CHAPTER

12

RADIATION HEAT TRANSFER

In Chapter 11, we considered the fundamental aspects of radiation and the radiation properties of surfaces. We are now in a position to consider radiation exchange between two or more surfaces, which is the primary quantity of interest in most radiation problems.

We start this chapter with a discussion of view factors and the rules associated with them. View factor expressions and charts for some common configurations are given, and the crossed-strings method is presented. We then discuss radiation heat transfer, first between black surfaces and then between nonblack surfaces using the radiation network approach. We continue with radiation shields and discuss the radiation effect on temperature measurements and comfort. Finally, we consider gas radiation, and discuss the effective emissivities and absorptivities of gas bodies of various shapes. We also discuss radiation exchange between the walls of combustion chambers and the high-temperature emitting and absorbing combustion gases inside.

CONTENTS

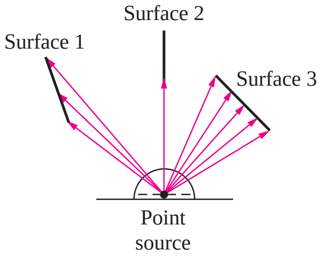


FIGURE 12–1
Radiation heat exchange between surfaces depends on the *orientation* of the surfaces relative to each other, and this dependence on orientation is accounted for by the *view factor.*

12–1 ▪ THE VIEW FACTOR

Radiation heat transfer between surfaces depends on the *orientation* of the surfaces relative to each other as well as their radiation properties and temperatures, as illustrated in Figure 12–1. For example, a camper will make the most use of a campfire on a cold night by standing as close to the fire as possible and by blocking as much of the radiation coming from the fire by turning her front to the fire instead of her side. Likewise, a person will maximize the amount of solar radiation incident on him and take a sunbath by lying down on his back instead of standing up on his feet.

To account for the effects of orientation on radiation heat transfer between two surfaces, we define a new parameter called the *view factor,* which is a purely geometric quantity and is independent of the surface properties and temperature. It is also called the *shape factor, configuration factor,* and *angle factor.* The view factor based on the assumption that the surfaces are diffuse emitters and diffuse reflectors is called the *diffuse view factor,* and the view factor based on the assumption that the surfaces are diffuse emitters but specular reflectors is called the *specular view factor.* In this book, we will consider radiation exchange between diffuse surfaces only, and thus the term *view factor* will simply mean *diffuse view factor.*

The view factor from a surface i to a surface j is denoted by $F_{i \to j}$ or just F_{ij}, and is defined as

F_{ij} = *the fraction of the radiation leaving surface i that strikes surface j directly*

The notation $F_{i \to j}$ is instructive for beginners, since it emphasizes that the view factor is for radiation that travels from surface i to surface j. However, this notation becomes rather awkward when it has to be used many times in a problem. In such cases, it is convenient to replace it by its *shorthand* version F_{ij}.

Therefore, the view factor F_{12} represents the fraction of radiation leaving surface 1 that strikes surface 2 directly, and F_{21} represents the fraction of the radiation leaving surface 2 that strikes surface 1 directly. Note that the radiation that strikes a surface does not need to be absorbed by that surface. Also, radiation that strikes a surface after being reflected by other surfaces is not considered in the evaluation of view factors.

To develop a general expression for the view factor, consider two differential surfaces dA_1 and dA_2 on two arbitrarily oriented surfaces A_1 and A_2, respectively, as shown in Figure 12–2. The distance between dA_1 and dA_2 is r, and the angles between the normals of the surfaces and the line that connects dA_1 and dA_2 are θ_1 and θ_2, respectively. Surface 1 emits and reflects radiation diffusely in all directions with a constant intensity of I_1, and the solid angle subtended by dA_2 when viewed by dA_1 is $d\omega_{21}$.

The rate at which radiation leaves dA_1 in the direction of θ_1 is $I_1 \cos \theta_1 dA_1$. Noting that $d\omega_{21} = dA_2 \cos \theta_2 / r^2$, the portion of this radiation that strikes dA_2 is

$$\dot{Q}_{dA_1 \to dA_2} = I_1 \cos \theta_1 dA_1 d\omega_{21} = I_1 \cos \theta_1 dA_1 \frac{dA_2 \cos \theta_2}{r^2} \qquad \textbf{(12-1)}$$

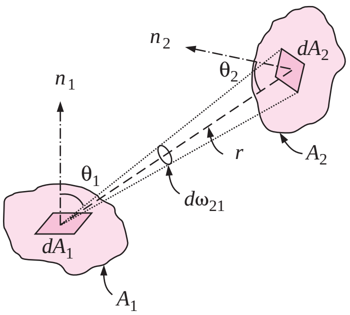


FIGURE 12–2
Geometry for the determination of the view factor between two surfaces.

