4 - (510\ \text{R})^4]$$
$$= 61.2\ \text{Btu/h}$$

Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{o,\ \text{total}} = \dot{Q}_{o,\ \text{conv}} + \dot{Q}_{o,\ \text{rad}} = 33.8 + 61.2 = 95.0\ \text{Btu/h}$$

which is much larger than 30 Btu/h. Therefore, the assumed temperature of 110°F for the glass cover is high. Repeating the calculations with lower temperatures (including the evaluation of properties), the glass cover temperature corresponding to 30 Btu/h is determined to be 78°F (it would be 106°F if radiation were ignored).

The temperature of the aluminum tube is determined in a similar manner using the natural convection and radiation relations for two horizontal concentric cylinders. The characteristic length in this case is the distance between the two cylinders, which is

$$L_c = (D_o - D_i)/2 = (4 - 2)/2 = 1\ \text{in.} = 1/12\ \text{ft}$$

Also,

$$A_i = A_{\text{tube}} = (\pi D_i L) = \pi(2\ /12\ \text{ft})(1\ \text{ft}) = 0.5236\ \text{ft}^2 \qquad \text{(per foot of tube)}$$

We start the calculations by assuming the tube temperature to be 122°F, and thus an average temperature of (78 + 122)/2 = 100°F = 640 R. Using properties at 100°F,

$$\text{Ra}_L = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr}$$
$$= \frac{(32.2\ \text{ft/s}^2)[1/(640\ \text{R})](122 - 78\ \text{R})(1/12\ \text{ft})^3}{(1.809 \times 10^{-4}\ \text{ft}^2/\text{s})^2}(0.726) = 3.249 \times 10^4$$

The effective thermal conductivity is

$$F_{\text{cyc}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3\,(D_i^{-3/5} + D_o^{-3/5})^5}$$
$$= \frac{[\ln(4/2)]^4}{(1/12\ \text{ft})^3\,[(2/12\ \text{ft})^{-3/5} + (4/12\ \text{ft})^{-3/5}]^5} = 0.1466$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{cyc}}\text{Ra}_L)^{1/4}$$
$$= 0.386(0.01529\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})\left(\frac{0.726}{0.861 + 0.726}\right)(0.1466 \times 3.249 \times 10^4)^{1/4}$$
$$= 0.04032\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$$

Then the rate of heat transfer between the cylinders by convection becomes

$$\dot{Q}_{i,\,\text{conv}} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o)$$
$$= \frac{2\pi(0.04032\ \text{Btu/h}\cdot\text{ft}\ °\text{F})}{\ln(4/2)}(122 - 78)°\text{F} = 16.1\ \text{Btu/h}$$

Also,

$$\dot{Q}_{i,\,\text{rad}} = \frac{\sigma A_i\,(T_i^4 - T_o^4)}{\dfrac{1}{\varepsilon_i} + \dfrac{1-\varepsilon_o}{\varepsilon_o}\left(\dfrac{D_i}{D_o}\right)}$$
$$= \frac{(0.1714 \times 10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(0.5236\ \text{ft}^2)[(582\ \text{R})^4 - (538\ \text{R})^4]}{\dfrac{1}{0.95} + \dfrac{1-0.9}{0.9}\left(\dfrac{2\ \text{in.}}{4\ \text{in.}}\right)}$$
$$= 25.1\ \text{Btu/h}$$

Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{i,\,\text{total}} = \dot{Q}_{i,\,\text{conv}} + \dot{Q}_{i,\,\text{rad}} = 16.1 + 25.1 = 41.1\ \text{Btu/h}$$

which is larger than 30 Btu/h. Therefore, the assumed temperature of 122°F for the tube is high. By trying other values, the tube temperature corresponding to 30 Btu/h is determined to be **112°F** (it would be 180°F if radiation were ignored). Therefore, the tube will reach an equilibrium temperature of 112°F when the pump fails.

Discussion It is clear from the results obtained that radiation should always be considered in systems that are heated or cooled by natural convection, unless the surfaces involved are polished and thus have very low emissivities.

12–5 ▪ RADIATION SHIELDS AND THE RADIATION EFFECT

Radiation heat transfer between two surfaces can be reduced greatly by inserting a thin, high-reflectivity (low-emissivity) sheet of material between the two surfaces. Such highly reflective thin plates or shells are called **radiation shields.** Multilayer radiation shields constructed of about 20 sheets per cm thickness separated by evacuated space are commonly used in cryogenic and space applications. Radiation shields are also used in temperature measurements of fluids to reduce the error caused by the radiation effect when the temperature sensor is exposed to surfaces that are much hotter or colder than the fluid itself. The role of the radiation shield is to reduce the rate of radiation heat transfer by placing additional resistances in the path of radiation heat flow. The lower the emissivity of the shield, the higher the resistance.

Radiation heat transfer between two large parallel plates of emissivities ε_1 and ε_2 maintained at uniform temperatures T_1 and T_2 is given by Eq. 12–38:

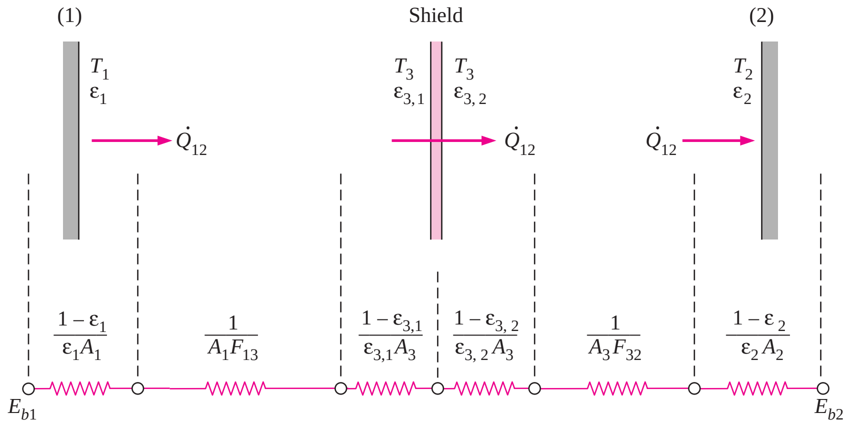


FIGURE 12–30
The radiation shield placed between two parallel plates and the radiation network associated with it.

