CHAPTER

9

NATURAL CONVECTION

In Chapters 7 and 8, we considered heat transfer by *forced convection,* where a fluid was *forced* to move over a surface or in a tube by external means such as a pump or a fan. In this chapter, we consider *natural convection,* where any fluid motion occurs by natural means such as buoyancy. The fluid motion in forced convection is quite *noticeable,* since a fan or a pump can transfer enough momentum to the fluid to move it in a certain direction. The fluid motion in natural convection, however, is often not noticeable because of the low velocities involved.

The convection heat transfer coefficient is a strong function of *velocity:* the higher the velocity, the higher the convection heat transfer coefficient. The fluid velocities associated with natural convection are low, typically less than 1 m/s. Therefore, the heat transfer coefficients encountered in natural convection are usually much lower than those encountered in forced convection. Yet several types of heat transfer equipment are designed to operate under natural convection conditions instead of forced convection, because natural convection does not require the use of a fluid mover.

We start this chapter with a discussion of the physical mechanism of *natural convection* and the *Grashof number.* We then present the correlations to evaluate heat transfer by natural convection for various geometries, including finned surfaces and enclosures. Finally, we discuss simultaneous forced and natural convection.

CONTENTS

9–1 ▪ PHYSICAL MECHANISM OF NATURAL CONVECTION

Many familiar heat transfer applications involve natural convection as the primary mechanism of heat transfer. Some examples are cooling of electronic equipment such as power transistors, TVs, and VCRs; heat transfer from electric baseboard heaters or steam radiators; heat transfer from the refrigeration coils and power transmission lines; and heat transfer from the bodies of animals and human beings. Natural convection in gases is usually accompanied by radiation of comparable magnitude except for low-emissivity surfaces.

We know that a hot boiled egg (or a hot baked potato) on a plate eventually cools to the surrounding air temperature (Fig. 9–1). The egg is cooled by transferring heat by convection to the air and by radiation to the surrounding surfaces. Disregarding heat transfer by radiation, the physical mechanism of cooling a hot egg (or any hot object) in a cooler environment can be explained as follows:

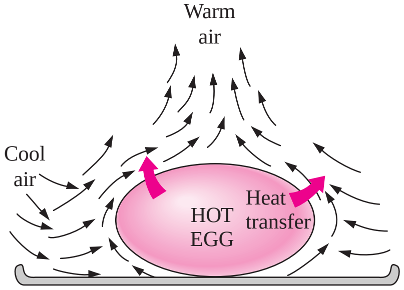


FIGURE 9–1
The cooling of a boiled egg in a cooler environment by natural convection.

As soon as the hot egg is exposed to cooler air, the temperature of the outer surface of the egg shell will drop somewhat, and the temperature of the air adjacent to the shell will rise as a result of heat conduction from the shell to the air. Consequently, the egg will soon be surrounded by a thin layer of warmer air, and heat will then be transferred from this warmer layer to the outer layers of air. The cooling process in this case would be rather slow since the egg would always be blanketed by warm air, and it would have no direct contact with the cooler air farther away. We may not notice any air motion in the vicinity of the egg, but careful measurements indicate otherwise.

The temperature of the air adjacent to the egg is higher, and thus its density is lower, since at constant pressure the density of a gas is inversely proportional to its temperature. Thus, we have a situation in which some low-density or "light" gas is surrounded by a high-density or "heavy" gas, and the natural laws dictate that *the light gas rise.* This is no different than the oil in a vinegar-and-oil salad dressing rising to the top (since $\rho_{oil} < \rho_{vinegar}$). This phenomenon is characterized incorrectly by the phrase "heat rises," which is understood to mean *heated air rises.* The space vacated by the warmer air in the vicinity of the egg is replaced by the cooler air nearby, and the presence of cooler air in the vicinity of the egg speeds up the cooling process. The rise of warmer air and the flow of cooler air into its place continues until the egg is cooled to the temperature of the surrounding air. The motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a **natural convection current,** and the heat transfer that is enhanced as a result of this natural convection current is called **natural convection heat transfer.** Note that in the absence of natural convection currents, heat transfer from the egg to the air surrounding it would be by conduction only, and the rate of heat transfer from the egg would be much lower.

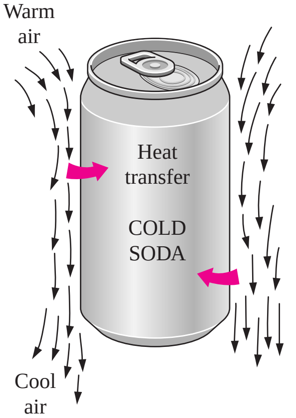


FIGURE 9–2
The warming up of a cold drink in a warmer environment by natural convection.

Natural convection is just as effective in the heating of cold surfaces in a warmer environment as it is in the cooling of hot surfaces in a cooler environment, as shown in Figure 9–2. Note that the direction of fluid motion is reversed in this case.

In a gravitational field, there is a net force that pushes upward a light fluid placed in a heavier fluid. The upward force exerted by a fluid on a body completely or partially immersed in it is called the **buoyancy force.** The magnitude of the buoyancy force is equal to the weight of the *fluid displaced* by the body. That is,

$$F_{\text{buoyancy}} = \rho_{\text{fluid}}\, gV_{\text{body}} \tag{9-1}$$

where ρ_{fluid} is the average density of the *fluid* (not the body), g is the gravitational acceleration, and V_{body} is the volume of the portion of the body immersed in the fluid (for bodies completely immersed in the fluid, it is the total volume of the body). In the absence of other forces, the net vertical force acting on a body is the difference between the weight of the body and the buoyancy force. That is,

$$\begin{aligned} F_{\text{net}} &= W - F_{\text{buoyancy}} \\ &= \rho_{\text{body}}\, gV_{\text{body}} - \rho_{\text{fluid}}\, gV_{\text{body}} \\ &= (\rho_{\text{body}} - \rho_{\text{fluid}})\, gV_{\text{body}} \end{aligned} \tag{9-2}$$

Note that this force is proportional to the difference in the densities of the fluid and the body immersed in it. Thus, a body immersed in a fluid will experience a "weight loss" in an amount equal to the weight of the fluid it displaces. This is known as *Archimedes' principle.*

To have a better understanding of the buoyancy effect, consider an egg dropped into water. If the average density of the egg is greater than the density of water (a sign of freshness), the egg will settle at the bottom of the container. Otherwise, it will rise to the top. When the density of the egg equals the density of water, the egg will settle somewhere in the water while remaining completely immersed, acting like a "weightless object" in space. This occurs when the upward buoyancy force acting on the egg equals the weight of the egg, which acts downward.

The *buoyancy effect* has far-reaching implications in life. For one thing, without buoyancy, heat transfer between a hot (or cold) surface and the fluid surrounding it would be by *conduction* instead of by *natural convection.* The natural convection currents encountered in the oceans, lakes, and the atmosphere owe their existence to buoyancy. Also, light boats as well as heavy warships made of steel float on water because of buoyancy (Fig. 9–3). Ships are designed on the basis of the principle that the entire weight of a ship and its contents is equal to the weight of the water that the submerged volume of the ship can contain. The "chimney effect" that induces the upward flow of hot combustion gases through a chimney is also due to the buoyancy effect, and the upward force acting on the gases in the chimney is proportional to the difference between the densities of the hot gases in the chimney and the cooler air outside. Note that there is *no gravity* in space, and thus there can be no natural convection heat transfer in a spacecraft, even if the spacecraft is filled with atmospheric air.

In heat transfer studies, the primary variable is *temperature,* and it is desirable to express the net buoyancy force (Eq. 9-2) in terms of temperature differences. But this requires expressing the density difference in terms of a temperature difference, which requires a knowledge of a property that represents the *variation of the density of a fluid with temperature at constant pressure. The property that provides that information is the* **volume expansion coefficient** β, defined as (Fig. 9–4)

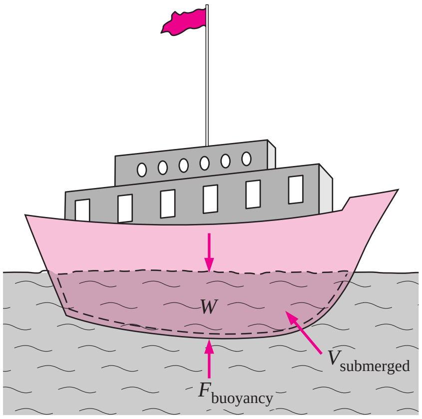


FIGURE 9–3
It is the buoyancy force that keeps the ships afloat in water ($W = F_{\text{buoyancy}}$ for floating objects).