The total rate at which radiation leaves dA_1 (via emission and reflection) in all directions is the radiosity (which is $J_1 = \pi I_1$) times the surface area,

$$\dot{Q}_{dA_1} = J_1 dA_1 = \pi I_1 dA_1 \quad \textbf{(12-2)}$$

Then the *differential view factor* $dF_{dA_1 \rightarrow dA_2}$, which is the fraction of radiation leaving dA_1 that strikes dA_2 directly, becomes

$$dF_{dA_1 \rightarrow dA_2} = \frac{\dot{Q}_{dA_1 \rightarrow dA_2}}{\dot{Q}_{dA_1}} = \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_2 \quad \textbf{(12-3)}$$

The differential view factor $dF_{dA_2 \rightarrow dA_1}$ can be determined from Eq. 12–3 by interchanging the subscripts 1 and 2.

The view factor from a differential area dA_1 to a finite area A_2 can be determined from the fact that the fraction of radiation leaving dA_1 that strikes A_2 is the sum of the fractions of radiation striking the differential areas dA_2. Therefore, the view factor $F_{dA_1 \rightarrow A_2}$ is determined by integrating $dF_{dA_1 \rightarrow dA_2}$ over A_2,

$$F_{dA_1 \rightarrow A_2} = \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_2 \quad \textbf{(12-4)}$$

The total rate at which radiation leaves the entire A_1 (via emission and reflection) in all directions is

$$\dot{Q}_{A_1} = J_1 A_1 = \pi I_1 A_1 \quad \textbf{(12-5)}$$

The portion of this radiation that strikes dA_2 is determined by considering the radiation that leaves dA_1 and strikes dA_2 (given by Eq. 12–1), and integrating it over A_1,

$$\dot{Q}_{A_1 \rightarrow dA_2} = \int_{A_1} \dot{Q}_{dA_1 \rightarrow dA_2} = \int_{A_1} \frac{I_1 \cos\theta_1 \cos\theta_2 \, dA_2}{r^2} dA_1 \quad \textbf{(12-6)}$$

Integration of this relation over A_2 gives the radiation that strikes the entire A_2,

$$\dot{Q}_{A_1 \rightarrow A_2} = \int_{A_2} \dot{Q}_{A_1 \rightarrow dA_2} = \int_{A_2} \int_{A_1} \frac{I_1 \cos\theta_1 \cos\theta_2}{r^2} dA_1 \, dA_2 \quad \textbf{(12-7)}$$

Dividing this by the total radiation leaving A_1 (from Eq. 12–5) gives the fraction of radiation leaving A_1 that strikes A_2, which is the view factor $F_{A_1 \rightarrow A_2}$ (or F_{12} for short),

$$F_{12} = F_{A_1 \rightarrow A_2} = \frac{\dot{Q}_{A_1 \rightarrow A_2}}{\dot{Q}_{A_1}} = \frac{1}{A_1} \int_{A_2} \int_{A_1} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1 \, dA_2 \quad \textbf{(12-8)}$$

The view factor $F_{A_2 \rightarrow A_1}$ is readily determined from Eq. 12–8 by interchanging the subscripts 1 and 2,

$$F_{21} = F_{A_2 \rightarrow A_1} = \frac{\dot{Q}_{A_2 \rightarrow A_1}}{\dot{Q}_{A_2}} = \frac{1}{A_2} \int_{A_2} \int_{A_1} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1 dA_2 \quad \textbf{(12-9)}$$

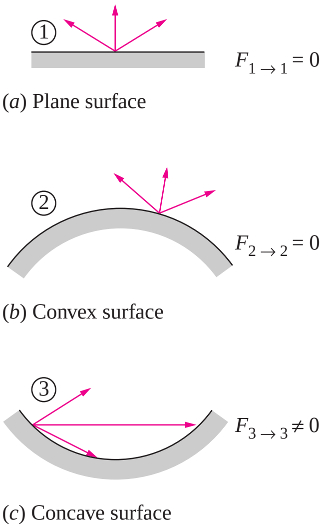


FIGURE 12–3
The view factor from a surface to itself is *zero* for *plane* or *convex* surfaces and *nonzero* for *concave* surfaces.