$$\dot{Q}_{12,\text{ no shield}} = \frac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

Now consider a radiation shield placed between these two plates, as shown in Figure 12–30. Let the emissivities of the shield facing plates 1 and 2 be $\varepsilon_{3,1}$ and $\varepsilon_{3,2}$, respectively. Note that the emissivity of different surfaces of the shield may be different. The radiation network of this geometry is constructed, as usual, by drawing a surface resistance associated with each surface and connecting these surface resistances with space resistances, as shown in the figure. The resistances are connected in series, and thus the rate of radiation heat transfer is

$$\dot{Q}_{12,\text{ one shield}} = \frac{E_{b1} - E_{b2}}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_{3,1}}{A_3\varepsilon_{3,1}} + \dfrac{1-\varepsilon_{3,2}}{A_3\varepsilon_{3,2}} + \dfrac{1}{A_3F_{32}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} \qquad \textbf{(12-42)}$$

Noting that $F_{13} = F_{23} = 1$ and $A_1 = A_2 = A_3 = A$ for infinite parallel plates, Eq. 12–42 simplifies to

$$\dot{Q}_{12,\text{ one shield}} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_{3,1}} + \dfrac{1}{\varepsilon_{3,2}} - 1\right)} \qquad \textbf{(12-43)}$$

where the terms in the second set of parentheses in the denominator represent the additional resistance to radiation introduced by the shield. The appearance of the equation above suggests that parallel plates involving multiple radiation shields can be handled by adding a group of terms like those in the second set of parentheses to the denominator for each radiation shield. Then the radiation heat transfer through large parallel plates separated by N radiation shields becomes

$$\dot{Q}_{12,\ N\text{ shields}} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_{3,1}} + \dfrac{1}{\varepsilon_{3,2}} - 1\right) + \cdots + \left(\dfrac{1}{\varepsilon_{N,1}} + \dfrac{1}{\varepsilon_{N,2}} - 1\right)} \qquad \textbf{(12-44)}$$

If the emissivities of all surfaces are equal, Eq. 12–44 reduces to

$$\dot{Q}_{12,\,N\text{ shields}} = \frac{A\sigma(T_1^4 - T_2^4)}{(N+1)\left(\dfrac{1}{\varepsilon} + \dfrac{1}{\varepsilon} - 1\right)} = \frac{1}{N+1}\,\dot{Q}_{12,\,\text{no shield}} \tag{12-45}$$

Therefore, when all emissivities are equal, 1 shield reduces the rate of radiation heat transfer to one-half, 9 shields reduce it to one-tenth, and 19 shields reduce it to one-twentieth (or 5 percent) of what it was when there were no shields.

The equilibrium temperature of the radiation shield T_3 in Figure 12–30 can be determined by expressing Eq. 12–43 for $\dot{Q}_{13}$ or $\dot{Q}_{23}$ (which involves T_3) after evaluating $\dot{Q}_{12}$ from Eq. 12–43 and noting that $\dot{Q}_{12} = \dot{Q}_{13} = \dot{Q}_{23}$ when steady conditions are reached.

Radiation shields used to reduce the rate of radiation heat transfer between concentric cylinders and spheres can be handled in a similar manner. In case of one shield, Eq. 12–42 can be used by taking $F_{13} = F_{23} = 1$ for both cases and by replacing the A's by the proper area relations.

Radiation Effect on Temperature Measurements

A temperature measuring device indicates the temperature of its *sensor,* which is supposed to be, but is not necessarily, the temperature of the medium that the sensor is in. When a thermometer (or any other temperature measuring device such as a thermocouple) is placed in a medium, heat transfer takes place between the sensor of the thermometer and the medium by convection until the sensor reaches the temperature of the medium. But when the sensor is surrounded by surfaces that are at a different temperature than the fluid, radiation exchange will take place between the sensor and the surrounding surfaces. When the heat transfers by convection and radiation balance each other, the sensor will indicate a temperature that falls between the fluid and surface temperatures. Below we develop a procedure to account for the radiation effect and to determine the actual fluid temperature.

Consider a thermometer that is used to measure the temperature of a fluid flowing through a large channel whose walls are at a lower temperature than the fluid (Fig. 12–31). Equilibrium will be established and the reading of the thermometer will stabilize when heat gain by convection, as measured by the sensor, equals heat loss by radiation (or vice versa). That is, on a unit-area basis,

$$\dot{q}_{\text{conv, to sensor}} = \dot{q}_{\text{rad, from sensor}}$$
$$h(T_f - T_{\text{th}}) = \varepsilon_{\text{th}}\sigma(T_{\text{th}}^4 - T_w^4)$$

or

$$T_f = T_{\text{th}} + \frac{\varepsilon_{\text{th}}\,\sigma(T_{\text{th}}^4 - T_w^4)}{h} \qquad \text{(K)} \tag{12-46}$$

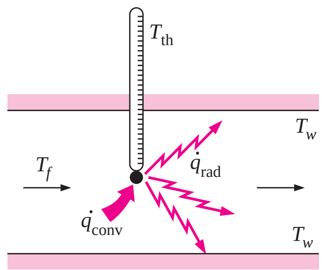


FIGURE 12–31
A thermometer used to measure the temperature of a fluid in a channel.

where

T_f = actual temperature of the fluid, K
T_{th} = temperature value measured by the thermometer, K
T_w = temperature of the surrounding surfaces, K
h = convection heat transfer coefficient, $W/m^2 \cdot K$
ε = emissivity of the sensor of the thermometer

The last term in Eq. 12–46 is due to the *radiation effect* and represents the *radiation correction.* Note that the radiation correction term is most significant when the convection heat transfer coefficient is small and the emissivity of the surface of the sensor is large. Therefore, the sensor should be coated with a material of high reflectivity (low emissivity) to reduce the radiation effect.