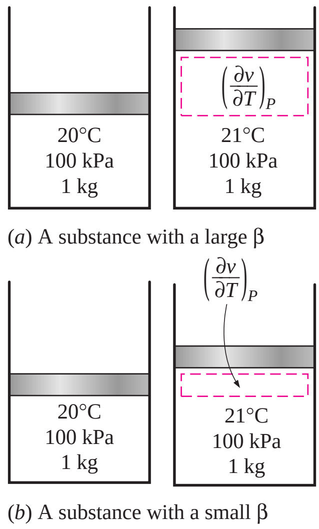


FIGURE 9–4
The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure.

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad (1/\text{K}) \tag{9-3}$$

In natural convection studies, the condition of the fluid sufficiently far from the hot or cold surface is indicated by the subscript "infinity" to serve as a reminder that this is the value at a distance where the presence of the surface is not felt. In such cases, the volume expansion coefficient can be expressed approximately by replacing differential quantities by differences as

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \qquad (\text{at constant } P) \tag{9-4}$$

or

$$\rho_\infty - \rho = \rho\beta(T - T_\infty) \qquad (\text{at constant } P) \tag{9-5}$$

where ρ_∞ is the density and T_∞ is the temperature of the quiescent fluid away from the surface.

We can show easily that the volume expansion coefficient β of an *ideal gas* ($P = \rho RT$) at a temperature T is equivalent to the inverse of the temperature:

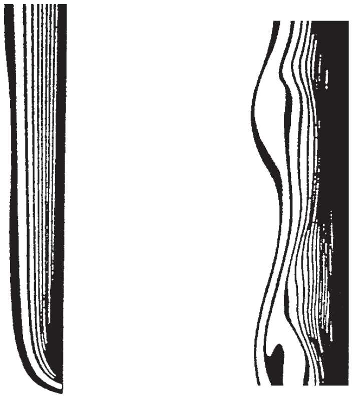
$$\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\text{K}) \tag{9-6}$$

where T is the *absolute* temperature. Note that a large value of β for a fluid means a large change in density with temperature, and that the product $\beta\,\Delta T$ represents the fraction of volume change of a fluid that corresponds to a temperature change ΔT at constant pressure. Also note that the buoyancy force is proportional to the *density difference,* which is proportional to the *temperature difference* at constant pressure. Therefore, the larger the temperature difference between the fluid adjacent to a hot (or cold) surface and the fluid away from it, the *larger* the buoyancy force and the *stronger* the natural convection currents, and thus the *higher* the heat transfer rate.

The magnitude of the natural convection heat transfer between a surface and a fluid is directly related to the *flow rate* of the fluid. The higher the flow rate, the higher the heat transfer rate. In fact, it is the very high flow rates that increase the heat transfer coefficient by orders of magnitude when forced convection is used. In natural convection, no blowers are used, and therefore the flow rate cannot be controlled externally. The flow rate in this case is established by the dynamic balance of *buoyancy* and *friction.*

As we have discussed earlier, the buoyancy force is caused by the density difference between the heated (or cooled) fluid adjacent to the surface and the fluid surrounding it, and is proportional to this density difference and the volume occupied by the warmer fluid. It is also well known that whenever two bodies in contact (solid–solid, solid–fluid, or fluid–fluid) move relative to each other, a *friction force* develops at the contact surface in the direction opposite to that of the motion. This opposing force slows down the fluid and thus reduces the flow rate of the fluid. Under steady conditions, the air flow rate driven by buoyancy is established at the point where these two effects *balance* each other. The friction force increases as more and more solid surfaces are introduced, seriously disrupting the fluid flow and heat transfer. For that reason, heat sinks with closely spaced fins are not suitable for natural convection cooling.

Most heat transfer correlations in natural convection are based on experimental measurements. The instrument often used in natural convection experiments is the *Mach–Zehnder interferometer,* which gives a plot of isotherms in the fluid in the vicinity of a surface. The operation principle of interferometers is based on the fact that at low pressure, the lines of constant temperature for a gas correspond to the lines of constant density, and that the index of refraction of a gas is a function of its density. Therefore, the degree of refraction of light at some point in a gas is a measure of the temperature gradient at that point. An interferometer produces a map of interference fringes, which can be interpreted as lines of *constant temperature* as shown in Figure 9–5. The smooth and parallel lines in (*a*) indicate that the flow is *laminar,* whereas the eddies and irregularities in (*b*) indicate that the flow is *turbulent.* Note that the lines are closest near the surface, indicating a *higher temperature gradient.*



(*a*) Laminar flow (*b*) Turbulent flow

FIGURE 9–5
Isotherms in natural convection over a hot plate in air.

9–2 ▪ EQUATION OF MOTION AND THE GRASHOF NUMBER

In this section we derive the equation of motion that governs the natural convection flow in laminar boundary layer. The conservation of mass and energy equations derived in Chapter 6 for forced convection are also applicable for natural convection, but the momentum equation needs to be modified to incorporate buoyancy.

Consider a vertical hot flat plate immersed in a quiescent fluid body. We assume the natural convection flow to be steady, laminar, and two-dimensional, and the fluid to be Newtonian with constant properties, including density, with one exception: the density difference $\rho - \rho_\infty$ is to be considered since it is this density difference between the inside and the outside of the boundary layer that gives rise to buoyancy force and sustains flow. (This is known as the *Boussinesq approximation.*) We take the upward direction along the plate to be *x,* and the direction normal to surface to be *y,* as shown in Figure 9–6. Therefore, gravity acts in the $-x$-direction. Noting that the flow is steady and two-dimensional, the *x*- and *y*-components of velocity within boundary layer are $u = u(x, y)$ and $v = v(x, y)$, respectively.

The velocity and temperature profiles for natural convection over a vertical hot plate are also shown in Figure 9–6. Note that as in forced convection, the thickness of the boundary layer increases in the flow direction. Unlike forced convection, however, the fluid velocity is *zero* at the outer edge of the velocity boundary layer as well as at the surface of the plate. This is expected since the fluid beyond the boundary layer is motionless. Thus, the fluid velocity increases with distance from the surface, reaches a maximum, and gradually decreases to zero at a distance sufficiently far from the surface. At the surface, the fluid temperature is equal to the plate temperature, and gradually decreases to the temperature of the surrounding fluid at a distance sufficiently far from the surface, as shown in the figure. In the case of *cold surfaces,* the shape of the velocity and temperature profiles remains the same but their direction is reversed.

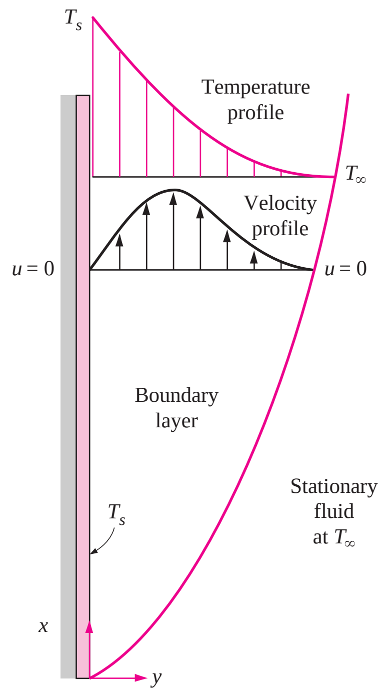


FIGURE 9–6
Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature T_s inserted in a fluid at temperature T_∞.

Consider a differential volume element of height *dx,* length *dy,* and unit depth in the *z*-direction (normal to the paper) for analysis. The forces acting on this volume element are shown in Figure 9–7. Newton's second law of motion for this control volume can be expressed as

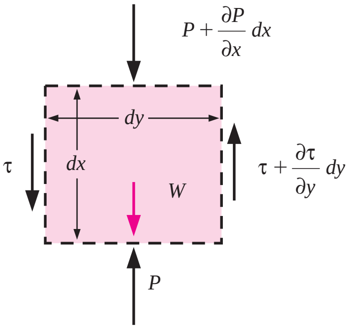


FIGURE 9–7
Forces acting on a differential control volume in the natural convection boundary layer over a vertical flat plate.

$$\delta m \cdot a_x = F_x \tag{9-7}$$

where $\delta m = \rho(dx \cdot dy \cdot 1)$ is the mass of the fluid element within the control volume. The acceleration in the x-direction is obtained by taking the total differential of $u(x, y)$, which is $du = (\partial u/\partial x)dx + (\partial u/\partial y)dy$, and dividing it by dt. We get