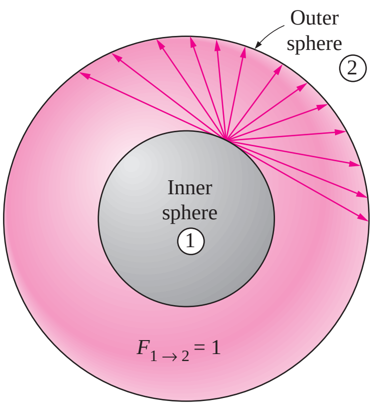


FIGURE 12–4
In a geometry that consists of two concentric spheres, the view factor $F_{1 \to 2} = 1$ since the entire radiation leaving the surface of the smaller sphere will be intercepted by the larger sphere.

Note that I_1 is constant but r, θ_1, and θ_2 are variables. Also, integrations can be performed in any order since the integration limits are constants. These relations confirm that the view factor between two surfaces depends on their relative orientation and the distance between them.

Combining Eqs. 12–8 and 12–9 after multiplying the former by A_1 and the latter by A_2 gives

$$A_1F_{12} = A_2F_{21} \tag{12-10}$$

which is known as the **reciprocity relation** for view factors. It allows the calculation of a view factor from a knowledge of the other.

The view factor relations developed above are applicable to any two surfaces i and j provided that the surfaces are diffuse emitters and diffuse reflectors (so that the assumption of constant intensity is valid). For the special case of $j = i$, we have

$F_{i \to i}$ = *the fraction of radiation leaving surface i that strikes itself directly*

Noting that in the absence of strong electromagnetic fields radiation beams travel in straight paths, the view factor from a surface to itself will be zero unless the surface "sees" itself. Therefore, $F_{i \to i} = 0$ for *plane* or *convex* surfaces and $F_{i \to i} \neq 0$ for concave surfaces, as illustrated in Figure 12–3.

The value of the view factor ranges between *zero* and *one*. The limiting case $F_{i \to j} = 0$ indicates that the two surfaces do not have a direct view of each other, and thus radiation leaving surface i cannot strike surface j directly. The other limiting case $F_{i \to j} = 1$ indicates that surface j completely surrounds surface i, so that the entire radiation leaving surface i is intercepted by surface j. For example, in a geometry consisting of two concentric spheres, the entire radiation leaving the surface of the smaller sphere (surface 1) will strike the larger sphere (surface 2), and thus $F_{1 \to 2} = 1$, as illustrated in Figure 12–4.

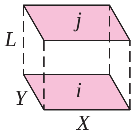
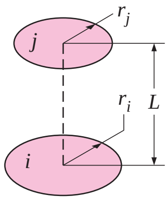
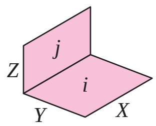
The view factor has proven to be very useful in radiation analysis because it allows us to express the *fraction of radiation* leaving a surface that strikes another surface in terms of the orientation of these two surfaces relative to each other. The underlying assumption in this process is that the radiation a surface receives from a source is directly proportional to the angle the surface subtends when viewed from the source. This would be the case only if the radiation coming off the source is *uniform* in all directions throughout its surface and the medium between the surfaces does not *absorb*, *emit*, or *scatter* radiation. That is, it will be the case when the surfaces are *isothermal* and *diffuse* emitters and reflectors and the surfaces are separated by a *nonparticipating* medium such as a vacuum or air.

The view factor $F_{1 \to 2}$ between two surfaces A_1 and A_2 can be determined in a systematic manner first by expressing the view factor between two differential areas dA_1 and dA_2 in terms of the spatial variables and then by performing the necessary integrations. However, this approach is not practical, since, even for simple geometries, the resulting integrations are usually very complex and difficult to perform.

View factors for hundreds of common geometries are evaluated and the results are given in analytical, graphical, and tabular form in several publications. View factors for selected geometries are given in Tables 12–1 and 12–2 in *analytical* form and in Figures 12–5 to 12–8 in *graphical* form. The view

TABLE 12–1

View factor expressions for some common geometries of finite size (3D)

Geometry	Relation
Aligned parallel rectangles 	$\bar{X} = X/L$ and $\bar{Y} = Y/L$ $F_{i \to j} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$
Coaxial parallel disks 	$R_i = r_i/L$ and $R_j = r_j/L$ $S = 1 + \frac{1+R_j^2}{R_i^2}$ $F_{i \to j} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{r_j}{r_i}\right)^2\right]^{1/2}\right\}$
Perpendicular rectangles with a common edge 	$H = Z/X$ and $W = Y/X$ $F_{i \to j} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2} \times \left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$