Placing the sensor in a radiation shield without interfering with the fluid flow also reduces the radiation effect. The sensors of temperature measurement devices used outdoors must be protected from direct sunlight since the radiation effect in that case is sure to reach unacceptable levels.

The radiation effect is also a significant factor in *human comfort* in heating and air-conditioning applications. A person who feels fine in a room at a specified temperature may feel chilly in another room at the same temperature as a result of the radiation effect if the walls of the second room are at a considerably lower temperature. For example, most people will feel comfortable in a room at 22°C if the walls of the room are also roughly at that temperature. When the wall temperature drops to 5°C for some reason, the interior temperature of the room must be raised to at least 27°C to maintain the same level of comfort. Therefore, well-insulated buildings conserve energy not only by reducing the heat loss or heat gain, but also by allowing the thermostats to be set at a lower temperature in winter and at a higher temperature in summer without compromising the comfort level.

EXAMPLE 12–11 Radiation Shields

A thin aluminum sheet with an emissivity of 0.1 on both sides is placed between two very large parallel plates that are maintained at uniform temperatures $T_1 = 800$ K and $T_2 = 500$ K and have emissivities $\varepsilon_1 = 0.2$ and $\varepsilon_2 = 0.7$, respectively, as shown in Fig. 12–32. Determine the net rate of radiation heat transfer between the two plates per unit surface area of the plates and compare the result to that without the shield.

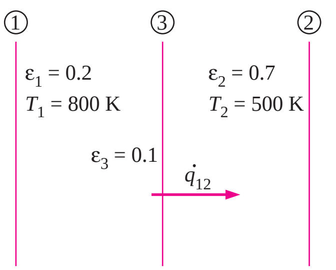


FIGURE 12–32
Schematic for Example 12–11.

SOLUTION A thin aluminum sheet is placed between two large parallel plates maintained at uniform temperatures. The net rates of radiation heat transfer between the two plates with and without the radiation shield are to be determined.

Assumptions The surfaces are opaque, diffuse, and gray.

Analysis The net rate of radiation heat transfer between these two plates without the shield was determined in Example 12–7 to be 3625 W/m^2. Heat transfer in the presence of one shield is determined from Eq. 12–43 to be

$$\dot{q}_{12,\text{ one shield}} = \frac{\dot{Q}_{12,\text{ one shield}}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_{3,1}} + \dfrac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$= \frac{(5.67 \times 10^{-8}\text{ W/m}^2 \cdot \text{K}^4)[(800\text{ K})^4 - (500\text{ K})^4]}{\left(\dfrac{1}{0.2} + \dfrac{1}{0.7} - 1\right) + \left(\dfrac{1}{0.1} + \dfrac{1}{0.1} - 1\right)}$$

$$= \mathbf{806\ W/m^2}$$

Discussion Note that the rate of radiation heat transfer reduces to about one-fourth of what it was as a result of placing a radiation shield between the two parallel plates.

EXAMPLE 12–12 Radiation Effect on Temperature Measurements

A thermocouple used to measure the temperature of hot air flowing in a duct whose walls are maintained at T_w = 400 K shows a temperature reading of T_{th} = 650 K (Fig. 12–33). Assuming the emissivity of the thermocouple junction to be ε = 0.6 and the convection heat transfer coefficient to be h = 80 W/m² · °C, determine the actual temperature of the air.

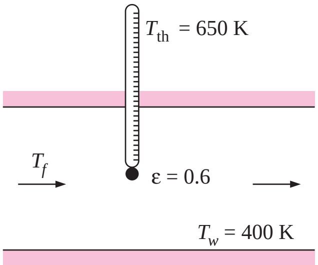


FIGURE 12–33
Schematic for Example 12–12.

SOLUTION The temperature of air in a duct is measured. The radiation effect on the temperature measurement is to be quantified, and the actual air temperature is to be determined.

Assumptions The surfaces are opaque, diffuse, and gray.

Analysis The walls of the duct are at a considerably lower temperature than the air in it, and thus we expect the thermocouple to show a reading lower than the actual air temperature as a result of the radiation effect. The actual air temperature is determined from Eq. 12–46 to be

$$T_f = T_{th} + \frac{\varepsilon_{th}\,\sigma(T_{th}^4 - T_w^4)}{h}$$

$$= (650\text{ K}) + \frac{0.6 \times (5.67 \times 10^{-8}\text{ W/m}^2 \cdot \text{K}^4)[(650\text{ K})^4 - (400\text{ K})^4]}{80\text{ W/m}^2 \cdot {}^\circ\text{C}}$$

$$= \mathbf{715\ K}$$

Note that the radiation effect causes a difference of 65°C (or 65 K since °C ≡ K for temperature differences) in temperature reading in this case.

12–6 ▪ RADIATION EXCHANGE WITH EMITTING AND ABSORBING GASES

So far we considered radiation heat transfer between surfaces separated by a medium that does not emit, absorb, or scatter radiation—a nonparticipating medium that is completely transparent to thermal radiation. A vacuum satisfies this condition perfectly, and air at ordinary temperatures and pressures comes very close. Gases that consist of monatomic molecules such as Ar and He and symmetric diatomic molecules such as N_2 and O_2 are essentially transparent to radiation, except at extremely high temperatures at which ionization occurs. Therefore, atmospheric air can be considered to be a nonparticipating medium in radiation calculations.

Gases with asymmetric molecules such as H_2O, CO_2, CO, SO_2, and hydrocarbons H_nC_m may participate in the radiation process by absorption at moderate temperatures, and by absorption and emission at high temperatures such as those encountered in combustion chambers. Therefore, air or any other medium that contains such gases with asymmetric molecules at sufficient concentrations must be treated as a participating medium in radiation calculations. Combustion gases in a furnace or a combustion chamber, for example, contain sufficient amounts of H_2O and CO_2, and thus the emission and absorption of gases in furnaces must be taken into consideration.