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \tag{9-8}$$

The forces acting on the differential volume element in the vertical direction are the pressure forces acting on the top and bottom surfaces, the shear stresses acting on the side surfaces (the normal stresses acting on the top and bottom surfaces are small and are disregarded), and the force of gravity acting on the entire volume element. Then the net surface force acting in the x-direction becomes

$$\begin{aligned} F_x &= \left(\frac{\partial \tau}{\partial y} dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x} dx\right)(dy \cdot 1) - \rho g(dx \cdot dy \cdot 1) \\ &= \left(\mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g\right)(dx \cdot dy \cdot 1) \end{aligned} \tag{9-9}$$

since $\tau = \mu(\partial u/\partial y)$. Substituting Eqs. 9-8 and 9-9 into Eq. 9-7 and dividing by $\rho \cdot dx \cdot dy \cdot 1$ gives the *conservation of momentum* in the x-direction as

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g \tag{9-10}$$

The x-momentum equation in the quiescent fluid outside the boundary layer can be obtained from the relation above as a special case by setting $u = 0$. It gives

$$\frac{\partial P_\infty}{\partial x} = -\rho_\infty g \tag{9-11}$$

which is simply the relation for the variation of hydrostatic pressure in a quiescent fluid with height, as expected. Also, noting that $v \ll u$ in the boundary layer and thus $\partial v/\partial x \approx \partial v/\partial y \approx 0$, and that there are no body forces (including gravity) in the y-direction, the force balance in that direction gives $\partial P/\partial y = 0$. That is, the variation of pressure in the direction normal to the surface is negligible, and for a given x the pressure in the boundary layer is equal to the pressure in the quiescent fluid. Therefore, $P = P(x) = P_\infty(x)$ and $\partial P/\partial x = \partial P_\infty/\partial x = -\rho_\infty g$. Substituting into Eq. 9-10,

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g \tag{9-12}$$

The last term represents the net upward force per unit volume of the fluid (the difference between the buoyant force and the fluid weight). This is the force that initiates and sustains convection currents.

From Eq. 9-5, we have $\rho_\infty - \rho = \rho\beta(T - T_\infty)$. Substituting it into the last equation and dividing both sides by ρ gives the desired form of the x-momentum equation,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \tag{9-13}$$

This is the equation that governs the fluid motion in the boundary layer due to the effect of buoyancy. Note that the momentum equation involves the temperature, and thus the momentum and energy equations must be solved simultaneously.

The set of three partial differential equations (the continuity, momentum, and the energy equations) that govern natural convection flow over vertical isothermal plates can be reduced to a set of two ordinary nonlinear differential equations by the introduction of a similarity variable. But the resulting equations must still be solved numerically [Ostrach (1953), Ref. 27]. Interested reader is referred to advanced books on the topic for detailed discussions [e.g., Kays and Crawford (1993), Ref. 23].

The Grashof Number

The governing equations of natural convection and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by suitable constant quantities: all lengths by a characteristic length L_c, all velocities by an arbitrary reference velocity $\mathcal{V}$ (which, from the definition of Reynolds number, is taken to be $\mathcal{V} = \text{Re}_L\, \nu/L_c$), and temperature by a suitable temperature difference (which is taken to be $T_s - T_\infty$) as

$$x^* = \frac{x}{L_c} \qquad y^* = \frac{y}{L_c} \qquad u^* = \frac{u}{\mathcal{V}} \qquad v^* = \frac{v}{\mathcal{V}} \qquad \text{and} \qquad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

where asterisks are used to denote nondimensional variables. Substituting them into the momentum equation and simplifying give

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\right]\frac{T^*}{\text{Re}_L^2} + \frac{1}{\text{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} \tag{9-14}$$

The dimensionless parameter in the brackets represents the natural convection effects, and is called the **Grashof number** Gr_L,

$$\text{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \tag{9-15}$$

where

- g = gravitational acceleration, m/s^2
- β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
- T_s = temperature of the surface, °C
- T_∞ = temperature of the fluid sufficiently far from the surface, °C
- L_c = characteristic length of the geometry, m
- ν = kinematic viscosity of the fluid, m^2/s

We mentioned in the preceding chapters that the flow regime in forced convection is governed by the dimensionless *Reynolds number,* which represents the ratio of inertial forces to viscous forces acting on the fluid. The flow regime in natural convection is governed by the dimensionless *Grashof number,* which represents the ratio of the *buoyancy force* to the *viscous force* acting on the fluid (Fig. 9–8).

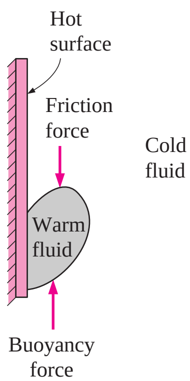


FIGURE 9–8
The Grashof number Gr is a measure of the relative magnitudes of the *buoyancy force* and the opposing *viscous force* acting on the fluid.

The role played by the Reynolds number in forced convection is played by the Grashof number in natural convection. As such, the Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection. For vertical plates, for example, the critical Grashof number is observed to be about 10^9. Therefore, the flow regime on a vertical plate becomes *turbulent* at Grashof numbers greater than 10^9.

When a surface is subjected to external flow, the problem involves both natural and forced convection. The relative importance of each mode of heat transfer is determined by the value of the coefficient $\text{Gr}_L/\text{Re}_L^2$: Natural convection effects are negligible if $\text{Gr}_L/\text{Re}_L^2 \ll 1$, free convection dominates and the forced convection effects are negligible if $\text{Gr}_L/\text{Re}_L^2 \gg 1$, and both effects are significant and must be considered if $\text{Gr}_L/\text{Re}_L^2 \approx 1$.

9–3 ▪ NATURAL CONVECTION OVER SURFACES

Natural convection heat transfer on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermophysical properties of the fluid involved.

Although we understand the mechanism of natural convection well, the complexities of fluid motion make it very difficult to obtain simple analytical relations for heat transfer by solving the governing equations of motion and energy. Some analytical solutions exist for natural convection, but such solutions lack generality since they are obtained for simple geometries under some simplifying assumptions. Therefore, with the exception of some simple cases, heat transfer relations in natural convection are based on experimental studies. Of the numerous such correlations of varying complexity and claimed accuracy available in the literature for any given geometry, we present here the ones that are best known and widely used.

The simple empirical correlations for the average *Nusselt number* Nu in natural convection are of the form (Fig. 9–9)

$$\text{Nu} = \frac{hL_c}{k} = C(\text{Gr}_L\,\text{Pr})^n = C\,\text{Ra}_L^n \tag{9-16}$$

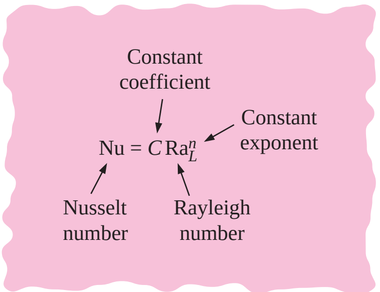


FIGURE 9–9
Natural convection heat transfer correlations are usually expressed in terms of the Rayleigh number raised to a constant *n* multiplied by another constant *C*, both of which are determined experimentally.

where Ra_L is the **Rayleigh number,** which is the product of the Grashof and Prandtl numbers:

$$\text{Ra}_L = \text{Gr}_L\,\text{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\,\text{Pr} \tag{9-17}$$

The values of the constants C and n depend on the *geometry* of the surface and the *flow regime*, which is characterized by the range of the Rayleigh number. The value of n is usually $\frac{1}{4}$ for laminar flow and $\frac{1}{3}$ for turbulent flow. The value of the constant C is normally less than 1.

Simple relations for the average Nusselt number for various geometries are given in Table 9–1, together with sketches of the geometries. Also given in this table are the characteristic lengths of the geometries and the ranges of Rayleigh number in which the relation is applicable. All fluid properties are to be evaluated at the film temperature $T_f = \frac{1}{2}(T_s + T_\infty)$.

When the average Nusselt number and thus the average convection coefficient is known, the rate of heat transfer by natural convection from a solid surface at a uniform temperature T_s to the surrounding fluid is expressed by Newton's law of cooling as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \qquad \text{(W)} \tag{9-18}$$

where A_s is the heat transfer surface area and h is the average heat transfer coefficient on the surface.

Vertical Plates (T_s = constant)

For a vertical flat plate, the characteristic length is the plate height L. In Table 9–1 we give three relations for the average Nusselt number for an isothermal vertical plate. The first two relations are very simple. Despite its complexity, we suggest using the third one (Eq. 9-21) recommended by Churchill and Chu (1975, Ref. 13) since it is applicable over the entire range of Rayleigh number. This relation is most accurate in the range of $10^{-1} < \text{Ra}_L < 10^9$.