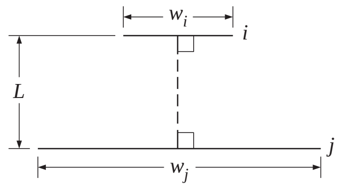
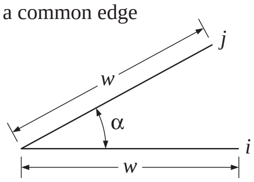
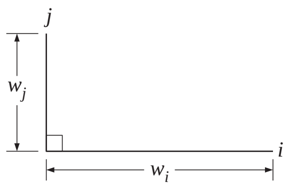
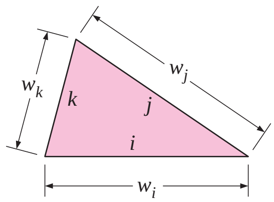
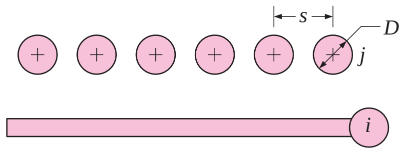
factors in Table 12–1 are for three-dimensional geometries. The view factors in Table 12–2, on the other hand, are for geometries that are *infinitely long* in the direction perpendicular to the plane of the paper and are therefore two-dimensional.

12–2 ▪ VIEW FACTOR RELATIONS

Radiation analysis on an enclosure consisting of N surfaces requires the evaluation of N^2 view factors, and this evaluation process is probably the most time-consuming part of a radiation analysis. However, it is neither practical nor necessary to evaluate all of the view factors directly. Once a sufficient number of view factors are available, the rest of them can be determined by utilizing some fundamental relations for view factors, as discussed next.

TABLE 12–2

View factor expressions for some infinitely long (2D) geometries

Geometry	Relation
Parallel plates with midlines connected by perpendicular line 	$W_i = w_i/L$ and $W_j = w_j/L$ $$F_{i\to j} = \frac{[(W_i + W_j)^2 + 4]^{1/2} - (W_j - W_i)^2 + 4]^{1/2}}{2W_i}$$
Inclined plates of equal width and with a common edge 	$$F_{i\to j} = 1 - \sin\frac{1}{2}\alpha$$
Perpendicular plates with a common edge 	$$F_{i\to j} = \frac{1}{2}\left\{1 + \frac{w_j}{w_i} - \left[1 + \left(\frac{w_j}{w_i}\right)^2\right]^{1/2}\right\}$$
Three-sided enclosure 	$$F_{i\to j} = \frac{w_i + w_j - w_k}{2w_i}$$
Infinite plane and row of cylinders 	$$F_{i\to j} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \frac{D}{s}\tan^{-1}\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}$$

1 The Reciprocity Relation

The view factors $F_{i\to j}$ and $F_{j\to i}$ are *not* equal to each other unless the areas of the two surfaces are. That is,

$$F_{j\to i} = F_{i\to j} \quad \text{when} \quad A_i = A_j$$
$$F_{j\to i} \neq F_{i\to j} \quad \text{when} \quad A_i \neq A_j$$