The presence of a participating medium complicates the radiation analysis considerably for several reasons:

- A participating medium emits and absorbs radiation throughout its entire volume. That is, gaseous radiation is a *volumetric phenomena,* and thus it depends on the size and shape of the body. This is the case even if the temperature is uniform throughout the medium.
- Gases emit and absorb radiation at a number of narrow wavelength bands. This is in contrast to solids, which emit and absorb radiation over the entire spectrum. Therefore, the gray assumption may not always be appropriate for a gas even when the surrounding surfaces are gray.
- The emission and absorption characteristics of the constituents of a gas mixture also depends on the temperature, pressure, and composition of the gas mixture. Therefore, the presence of other participating gases affects the radiation characteristics of a particular gas.

The propagation of radiation through a medium can be complicated further by presence of *aerosols* such as dust, ice particles, liquid droplets, and soot (unburned carbon) particles that *scatter* radiation. Scattering refers to the change of direction of radiation due to reflection, refraction, and diffraction. Scattering caused by gas molecules themselves is known as the *Rayleigh scattering*, and it has negligible effect on heat transfer. Radiation transfer in scattering media is considered in advanced books such as the ones by Modest (1993, Ref. 12) and Siegel and Howell (1992, Ref. 14).

The participating medium can also be semitransparent liquids or solids such as water, glass, and plastics. To keep complexities to a manageable level, we will limit our consideration to gases that emit and absorb radiation. In particular, we will consider the emission and absorption of radiation by H_2O and CO_2 only since they are the participating gases most commonly encountered in practice (combustion products in furnaces and combustion chambers burning hydrocarbon fuels contain both gases at high concentrations), and they are sufficient to demonstrate the basic principles involved.

Radiation Properties of a Participating Medium

Consider a participating medium of thickness L. A spectral radiation beam of intensity $I_{\lambda,0}$ is incident on the medium, which is attenuated as it propagates due to absorption. The decrease in the intensity of radiation as it passes through a layer of thickness dx is proportional to the intensity itself and the thickness dx. This is known as **Beer's law,** and is expressed as (Fig. 12–34)

$$dI_\lambda(x) = -\kappa_\lambda I_\lambda(x)dx \tag{12-47}$$

where the constant of proportionality κ_λ is the **spectral absorption coefficient** of the medium whose unit is m^{-1} (from the requirement of dimensional homogeneity). This is just like the amount of interest earned by a bank account during a time interval being proportional to the amount of money in the account and the time interval, with the interest rate being the constant of proportionality.

Separating the variables and integrating from $x = 0$ to $x = L$ gives

$$\frac{I_{\lambda,L}}{I_{\lambda,0}} = e^{-\kappa_\lambda L} \tag{12-48}$$

where we have assumed the absorptivity of the medium to be independent of x. Note that radiation intensity decays exponentially in accordance with Beer's law.

The **spectral transmissivity** of a medium can be defined as the ratio of the intensity of radiation leaving the medium to that entering the medium. That is,

$$\tau_\lambda = \frac{I_{\lambda,L}}{I_{\lambda,0}} = e^{-\kappa_\lambda L} \tag{12-49}$$

Note that $\tau_\lambda = 1$ when no radiation is absorbed and thus radiation intensity remains constant. Also, the spectral transmissivity of a medium represents the fraction of radiation transmitted by the medium at a given wavelength.

Radiation passing through a nonscattering (and thus nonreflecting) medium is either absorbed or transmitted. Therefore $\alpha_\lambda + \tau_\lambda = 1$, and the **spectral absorptivity** of a medium of thickness L is

$$\alpha_\lambda = 1 - \tau_\lambda = 1 - e^{-\kappa_\lambda L} \tag{12-50}$$

From Kirchoff's law, the **spectral emissivity** of the medium is

$$\varepsilon_\lambda = \alpha_\lambda = 1 - e^{-\kappa_\lambda L} \tag{12-51}$$

Note that the spectral absorptivity, transmissivity, and emissivity of a medium are dimensionless quantities, with values less than or equal to 1. The spectral absorption coefficient of a medium (and thus ε_λ, α_λ, and τ_λ), in general, vary with wavelength, temperature, pressure, and composition.

For an *optically thick* medium (a medium with a large value of $\kappa_\lambda L$), Eq. 12–51 gives $\varepsilon_\lambda \approx \alpha_\lambda \approx 1$. For $\kappa_\lambda L = 5$, for example, $\varepsilon_\lambda = \alpha_\lambda = 0.993$. Therefore, an optically thick medium emits like a blackbody at the given wavelength. As a result, an optically thick absorbing-emitting medium with no significant scattering at a given temperature T_g can be viewed as a "black surface" at T_g since it will absorb essentially all the radiation passing through it, and it will emit the maximum possible radiation that can be emitted by a surface at T_g, which is $E_{b\lambda}(T_g)$.

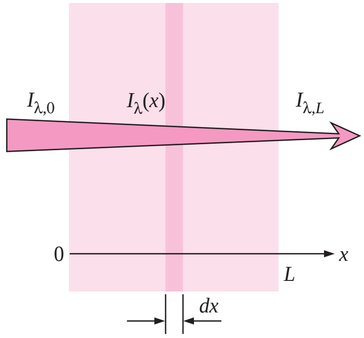


FIGURE 12–34
The attenuation of a radiation beam while passing through an absorbing medium of thickness L.

Emissivity and Absorptivity of Gases and Gas Mixtures

The spectral absorptivity of CO_2 is given in Figure 12–35 as a function of wavelength. The various peaks and dips in the figure together with discontinuities show clearly the band nature of absorption and the strong nongray characteristics. The shape and the width of these absorption bands vary with temperature and pressure, but the magnitude of absorptivity also varies with the thickness of the gas layer. Therefore, absorptivity values without specified thickness and pressure are meaningless.

The nongray nature of properties should be considered in radiation calculations for high accuracy. This can be done using a band model, and thus performing calculations for each absorption band. However, satisfactory results can be obtained by assuming the gas to be gray, and using an effective total absorptivity and emissivity determined by some averaging process. Charts for the total emissivities of gases are first presented by Hottel (Ref. 6), and they have been widely used in radiation calculations with reasonable accuracy. Alternative emissivity charts and calculation procedures have been developed more recently by Edwards and Matavosian (Ref. 2). Here we present the Hottel approach because of its simplicity.