Vertical Plates ($\dot{q}_s$ = constant)

In the case of constant surface heat flux, the rate of heat transfer is known (it is simply $\dot{Q} = \dot{q}_s A_s$), but the surface temperature T_s is not. In fact, T_s increases with height along the plate. It turns out that the Nusselt number relations for the constant surface temperature and constant surface heat flux cases are nearly identical [Churchill and Chu (1975), Ref. 13]. Therefore, the relations for isothermal plates can also be used for plates subjected to uniform heat flux, provided that the plate midpoint temperature $T_{L/2}$ is used for T_s in the evaluation of the film temperature, Rayleigh number, and the Nusselt number. Noting that $h = \dot{q}_s/(T_{L/2} - T_\infty)$, the average Nusselt number in this case can be expressed as

$$\text{Nu} = \frac{hL}{k} = \frac{\dot{q}_s L}{k(T_{L/2} - T_\infty)} \tag{9-27}$$

The midpoint temperature $T_{L/2}$ is determined by iteration so that the Nusselt numbers determined from Eqs. 9-21 and 9-27 match.

Vertical Cylinders

An outer surface of a vertical cylinder can be treated as a vertical plate when the diameter of the cylinder is sufficiently large so that the curvature effects are negligible. This condition is satisfied if

$$D \geq \frac{35L}{\text{Gr}_L^{1/4}} \tag{9-28}$$

When this criteria is met, the relations for vertical plates can also be used for vertical cylinders. Nusselt number relations for slender cylinders that do not meet this criteria are available in the literature [e.g., Cebeci (1974), Ref. 8].

Inclined Plates

Consider an inclined hot plate that makes an angle θ from the vertical, as shown in Figure 9–10, in a cooler environment. The net force $F = g(\rho_\infty - \rho)$ (the difference between the buoyancy and gravity) acting on a unit volume of the fluid in the boundary layer is always in the vertical direction. In the case

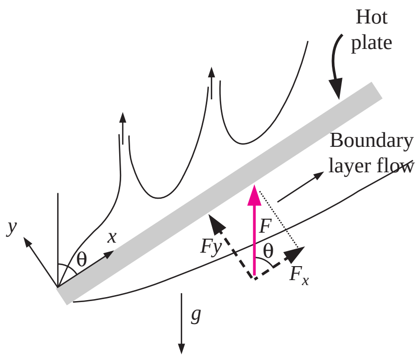
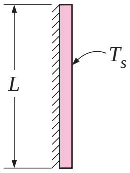
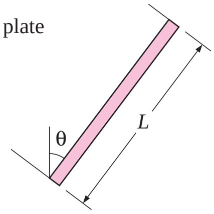
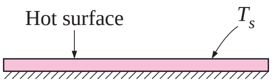


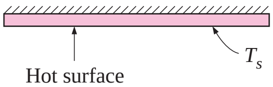
FIGURE 9–10
Natural convection flows on the upper and lower surfaces of an inclined hot plate.

TABLE 9–1

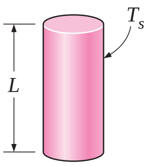
Empirical correlations for the average Nusselt number for natural convection over surfaces

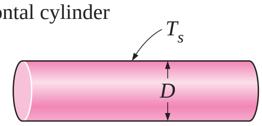
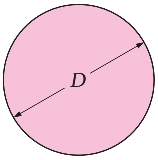
<table>
<tr><th>Geometry</th><th>Characteristic length L_c</th><th>Range of Ra</th><th>Nu</th></tr>
<tr><td>Vertical plate

</td><td>L</td><td>10^4–10^9
10^9–10^{13}
Entire range</td><td>$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ (9-19)
$\text{Nu} = 0.1\text{Ra}_L^{1/3}$ (9-20)
$\text{Nu} = \left\{0.825 + \dfrac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-21)
(complex but more accurate)</td></tr>
<tr><td>Inclined plate

</td><td>L</td><td></td><td>Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate

Replace g by $g \cos\theta$ for $\text{Ra} < 10^9$</td></tr>
<tr><td>Horiontal plate
(Surface area A and perimeter p)
(*a*) Upper surface of a hot plate
(or lower surface of a cold plate)


(*b*) Lower surface of a hot plate
(or upper surface of a cold plate)

</td><td>A_s/p</td><td>10^4–10^7
10^7–10^{11}

10^5–10^{11}</td><td>$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ (9-22)
$\text{Nu} = 0.15\text{Ra}_L^{1/3}$ (9-23)

$\text{Nu} = 0.27\text{Ra}_L^{1/4}$ (9-24)</td></tr>
<tr><td>Vertical cylinder

</td><td>L</td><td></td><td>A vertical cylinder can be treated as a vertical plate when

$D \geq \dfrac{35L}{\text{Gr}_L^{1/4}}$</td></tr>
<tr><td>Horizontal cylinder

</td><td>D</td><td>$\text{Ra}_D \leq 10^{12}$</td><td>$\text{Nu} = \left\{0.6 + \dfrac{0.387\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (9-25)</td></tr>
<tr><td>Sphere

</td><td>D</td><td>$\text{Ra}_D \leq 10^{11}$
$(\text{Pr} \geq 0.7)$</td><td>$\text{Nu} = 2 + \dfrac{0.589\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$ (9-26)</td></tr>
</table>

of inclined plate, this force can be resolved into two components: $F_y = F \cos \theta$ parallel to the plate that drives the flow along the plate, and $F_y = F \sin \theta$ normal to the plate. Noting that the force that drives the motion is reduced, we expect the convection currents to be weaker, and the rate of heat transfer to be lower relative to the vertical plate case.

The experiments confirm what we suspect for the lower surface of a hot plate, but the opposite is observed on the upper surface. The reason for this curious behavior for the upper surface is that the force component F_y initiates upward motion in addition to the parallel motion along the plate, and thus the boundary layer breaks up and forms plumes, as shown in the figure. As a result, the thickness of the boundary layer and thus the resistance to heat transfer decreases, and the rate of heat transfer increases relative to the vertical orientation.

In the case of a cold plate in a warmer environment, the opposite occurs as expected: The boundary layer on the upper surface remains intact with weaker boundary layer flow and thus lower rate of heat transfer, and the boundary layer on the lower surface breaks apart (the colder fluid falls down) and thus enhances heat transfer.

When the boundary layer remains intact (the lower surface of a hot plate or the upper surface of a cold plate), the Nusselt number can be determined from the vertical plate relations provided that g in the Rayleigh number relation is replaced by $g \cos \theta$ for $\theta < 60°$. Nusselt number relations for the other two surfaces (the upper surface of a hot plate or the lower surface of a cold plate) are available in the literature [e.g., Fujiii and Imura (1972), Ref. 18].

Horizontal Plates

The rate of heat transfer to or from a horizontal surface depends on whether the surface is facing upward or downward. For a hot surface in a cooler environment, the net force acts upward, forcing the heated fluid to rise. If the hot surface is facing upward, the heated fluid rises freely, inducing strong natural convection currents and thus effective heat transfer, as shown in Figure 9–11. But if the hot surface is facing downward, the plate will block the heated fluid that tends to rise (except near the edges), impeding heat transfer. The opposite is true for a cold plate in a warmer environment since the net force (weight minus buoyancy force) in this case acts downward, and the cooled fluid near the plate tends to descend.

The average Nusselt number for horizontal surfaces can be determined from the simple power-law relations given in Table 9–1. The characteristic length for horizontal surfaces is calculated from

$$L_c = \frac{A_s}{p} \tag{9-29}$$

where A_s is the surface area and p is the perimeter. Note that $L_c = a/4$ for a horizontal square surface of length a, and $D/4$ for a horizontal circular surface of diameter D.

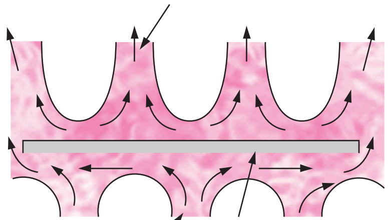


FIGURE 9–11
Natural convection flows on the upper and lower surfaces of a horizontal hot plate.

Horizontal Cylinders and Spheres

The boundary layer over a hot horizontal cylinder start to develop at the bottom, increasing in thickness along the circumference, and forming a rising

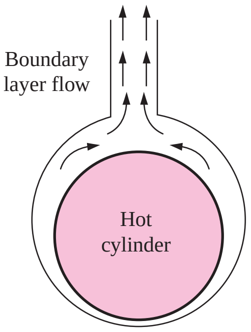


FIGURE 9–12
Natural convection flow over a horizontal hot cylinder.