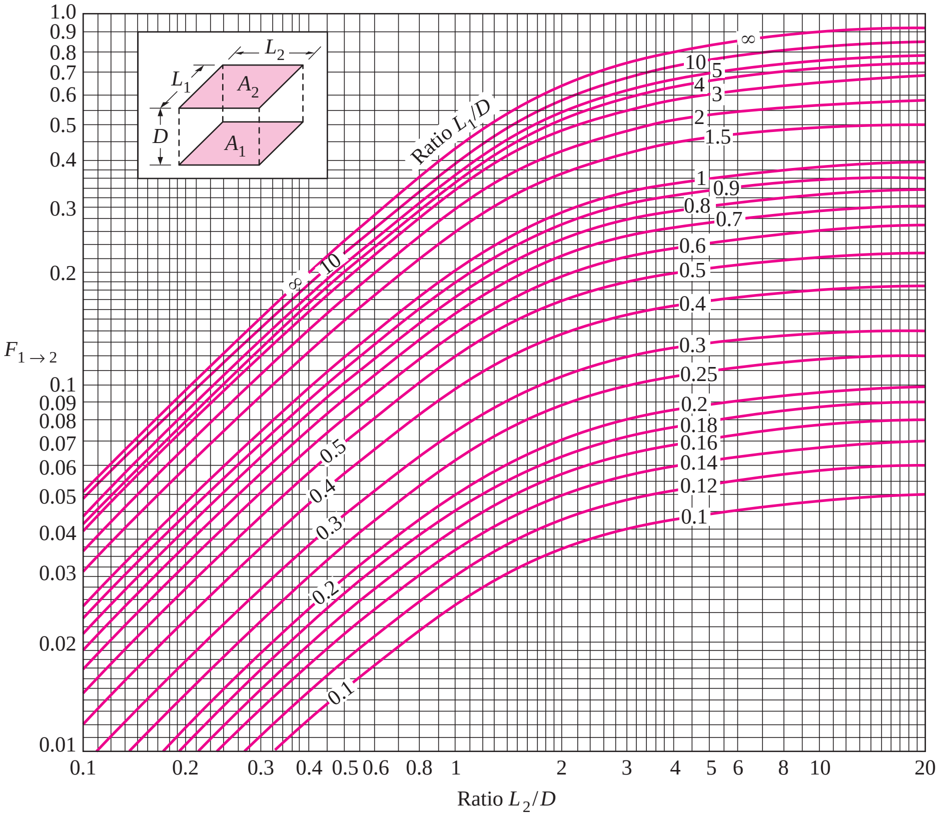


FIGURE 12–5
View factor between two aligned parallel rectangles of equal size.

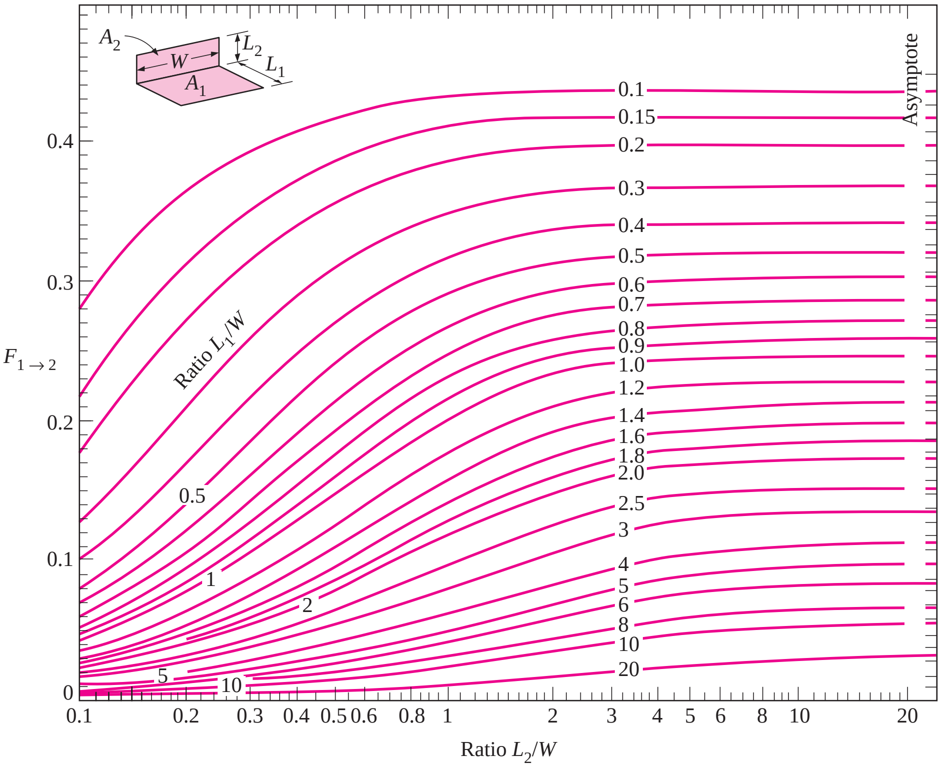


FIGURE 12–6
View factor between two perpendicular rectangles with a common edge.