Even with gray assumption, the total emissivity and absorptivity of a gas depends on the geometry of the gas body as well as the temperature, pressure, and composition. Gases that participate in radiation exchange such as CO_2 and H_2O typically coexist with nonparticipating gases such as N_2 and O_2, and thus radiation properties of an absorbing and emitting gas are usually reported for a mixture of the gas with nonparticipating gases rather than the pure gas. The emissivity and absorptivity of a gas component in a mixture depends primarily on its density, which is a function of temperature and partial pressure of the gas.

The emissivity of H_2O vapor in a mixture of nonparticipating gases is plotted in Figure 12–36*a* for a total pressure of $P = 1$ atm as a function of gas temperature T_g for a range of values for $P_w L$, where P_w is the partial pressure of water vapor and L is the mean distance traveled by the radiation beam.

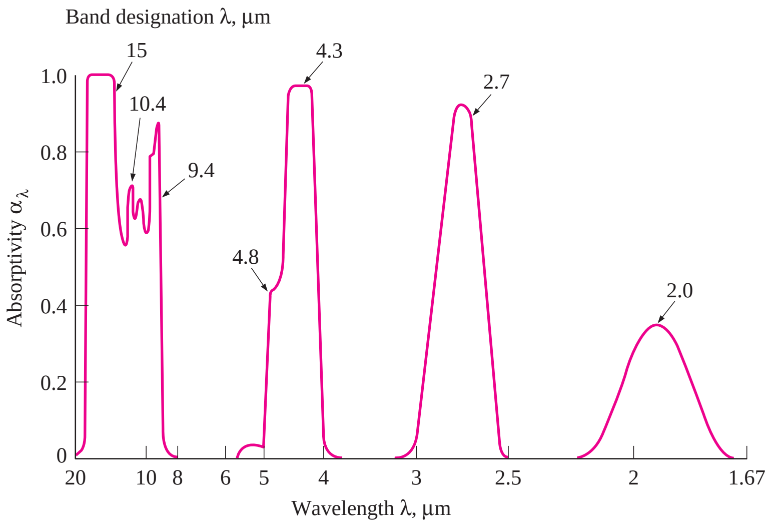


FIGURE 12–35
Spectral absorptivity of CO_2 at 830 K and 10 atm for a path length of 38.8 cm (from Siegel and Howell, 1992).

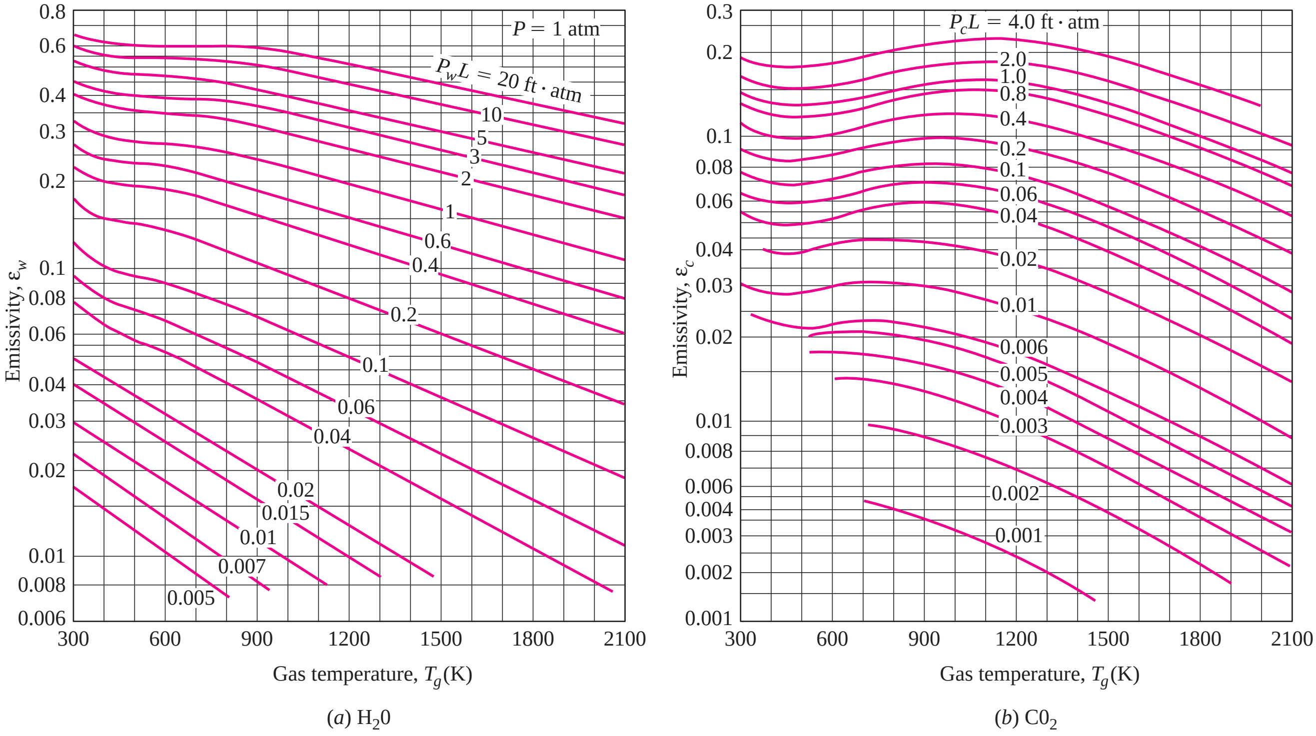


FIGURE 12–36

Emissivities of H_2O and CO_2 gases in a mixture of nonparticipating gases at a total pressure of 1 atm for a mean beam length of L (1 m · atm = 3.28 ft · atm) (from Hottel, 1954, Ref. 6).