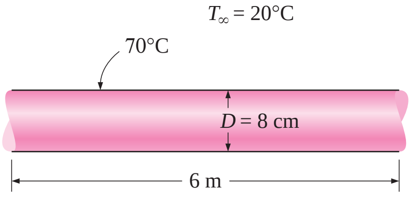


FIGURE 9–13
Schematic for Example 9–1.

plume at the top, as shown in Figure 9–12. Therefore, the local Nusselt number is highest at the bottom, and lowest at the top of the cylinder when the boundary layer flow remains laminar. The opposite is true in the case of a cold horizontal cylinder in a warmer medium, and the boundary layer in this case starts to develop at the top of the cylinder and ending with a descending plume at the bottom.

The average Nusselt number over the entire surface can be determined from Eq. 9-26 [Churchill and Chu (1975), Ref. 13] for an isothermal horizontal cylinder, and from Eq. 9-27 for an isothermal sphere [Churchill (1983}, Ref. 11] both given in Table 9–1.

EXAMPLE 9–1 Heat Loss from Hot Water Pipes

A 6-m-long section of an 8-cm-diameter horizontal hot water pipe shown in Figure 9–13 passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection.

SOLUTION A horizontal hot water pipe passes through a large room. The rate of heat loss from the pipe by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (70 + 20)/2 = 45°C$ and 1 atm are (Table A–15)

$$k = 0.02699 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7241$$

$$\nu = 1.749 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = \frac{1}{T_f} = \frac{1}{318 \text{ K}}$$

Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.08$ m. Then the Rayleigh number becomes

$$\text{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81 \text{ m/s}^2)[1/(318 \text{ K})](70 - 20 \text{ K})(0.08 \text{ m})^3}{(1.749 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.7241) = 1.869 \times 10^6$$

The natural convection Nusselt number in this case can be determined from Eq. 9-25 to be

$$\text{Nu} = \left\{0.6 + \frac{0.387 \text{ Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1869 \times 10^6)^{1/6}}{[1 + (0.559/0.7241)^{9/16}]^{8/27}}\right\}^2$$
$$= 17.40$$

Then,

$$h = \frac{k}{D}\text{Nu} = \frac{0.02699 \text{ W/m} \cdot °\text{C}}{0.08 \text{ m}}(17.40) = 5.869 \text{ W/m} \cdot °\text{C}$$
$$A_s = \pi DL = \pi(0.08 \text{ m})(6 \text{ m}) = 1.508 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.869 \text{ W/m}^2 \cdot °\text{C})(1.508 \text{ m}^2)(70 - 20)°\text{C} = \mathbf{443 \text{ W}}$$

Therefore, the pipe will lose heat to the air in the room at a rate of 443 W by natural convection.

Discussion The pipe will lose heat to the surroundings by radiation as well as by natural convection. Assuming the outer surface of the pipe to be black (emissivity $\varepsilon = 1$) and the inner surfaces of the walls of the room to be at room temperature, the radiation heat transfer is determined to be (Fig. 9–14)

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s \sigma (T_s^4 - T_{surr}^4)\\ &= (1)(1.508\ m^2)(5.67 \times 10^{-8}\ W/m^2 \cdot K^4)[(70 + 273\ K)^4 - (20 + 273\ K)^4]\\ &= 553\ W\end{aligned}$$

which is larger than natural convection. The emissivity of a real surface is less than 1, and thus the radiation heat transfer for a real surface will be less. But radiation will still be significant for most systems cooled by natural convection. Therefore, a radiation analysis should normally accompany a natural convection analysis unless the emissivity of the surface is low.

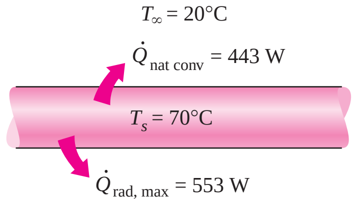


FIGURE 9–14
Radiation heat transfer is usually comparable to natural convection in magnitude and should be considered in heat transfer analysis.

EXAMPLE 9–2 Cooling of a Plate in Different Orientations

Consider a 0.6-m × 0.6-m thin square plate in a room at 30°C. One side of the plate is maintained at a temperature of 90°C, while the other side is insulated, as shown in Figure 9–15. Determine the rate of heat transfer from the plate by natural convection if the plate is (*a*) vertical, (*b*) horizontal with hot surface facing up, and (*c*) horizontal with hot surface facing down.

SOLUTION A hot plate with an insulated back is considered. The rate of heat loss by natural convection is to be determined for different orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (90 + 30)/2 = 60°C$ and 1 atm are (Table A-15)

$$k = 0.02808\ W/m \cdot °C \qquad Pr = 0.7202$$

$$\nu = 1.896 \times 10^{-5}\ m^2/s \qquad \beta = \frac{1}{T_f} = \frac{1}{333\ K}$$

Analysis (*a*) *Vertical.* The characteristic length in this case is the height of the plate, which is $L = 0.6$ m. The Rayleigh number is

$$\begin{aligned}Ra_L &= \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}Pr\\ &= \frac{(9.81\ m/s^2)[1/(333\ K)](90 - 30\ K)(0.6\ m)^3}{(1.896 \times 10^{-5}\ m^2/s)^2}(0.722) = 7.656 \times 10^8\end{aligned}$$

Then the natural convection Nusselt number can be determined from Eq. 9-21 to be

$$\begin{aligned}Nu &= \left\{0.825 + \frac{0.387\ Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2\\ &= \left\{0.825 + \frac{0.387(7.656 \times 10^8)^{1/6}}{1 + (0.492/0.7202)^{9/16}]^{8/27}}\right\}^2 = 113.4\end{aligned}$$

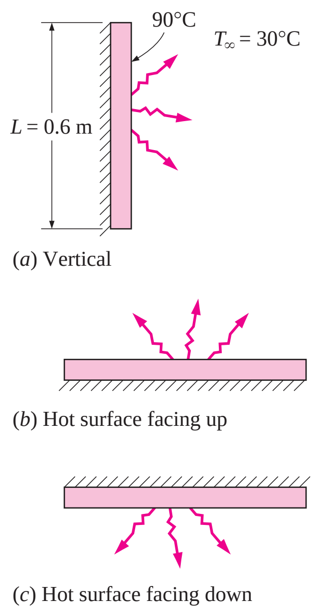


FIGURE 9–15
Schematic for Example 9–2.

Note that the simpler relation Eq. 9-19 would give Nu = 0.59 $\text{Ra}_L^{1/4}$ = 98.14, which is 13 percent lower. Then,

$$h = \frac{k}{L}\text{Nu} = \frac{0.02808 \text{ W/m} \cdot {}^\circ\text{C}}{0.6 \text{ m}}(113.4) = 5.306 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

$$A_s = L^2 = (0.6 \text{ m})^2 = 0.36 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.306 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.36 \text{ m}^2)(90 - 30){}^\circ\text{C} = \mathbf{115 \text{ W}}$$

(*b*) *Horizontal with hot surface facing up.* The characteristic length and the Rayleigh number in this case are

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6 \text{ m}}{4} = 0.15 \text{ m}$$

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr}$$

$$= \frac{(9.81 \text{ m/s}^2)[1/(333 \text{ K})](90 - 30 \text{ K})(0.15 \text{ m})^3}{(1.896 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.7202) = 1.196 \times 10^7$$

The natural convection Nusselt number can be determined from Eq. 9-22 to be

$$\text{Nu} = 0.54 \text{ Ra}_L^{1/4} = 0.54(1.196 \times 10^7)^{1/4} = 31.76$$

Then,

$$h = \frac{k}{L_c}\text{Nu} = \frac{0.0280 \text{ W/m} \cdot {}^\circ\text{C}}{0.15 \text{ m}}(31.76) = 5.946 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

$$A_s = L^2 = (0.6 \text{ m})^2 = 0.36 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.946 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.36 \text{ m}^2)(90 - 30){}^\circ\text{C} = \mathbf{128 \text{ W}}$$

(*c*) *Horizontal with hot surface facing down.* The characteristic length, the heat transfer surface area, and the Rayleigh number in this case are the same as those determined in (*b*). But the natural convection Nusselt number is to be determined from Eq. 9-24,

$$\text{Nu} = 0.27 \text{ Ra}_L^{1/4} = 0.27(1.196 \times 10^7)^{1/4} = 15.86$$

Then,

$$h = \frac{k}{L_c}\text{Nu} = \frac{0.02808 \text{ W/m} \cdot {}^\circ\text{C}}{0.15 \text{ m}}(15.86) = 2.973 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (2.973 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.36 \text{ m}^2)(90 - 30){}^\circ\text{C} = \mathbf{64.2 \text{ W}}$$

Note that the natural convection heat transfer is the lowest in the case of the hot surface facing down. This is not surprising, since the hot air is "trapped" under the plate in this case and cannot get away from the plate easily. As a result, the cooler air in the vicinity of the plate will have difficulty reaching the plate, which results in a reduced rate of heat transfer.