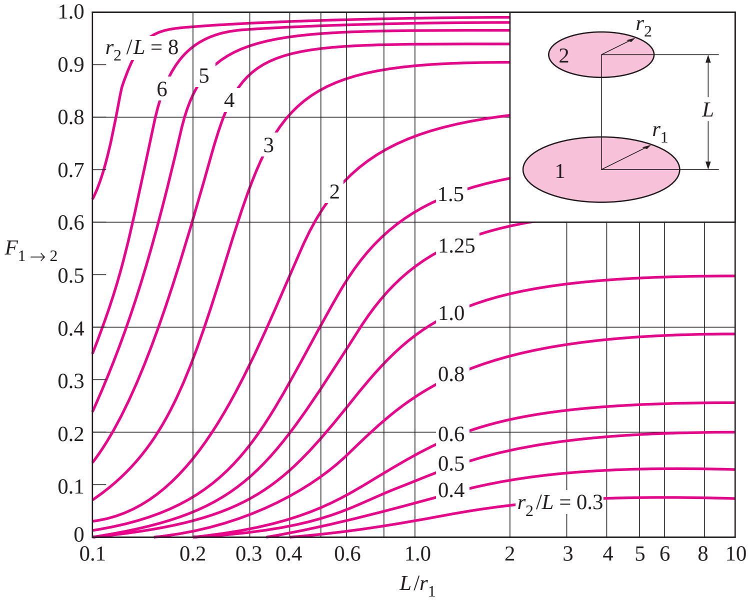


FIGURE 12–7
View factor between two coaxial parallel disks.

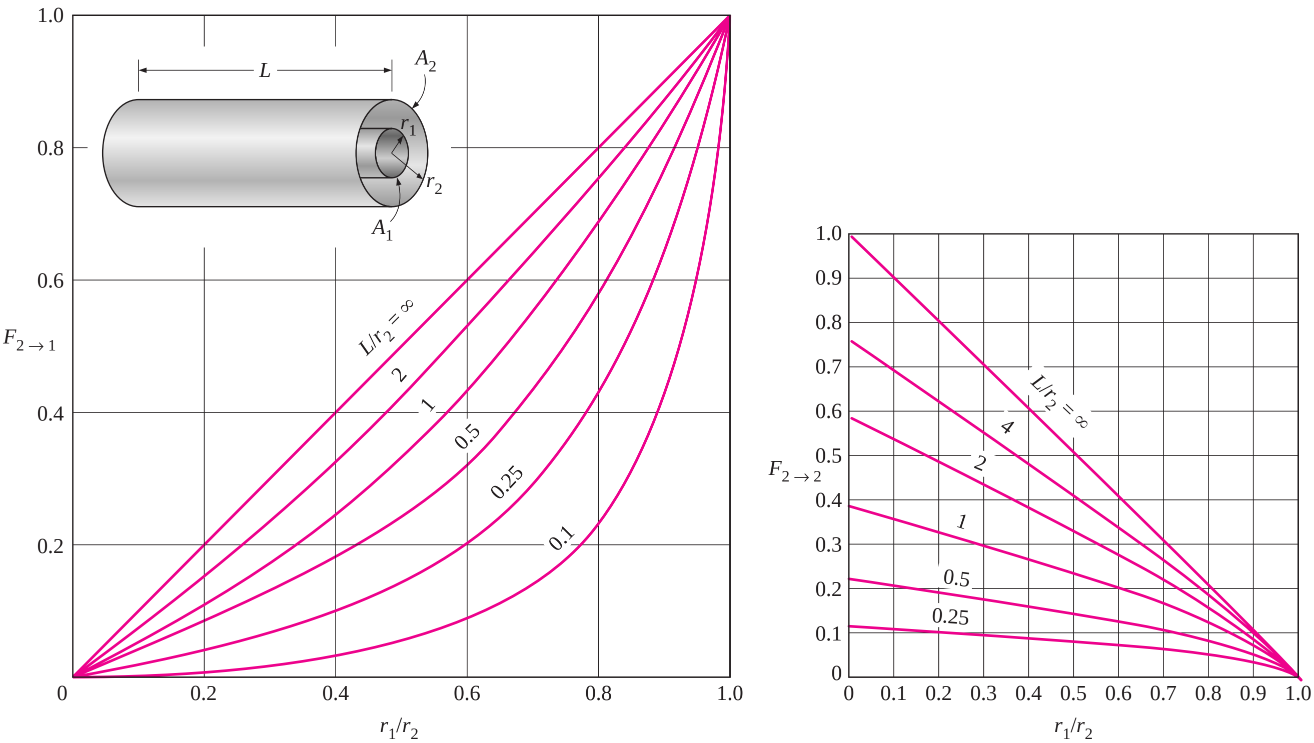


FIGURE 12–8
View factors for two concentric cylinders of finite length: (*a*) outer cylinder to inner cylinder; (*b*) outer cylinder to itself.