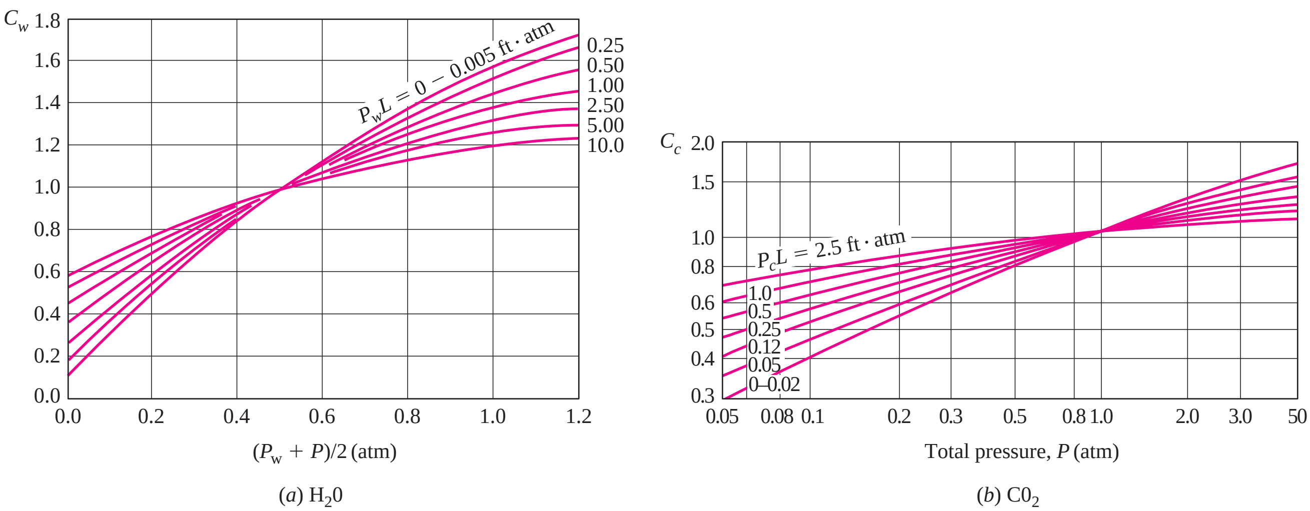


FIGURE 12–37

Correction factors for the emissivities of H_2O and CO_2 gases at pressures other than 1 atm for use in the relations $\varepsilon_w = C_w \varepsilon_{w,\,1\text{ atm}}$ and $\varepsilon_c = C_c \varepsilon_{c,\,1\text{ atm}}$ (1 m · atm = 3.28 ft · atm) (from Hottel, 1954, Ref. 6).

Emissivity at a total pressure P other than $P = 1$ atm is determined by multiplying the emissivity value at 1 atm by a **pressure correction factor** C_w obtained from Figure 12–37*a* for water vapor. That is,

$$\varepsilon_w = C_w \varepsilon_{w,\,1\,\text{atm}} \tag{12-52}$$

Note that $C_w = 1$ for $P = 1$ atm and thus $(P_w + P)/2 \cong 0.5$ (a very low concentration of water vapor is used in the preparation of the emissivity chart in Fig. 12–36*a* and thus P_w is very low). Emissivity values are presented in a similar manner for a mixture of CO_2 and nonparticipating gases in Fig. 12–36*b* and 12–37*b*.

Now the question that comes to mind is what will happen if the CO_2 and H_2O gases exist *together* in a mixture with nonparticipating gases. The emissivity of each participating gas can still be determined as explained above using its partial pressure, but the effective emissivity of the mixture cannot be determined by simply adding the emissivities of individual gases (although this would be the case if different gases emitted at different wavelengths). Instead, it should be determined from

$$\begin{aligned}\varepsilon_g &= \varepsilon_c + \varepsilon_w - \Delta\varepsilon \\ &= C_c \varepsilon_{c,\,1\,\text{atm}} + C_w \varepsilon_{w,\,1\,\text{atm}} - \Delta\varepsilon\end{aligned} \tag{12-53}$$

where $\Delta\varepsilon$ is the **emissivity correction factor,** which accounts for the overlap of emission bands. For a gas mixture that contains both CO_2 and H_2O gases, $\Delta\varepsilon$ is plotted in Figure 12–38.

The emissivity of a gas also depends on the *mean length* an emitted radiation beam travels in the gas before reaching a bounding surface, and thus the shape and the size of the gas body involved. During their experiments in the 1930s, Hottel and his coworkers considered the emission of radiation from a hemispherical gas body to a small surface element located at the center of the base of the hemisphere. Therefore, the given charts represent emissivity data for the emission of radiation from a hemispherical gas body of radius L toward the center of the base of the hemisphere. It is certainly desirable to extend the reported emissivity data to gas bodies of other geometries, and this

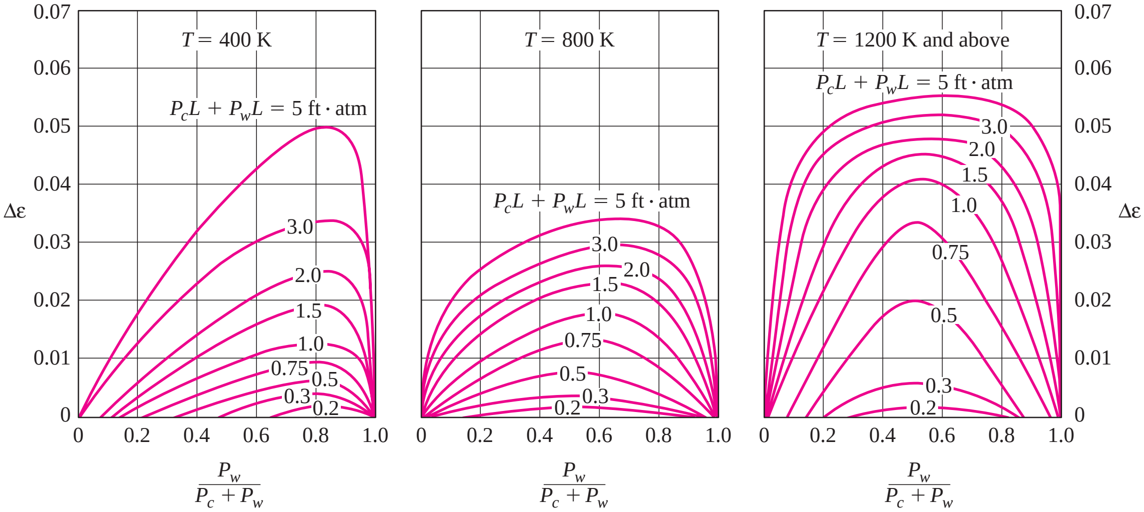


FIGURE 12–38

Emissivity correction $\Delta\varepsilon$ for use in $\varepsilon_g = \varepsilon_w + \varepsilon_c - \Delta\varepsilon$ when both CO_2 and H_2O vapor are present in a gas mixture (1 m · atm = 328 ft · atm) (from Hottel, 1954, Ref. 6).