Discussion The plate will lose heat to the surroundings by radiation as well as by natural convection. Assuming the surface of the plate to be black (emissivity

$\varepsilon = 1$) and the inner surfaces of the walls of the room to be at room temperature, the radiation heat transfer in this case is determined to be

$$\dot{Q}_{rad} = \varepsilon A_s \sigma (T_s^4 - T_{surr}^4)$$
$$= (1)(0.36\ m^2)(5.67 \times 10^{-8}\ W/m^2 \cdot K^4)[(90 + 273\ K)^4 - (30 + 273\ K)^4]$$
$$= 182\ W$$

which is larger than that for natural convection heat transfer for each case. Therefore, radiation can be significant and needs to be considered in surfaces cooled by natural convection.

9–4 ▪ NATURAL CONVECTION FROM FINNED SURFACES AND PCBs

Natural convection flow through a channel formed by two parallel plates as shown in Figure 9–16 is commonly encountered in practice. When the plates are hot ($T_s > T_\infty$), the ambient fluid at T_∞ enters the channel from the lower end, rises as it is heated under the effect of buoyancy, and the heated fluid leaves the channel from the upper end. The plates could be the fins of a finned heat sink, or the PCBs (printed circuit boards) of an electronic device. The plates can be approximated as being isothermal (T_s = constant) in the first case, and isoflux ($\dot{q}_s$ = constant) in the second case.

Boundary layers start to develop at the lower ends of opposing surfaces, and eventually merge at the midplane if the plates are vertical and sufficiently long. In this case, we will have fully developed channel flow after the merger of the boundary layers, and the natural convection flow is analyzed as channel flow. But when the plates are short or the spacing is large, the boundary layers of opposing surfaces never reach each other, and the natural convection flow on a surface is not affected by the presence of the opposing surface. In that case, the problem should be analyzed as natural convection from two independent plates in a quiescent medium, using the relations given for surfaces, rather than natural convection flow through a channel.

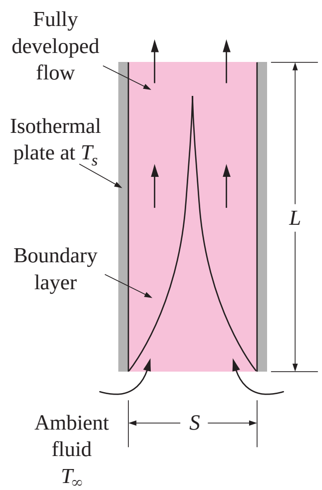


FIGURE 9–16
Natural convection flow through a channel between two isothermal vertical plates.

Natural Convection Cooling of Finned Surfaces (T_s = constant)

Finned surfaces of various shapes, called *heat sinks*, are frequently used in the cooling of electronic devices. Energy dissipated by these devices is transferred to the heat sinks by conduction and from the heat sinks to the ambient air by natural or forced convection, depending on the power dissipation requirements. Natural convection is the preferred mode of heat transfer since it involves no moving parts, like the electronic components themselves. However, in the natural convection mode, the components are more likely to run at a higher temperature and thus undermine reliability. A properly selected heat sink may considerably lower the operation temperature of the components and thus reduce the risk of failure.

Natural convection from vertical finned surfaces of rectangular shape has been the subject of numerous studies, mostly experimental. Bar-Cohen and

Rohsenow (1984, Ref. 5) have compiled the available data under various boundary conditions, and developed correlations for the Nusselt number and optimum spacing. The characteristic length for vertical parallel plates used as fins is usually taken to be the spacing between adjacent fins S, although the fin height L could also be used. The Rayleigh number is expressed as

$$\text{Ra}_S = \frac{g\beta(T_s - T_\infty)S^3}{\nu^2}\text{Pr} \quad \text{and} \quad \text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \text{Ra}_S\frac{L^3}{S^3} \tag{9-30}$$

The recommended relation for the average Nusselt number for vertical isothermal parallel plates is

$$T_s = \text{constant:} \qquad \text{Nu} = \frac{hS}{k} = \left[\frac{576}{(\text{Ra}_S S/L)^2} + \frac{2.873}{(\text{Ra}_S S/L)^{0.5}}\right]^{-0.5} \tag{9-31}$$

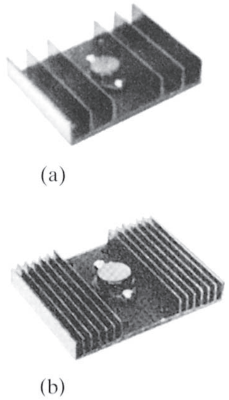


FIGURE 9–17
Heat sinks with (*a*) widely spaced and (*b*) closely packed fins (courtesy of Vemaline Products).

A question that often arises in the selection of a heat sink is whether to select one with *closely packed* fins or *widely spaced* fins for a given base area (Fig. 9–17). A heat sink with closely packed fins will have greater surface area for heat transfer but a smaller heat transfer coefficient because of the extra resistance the additional fins introduce to fluid flow through the interfin passages. A heat sink with widely spaced fins, on the other hand, will have a higher heat transfer coefficient but a smaller surface area. Therefore, there must be an *optimum spacing* that maximizes the natural convection heat transfer from the heat sink for a given base area WL, where W and L are the width and height of the base of the heat sink, respectively, as shown in Figure 9–18. When the fins are essentially isothermal and the fin thickness t is small relative to the fin spacing S, the optimum fin spacing for a vertical heat sink is determined by Bar-Cohen and Rohsenow to be

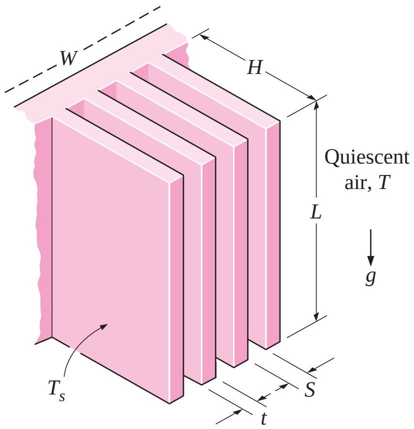


FIGURE 9–18
Various dimensions of a finned surface oriented vertically.

$$T_s = \text{constant:} \qquad S_{\text{opt}} = 2.714\left(\frac{S^3L}{\text{Ra}_S}\right)^{0.25} = 2.714\frac{L}{\text{Ra}_L^{0.25}} \tag{9-32}$$

It can be shown by combining the three equations above that when $S = S_{\text{opt}}$, the Nusselt number is a constant and its value is 1.307,

$$S = S_{\text{opt}}: \qquad \text{Nu} = \frac{hS_{\text{opt}}}{k} = 1.307 \tag{9-33}$$

The rate of heat transfer by natural convection from the fins can be determined from

$$\dot{Q} = h(2nLH)(T_s - T_\infty) \tag{9-34}$$

where $n = W/(S + t) \approx W/S$ is the number of fins on the heat sink and T_s is the surface temperature of the fins. All fluid properties are to be evaluated at the average temperature $T_{\text{ave}} = (T_s + T_\infty)/2$.

Natural Convection Cooling of Vertical PCBs ($\dot{q}_s$ = constant)

Arrays of printed circuit boards used in electronic systems can often be modeled as parallel plates subjected to uniform heat flux $\dot{q}_s$ (Fig. 9–19). The plate temperature in this case increases with height, reaching a maximum at the upper edge of the board. The modified Rayleigh number for uniform heat flux on both plates is expressed as

$$\text{Ra}_S^* = \frac{g\beta \dot{q}_s S^4}{kv^2}\,\text{Pr} \tag{9-35}$$

The Nusselt number at the upper edge of the plate where maximum temperature occurs is determined from [Bar-Cohen and Rohsenow (1984), Ref. 5]

$$\text{Nu}_L = \frac{h_L S}{k} = \left[\frac{48}{\text{Ra}_S^* S/L} + \frac{2.51}{(\text{Ra}_L^* S/L)^{0.4}}\right]^{-0.5} \tag{9-36}$$

The optimum fin spacing for the case of uniform heat flux on both plates is given as

$$\dot{q}_s = \text{constant:} \qquad S_{\text{opt}} = 2.12\left(\frac{S^4 L}{\text{Ra}_S^*}\right)^{0.2} \tag{9-37}$$

The total rate of heat transfer from the plates is

$$\dot{Q} = \dot{q}_s A_s = \dot{q}_s(2nLH) \tag{9-38}$$

where $n = W/(S + t) \approx W/S$ is the number of plates. The critical surface temperature T_L occurs at the upper edge of the plates, and it can be determined from

$$\dot{q}_s = h_L(T_L - T_\infty) \tag{9-39}$$

All fluid properties are to be evaluated at the average temperature $T_{\text{ave}} = (T_L + T_\infty)/2$.

Mass Flow Rate through the Space between Plates

As we mentioned earlier, the magnitude of the natural convection heat transfer is directly related to the mass flow rate of the fluid, which is established by the dynamic balance of two opposing effects: *buoyancy* and *friction.*

The fins of a heat sink introduce both effects: *inducing extra buoyancy* as a result of the elevated temperature of the fin surfaces and *slowing down the fluid* by acting as an added obstacle on the flow path. As a result, increasing the number of fins on a heat sink can either enhance or reduce natural convection, depending on which effect is dominant. The buoyancy-driven fluid flow rate is established at the point where these two effects balance each other. The friction force increases as more and more solid surfaces are introduced, seriously disrupting fluid flow and heat transfer. Under some conditions, the increase in friction may more than offset the increase in buoyancy. This in turn will tend to reduce the flow rate and thus the heat transfer. For that reason, heat sinks with closely spaced fills are not suitable for natural convection cooling.

When the heat sink involves closely spaced fins, the narrow channels formed tend to block or "suffocate" the fluid, especially when the heat sink is long. As a result, the blocking action produced overwhelms the extra buoyancy and downgrades the heat transfer characteristics of the heat sink. Then, at a fixed power setting, the heat sink runs at a higher temperature relative to the no-shroud case. When the heat sink involves widely spaced fins, the

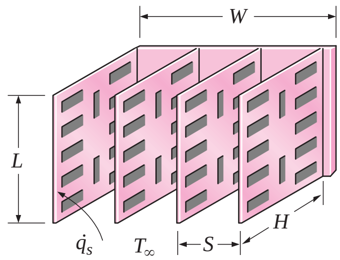


FIGURE 9–19
Arrays of vertical printed circuit boards (PCBs) cooled by natural convection.

shroud does not introduce a significant increase in resistance to flow, and the buoyancy effects dominate. As a result, heat transfer by natural convection may improve, and at a fixed power level the heat sink may run at a lower temperature.

When extended surfaces such as fins are used to enhance natural convection heat transfer between a solid and a fluid, the flow rate of the fluid in the vicinity of the solid adjusts itself to incorporate the changes in buoyancy and friction. It is obvious that this enhancement technique will work to advantage only when the increase in buoyancy is greater than the additional friction introduced. One does not need to be concerned with pressure drop or pumping power when studying natural convection since no pumps or blowers are used in this case. Therefore, an enhancement technique in natural convection is evaluated on heat transfer performance alone.

The failure rate of an electronic component increases almost exponentially with operating temperature. The cooler the electronic device operates, the more reliable it is. A rule of thumb is that the semiconductor failure rate is halved for each 10°C reduction in junction operating temperature. The desire to lower the operating temperature without having to resort to forced convection has motivated researchers to investigate enhancement techniques for natural convection. Sparrow and Prakash (Ref. 31) have demonstrated that, under certain conditions, the use of discrete plates in lieu of continuous plates of the same surface area increases heat transfer considerably. In other experimental work, using transistors as the heat source, Çengel and Zing (Ref. 9) have demonstrated that temperature recorded on the transistor case dropped by as much as 30°C when a shroud was used, as opposed to the corresponding no-shroud case.

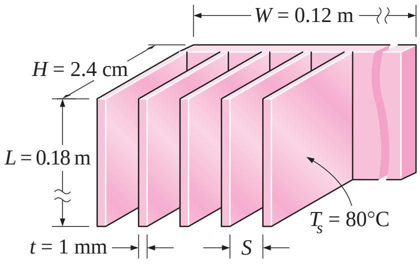


FIGURE 9–20
Schematic for Example 9–3.

EXAMPLE 9–3 Optimum Fin Spacing of a Heat Sink

A 12-cm-wide and 18-cm-high vertical hot surface in 30°C air is to be cooled by a heat sink with equally spaced fins of rectangular profile (Fig. 9–20). The fins are 0.1 cm thick and 18 cm long in the vertical direction and have a height of 2.4 cm from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 80°C.

SOLUTION A heat sink with equally spaced rectangular fins is to be used to cool a hot surface. The optimum fin spacing and the rate of heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** The atmospheric pressure at that location is 1 atm. **4** The thickness t of the fins is very small relative to the fin spacing S so that Eq. 9-32 for optimum fin spacing is applicable. **5** All fin surfaces are isothermal at base temperature.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (80 + 30)/2 = 55°C$ and 1 atm pressure are (Table A-15)

$$k = 0.02772 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7215$$
$$\nu = 1.846 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = 1/T_f = 1/328 \text{ K}$$

Analysis We take the characteristic length to be the length of the fins in the vertical direction (since we do not know the fin spacing). Then the Rayleigh number becomes

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81\text{ m/s}^2)[1/(328\text{ K})](80 - 30\text{ K})(0.18\text{ m})^3}{(1.846 \times 10^{-5}\text{ m}^2/\text{s})^2}(0.7215) = 1.846 \times 10^7$$

The optimum fin spacing is determined from Eq. 7-32 to be

$$S_{\text{opt}} = 2.714\frac{L}{\text{Ra}_L^{0.25}} = 2.714\frac{0.8\text{ m}}{(1.846 \times 10^7)^{0.25}} = 7.45 \times 10^{-3}\text{ m} = \mathbf{7.45\ mm}$$

which is about seven times the thickness of the fins. Therefore, the assumption of negligible fin thickness in this case is acceptable. The number of fins and the heat transfer coefficient for this optimum fin spacing case are

$$n = \frac{W}{S + t} = \frac{0.12\text{ m}}{(0.00745 + 0.0001)\text{ m}} \approx 15\text{ fins}$$

The convection coefficient for this optimum in spacing case is, from Eq. 9-33,

$$h = \text{Nu}_{\text{opt}}\frac{k}{S_{\text{opt}}} = 1.307\frac{0.02772\text{ W/m}\cdot{}^\circ\text{C}}{0.00745\text{ m}} = 0.2012\text{ W/m}^2\cdot{}^\circ\text{C}$$

Then the rate of natural convection heat transfer becomes

$$\dot{Q} = hA_s(T_s - T_\infty) = h(2nLH)(T_s - T_\infty)$$
$$= (0.2012\text{ W/m}^2\cdot{}^\circ\text{C})[2 \times 15(0.18\text{ m})(0.024\text{ m})](80 - 30)^\circ\text{C} = \mathbf{1.30\ W}$$

Therefore, this heat sink can dissipate heat by natural convection at a rate of 1.30 W.

9–5 ▪ NATURAL CONVECTION INSIDE ENCLOSURES

A considerable portion of heat loss from a typical residence occurs through the windows. We certainly would insulate the windows, if we could, in order to conserve energy. The problem is finding an insulating material that is transparent. An examination of the thermal conductivities of the insulting materials reveals that *air* is a *better insulator* than most common insulating materials. Besides, it is transparent. Therefore, it makes sense to insulate the windows with a layer of air. Of course, we need to use another sheet of glass to trap the air. The result is an *enclosure,* which is known as a *double-pane window* in this case. Other examples of enclosures include wall cavities, solar collectors, and cryogenic chambers involving concentric cylinders or spheres.

Enclosures are frequently encountered in practice, and heat transfer through them is of practical interest. Heat transfer in enclosed spaces is complicated by the fact that the fluid in the enclosure, in general, does not remain stationary. In a vertical enclosure, the fluid adjacent to the hotter surface rises and the fluid adjacent to the cooler one falls, setting off a rotationary motion within the enclosure that enhances heat transfer through the enclosure. Typical flow patterns in vertical and horizontal rectangular enclosures are shown in Figures 9–21 and 9–22.

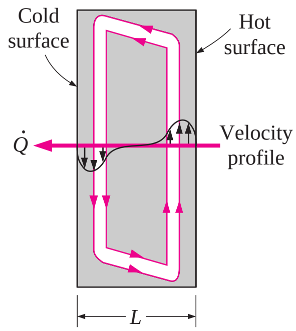


FIGURE 9–21
Convective currents in a vertical rectangular enclosure.

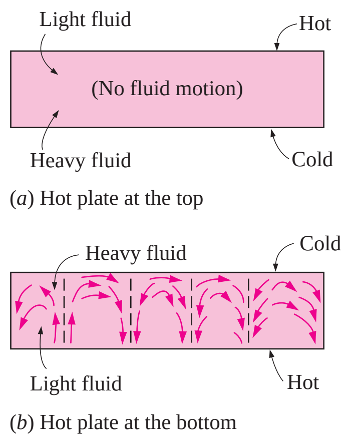


FIGURE 9–22
Convective currents in a horizontal enclosure with (*a*) hot plate at the top and (*b*) hot plate at the bottom.