is done by introducing the concept of **mean beam length** L, which represents the radius of an equivalent hemisphere. The mean beam lengths for various gas geometries are listed in Table 12–4. More extensive lists are available in the literature [such as Hottel (1954, Ref. 6), and Siegel and Howell, (1992, Ref. 14)]. The emissivities associated with these geometries can be determined from Figures 12–36 through 12–38 by using the appropriate mean beam length.

Following a procedure recommended by Hottel, the absorptivity of a gas that contains CO_2 and H_2O gases for radiation emitted by a source at temperature T_s can be determined similarly from

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha \tag{12-54}$$

where $\Delta\alpha = \Delta\varepsilon$ and is determined from Figure 12–38 at the source temperature T_s. The absorptivities of CO_2 and H_2O can be determined from the emissivity charts (Figs. 12–36 and 12–37) as

CO_2:
$$\alpha_c = C_c \times (T_g/T_s)^{0.65} \times \varepsilon_c(T_s, P_c L T_s/T_g) \tag{12-55}$$

and

H_2O:
$$\alpha_w = C_w \times (T_g/T_s)^{0.45} \times \varepsilon_w(T_s, P_w L T_s/T_g) \tag{12-56}$$

The notation indicates that the emissivities should be evaluated using T_s instead of T_g (both in K or R), $P_c L T_s/T_g$ instead of $P_c L$, and $P_w L T_s/T_g$ instead of $P_w L$. Note that the absorptivity of the gas depends on the source temperature T_s as well as the gas temperature T_g. Also, $\alpha = \varepsilon$ when $T_s = T_g$, as expected. The pressure correction factors C_c and C_w are evaluated using $P_c L$ and $P_w L$, as in emissivity calculations.

When the total emissivity of a gas ε_g at temperature T_g is known, the emissive power of the gas (radiation emitted by the gas per unit surface area) can

TABLE 12–4

Mean beam length L for various gas volume shapes

Gas Volume Geometry	L
Hemisphere of radius R radiating to the center of its base	R
Sphere of diameter D radiating to its surface	$0.65D$
Infinite circular cylinder of diameter D radiating to curved surface	$0.95D$
Semi-infinite circular cylinder of diameter D radiating to its base	$0.65D$
Semi-infinite circular cylinder of diameter D radiating to center of its base	$0.90D$
Infinite semicircular cylinder of radius R radiating to center of its base	$1.26R$
Circular cylinder of height equal to diameter D radiating to entire surface	$0.60D$
Circular cylinder of height equal to diameter D radiating to center of its base	$0.71D$
Infinite slab of thickness D radiating to either bounding plane	$1.80D$
Cube of side length L radiating to any face	$0.66L$
Arbitrary shape of volume V and surface area A_s radiating to surface	$3.6V/A_s$

be expressed as $E_g = \varepsilon_g \sigma T_g^4$. Then the rate of radiation energy emitted by a gas to a bounding surface of area A_s becomes

$$\dot{Q}_{g,e} = \varepsilon_g A_s \sigma T_g^4 \quad \textbf{(12-57)}$$

If the bounding surface is black at temperature T_s, the surface will emit radiation to the gas at a rate of $A_s \sigma T_s^4$ without reflecting any, and the gas will absorb this radiation at a rate of $\alpha_g A_s \sigma T_s^4$, where α_g is the absorptivity of the gas. Then the net rate of radiation heat transfer between the gas and a black surface surrounding it becomes

Black enclosure:
$$\dot{Q}_{net} = A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \quad \textbf{(12-58)}$$

If the surface is not black, the analysis becomes more complicated because of the radiation reflected by the surface. But for surfaces that are nearly black with an emissivity $\varepsilon_s > 0.7$, Hottel (1954, Ref. 6), recommends this modification,

$$\dot{Q}_{net,\,gray} = \frac{\varepsilon_s + 1}{2}\dot{Q}_{net,\,black} = \frac{\varepsilon_s + 1}{2} A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \quad \textbf{(12-59)}$$

The emissivity of wall surfaces of furnaces and combustion chambers are typically greater than 0.7, and thus the relation above provides great convenience for preliminary radiation heat transfer calculations.

EXAMPLE 12–13 Effective Emissivity of Combustion Gases

A cylindrical furnace whose height and diameter are 5 m contains combustion gases at 1200 K and a total pressure of 2 atm. The composition of the combustion gases is determined by volumetric analysis to be 80 percent N_2, 8 percent H_2O, 7 percent O_2, and 5 percent CO_2. Determine the effective emissivity of the combustion gases (Fig. 12–39).

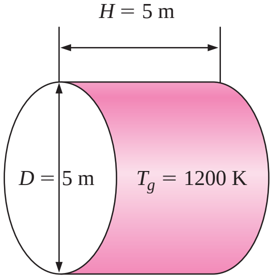


FIGURE 12–39
Schematic for Example 12–13.

SOLUTION The temperature, pressure, and composition of a gas mixture is given. The emissivity of the mixture is to be determined.

Assumptions **1** All the gases in the mixture are ideal gases. **2** The emissivity determined is the mean emissivity for radiation emitted to all surfaces of the cylindrical enclosure.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2} P = 0.05(2 \text{ atm}) = 0.10 \text{ atm}$$
$$P_w = y_{H_2O} P = 0.08(2 \text{ atm}) = 0.16 \text{ atm}$$

The mean beam length for a cylinder of equal diameter and height for radiation emitted to all surfaces is, from Table 12–4,

$$L = 0.60D = 0.60(5 \text{ m}) = 3 \text{ m}$$

Then,

$$P_c L = (0.10 \text{ atm})(3 \text{ m}) = 0.30 \text{ m} \cdot \text{atm} = 0.98 \text{ ft} \cdot \text{atm}$$
$$P_w L = (0.16 \text{ atm})(3 \text{ m}) = 0.48 \text{ m} \cdot \text{atm} = 1.57 \text{ ft} \cdot \text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of $T_g = 1200$ K and 1 atm are, from Figure 12–36,

$$\varepsilon_{c,\, 1 \text{ atm}} = 0.16 \qquad \text{and} \qquad \varepsilon_{w,\, 1 \text{ atm}} = 0.23$$

These are the base emissivity values at 1 atm, and they need to be corrected for the 2 atm total pressure. Noting that $(P_w + P)/2 = (0.16 + 2)/2 = 1.08$ atm, the pressure correction factors are, from Figure 12–37,

$$C_c = 1.1 \qquad \text{and} \qquad C_w = 1.4$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g =$ 1200 K is, from Figure 12–38,

$$\left.\begin{aligned} P_c L + P_w L &= 0.98 + 1.57 = 2.55 \\ \frac{P_w}{P_w + P_c} &= \frac{0.16}{0.16 + 0.10} = 0.615 \end{aligned}\right\} \qquad \Delta\varepsilon = 0.048$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c\, \varepsilon_{c,\, 1 \text{ atm}} + C_w\, \varepsilon_{w,\, 1 \text{ atm}} - \Delta\varepsilon = 1.1 \times 0.16 + 1.4 \times 0.23 - 0.048 = \mathbf{0.45}$$

Discussion This is the average emissivity for radiation emitted to all surfaces of the cylindrical enclosure. For radiation emitted towards the center of the base, the mean beam length is $0.71D$ instead of $0.60D$, and the emissivity value would be different.

EXAMPLE 12–14 **Radiation Heat Transfer in a Cylindrical Furnace**

Reconsider the cylindrical furnace discussed in Example 12–13. For a wall temperature of 600 K, determine the absorptivity of the combustion gases and the rate of radiation heat transfer from the combustion gases to the furnace walls (Fig. 12–40).

SOLUTION The temperatures for the wall surfaces and the combustion gases are given for a cylindrical furnace. The absorptivity of the gas mixture and the rate of radiation heat transfer are to be determined.

Assumptions **1** All the gases in the mixture are ideal gases. **2** All interior surfaces of furnace walls are black. **3** Scattering by soot and other particles is negligible.

Analysis The average emissivity of the combustion gases at the gas temperature of $T_g = 1200$ K was determined in the preceding example to be $\varepsilon_g = 0.45$.

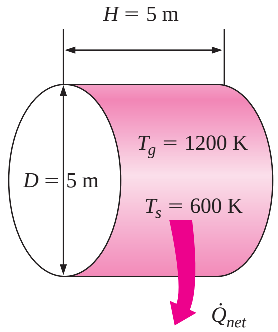


FIGURE 12–40
Schematic for Example 12–14.

For a source temperature of $T_s = 600$ K, the absorptivity of the gas is again determined using the emissivity charts as

$$P_c L \frac{T_s}{T_g} = (0.10 \text{ atm})(3 \text{ m}) \frac{600 \text{ K}}{1200 \text{ K}} = 0.15 \text{ m} \cdot \text{atm} = 0.49 \text{ ft} \cdot \text{atm}$$

$$P_w L \frac{T_s}{T_g} = (0.16 \text{ atm})(3 \text{ m}) \frac{600 \text{ K}}{1200 \text{ K}} = 0.24 \text{ m} \cdot \text{atm} = 0.79 \text{ ft} \cdot \text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of $T_s = 600$ K and 1 atm are, from Figure 12–36,

$$\varepsilon_{c,\,1\text{ atm}} = 0.11 \qquad \text{and} \qquad \varepsilon_{w,\,1\text{ atm}} = 0.25$$

The pressure correction factors were determined in the preceding example to be $C_c = 1.1$ and $C_w = 1.4$, and they do not change with surface temperature. Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c\left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,\,1\text{ atm}} = (1.1)\left(\frac{1200 \text{ K}}{600 \text{ K}}\right)^{0.65} (0.11) = 0.19$$

$$\alpha_w = C_w\left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,\,1\text{ atm}} = (1.4)\left(\frac{1200 \text{ K}}{600 \text{ K}}\right)^{0.45} (0.25) = 0.48$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Figure 12–38 at $T = T_s = 600$ K instead of $T_g = 1200$ K. There is no chart for 600 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and take their average. At $P_w/(P_w + P_c) = 0.615$ and $P_c L + P_w L = 2.55$ we read $\Delta\varepsilon = 0.027$. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.19 + 0.48 - 0.027 = \mathbf{0.64}$$

The surface area of the cylindrical surface is

$$A_s = \pi DH + 2\frac{\pi D^2}{4} = \pi(5 \text{ m})(5 \text{ m}) + 2\frac{\pi(5 \text{ m})^2}{4} = 118 \text{ m}^2$$

Then the net rate of radiation heat transfer from the combustion gases to the walls of the furnace becomes

$$\begin{aligned}\dot{Q}_{\text{net}} &= A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)\\ &= (118 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[0.45(1200 \text{ K})^4 - 0.64(600 \text{ K})^4]\\ &= \mathbf{2.79 \times 10^4\ W}\end{aligned}$$

Discussion The heat transfer rate determined above is for the case of black wall surfaces. If the surfaces are not black but the surface emissivity ε_s is greater than 0.7, the heat transfer rate can be determined by multiplying the rate of heat transfer already determined by $(\varepsilon_s + 1)/2$.