The characteristics of heat transfer through a horizontal enclosure depend on whether the hotter plate is at the top or at the bottom, as shown in Figure 9–22. When the hotter plate is at the *top*, no convection currents will develop in the enclosure, since the lighter fluid will always be on top of the heavier fluid. Heat transfer in this case will be by *pure conduction*, and we will have Nu $= 1$. When the hotter plate is at the *bottom*, the heavier fluid will be on top of the lighter fluid, and there will be a tendency for the lighter fluid to topple the heavier fluid and rise to the top, where it will come in contact with the cooler plate and cool down. Until that happens, however, the heat transfer is still by *pure conduction* and Nu $= 1$. When Ra > 1708, the buoyant force overcomes the fluid resistance and initiates natural convection currents, which are observed to be in the form of hexagonal cells called *Bénard cells*. For Ra $> 3 \times 10^5$, the cells break down and the fluid motion becomes turbulent.

The Rayleigh number for an enclosure is determined from

$$\text{Ra}_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\,\text{Pr} \qquad \textbf{(9-40)}$$

where the characteristic length L_c is the distance between the hot and cold surfaces, and T_1 and T_2 are the temperatures of the hot and cold surfaces, respectively. All fluid properties are to be evaluated at the average fluid temperature $T_{\text{ave}} = (T_1 + T_2)/2$.

Effective Thermal Conductivity

When the Nusselt number is known, the rate of heat transfer through the enclosure can be determined from

$$\dot{Q} = hA_s(T_1 - T_2) = k\text{Nu}A_s\frac{T_1 - T_2}{L_c} \qquad \textbf{(9-41)}$$

since $h = k\text{Nu}/L$. The rate of steady heat conduction across a layer of thickness L_c, area A_s, and thermal conductivity k is expressed as

$$\dot{Q}_{\text{cond}} = kA_s\frac{T_1 - T_2}{L_c} \qquad \textbf{(9-42)}$$

where T_1 and T_2 are the temperatures on the two sides of the layer. A comparison of this relation with Eq. 9-41 reveals that the convection heat transfer in an enclosure is analogous to heat conduction across the fluid layer in the enclosure provided that the thermal conductivity k is replaced by kNu. That is, *the fluid in an enclosure behaves like a fluid whose thermal conductivity is kNu as a result of convection currents*. Therefore, the quantity kNu is called the **effective thermal conductivity** of the enclosure. That is,

$$k_{\text{eff}} = k\text{Nu} \qquad \textbf{(9-43)}$$

Note that for the special case of Nu $= 1$, the effective thermal conductivity of the enclosure becomes equal to the conductivity of the fluid. This is expected since this case corresponds to pure conduction (Fig. 9–23).

Natural convection heat transfer in enclosed spaces has been the subject of many experimental and numerical studies, and numerous correlations for the Nusselt number exist. Simple power-law type relations in the form of

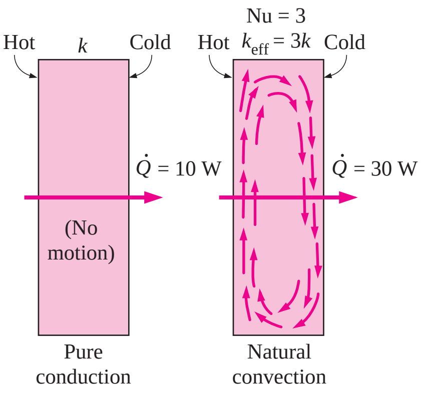


FIGURE 9–23
A Nusselt number of 3 for an enclosure indicates that heat transfer through the enclosure by *natural convection* is three times that by *pure conduction.*

$Nu = CRa_L^n$, where C and n are constants, are sufficiently accurate, but they are usually applicable to a narrow range of Prandtl and Rayleigh numbers and aspect ratios. The relations that are more comprehensive are naturally more complex. Next we present some widely used relations for various types of enclosures.

Horizontal Rectangular Enclosures

We need no Nusselt number relations for the case of the hotter plate being at the top, since there will be no convection currents in this case and heat transfer will be downward by conduction ($Nu = 1$). When the hotter plate is at the bottom, however, significant convection currents set in for $Ra_L > 1708$, and the rate of heat transfer increases (Fig. 9–24).

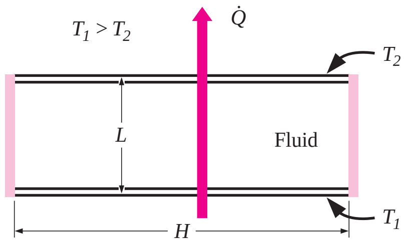


FIGURE 9–24
A horizontal rectangular enclosure with isothermal surfaces.

For horizontal enclosures that contain air, Jakob (1949, Ref. 22) recommends the following simple correlations

$$Nu = 0.195Ra_L^{1/4} \qquad 10^4 < Ra_L < 4 \times 10^5 \tag{9-44}$$

$$Nu = 0.068Ra_L^{1/3} \qquad 4 \times 10^5 < Ra_L < 10^7 \tag{9-45}$$

These relations can also be used for other gases with $0.5 < Pr < 2$. Using water, silicone oil, and mercury in their experiments, Globe and Dropkin (1959) obtained this correlation for horizontal enclosures heated from below,

$$Nu = 0.069Ra_L^{1/3}\,Pr^{0.074} \qquad 3 \times 10^5 < Ra_L < 7 \times 10^9 \tag{9-46}$$

Based on experiments with air, Hollands et al (1976, Ref. 19) recommend this correlation for horizontal enclosures,

$$Nu = 1 + 1.44\left[1 - \frac{1708}{Ra_L}\right]^+ + \left[\frac{Ra_L^{1/3}}{18} - 1\right]^+ \qquad Ra_L < 10^8 \tag{9-47}$$

The notation $[\ ]^+$ indicates that if the quantity in the bracket is negative, it should be set equal to zero. This relation also correlates data well for liquids with moderate Prandtl numbers for $Ra_L < 10^5$, and thus it can also be used for water.

Inclined Rectangular Enclosures

Air spaces between two inclined parallel plates are commonly encountered in flat-plate solar collectors (between the glass cover and the absorber plate) and the double-pane skylights on inclined roofs. Heat transfer through an inclined enclosure depends on the **aspect ratio** H/L as well as the tilt angle θ from the horizontal (Fig. 9–25).

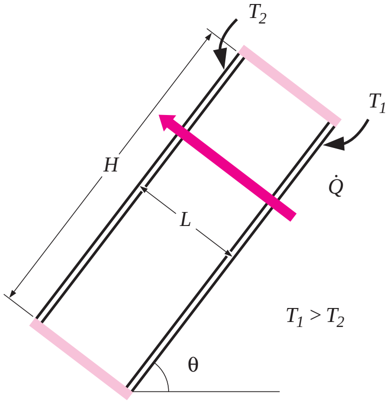


FIGURE 9–25
An inclined rectangular enclosure with isothermal surfaces.

For large aspect ratios ($H/L \geq 12$), this equation [Hollands et al., 1976, Ref. 19] correlates experimental data extremely well for tilt angles up to 70°,

$$Nu = 1 + 1.44\left[1 - \frac{1708}{Ra_L \cos\theta}\right]^+ \left(1 - \frac{1708(\sin 1.8\theta)^{1.6}}{Ra_L \cos\theta}\right) + \left[\frac{(Ra_L \cos\theta)^{1/3}}{18} - 1\right]^+ \tag{9-48}$$

for $Ra_L < 10^5$, $0 < \theta < 70°$, and $H/L \geq 12$. Again any quantity in $[\ ]^+$ should be set equal to zero if it is negative. This is to ensure that $Nu = 1$ for $Ra_L \cos\theta < 1708$. Note that this relation reduces to Eq. 9-47 for horizontal enclosures for $\theta = 0°$, as expected.

For enclosures with smaller aspect ratios ($H/L < 12$), the next correlation can be used provided that the tilt angle is less than the critical value θ_{cr} listed in Table 9–2 [Catton (1978), Ref. 7]

$$\mathrm{Nu} = \mathrm{Nu}_{\theta=0°}\left(\frac{\mathrm{Nu}_{\theta=90°}}{\mathrm{Nu}_{\theta=0°}}\right)^{\theta/\theta cr}(\sin\theta_{cr})^{\theta/(4\theta_{cr})} \qquad 0° < \theta < \theta_{cr} \tag{9-49}$$

For tilt angles greater than the critical value ($\theta_{cr} < \theta < 90°$), the Nusselt number can be obtained by multiplying the Nusselt number for a vertical enclosure by $(\sin\theta)^{1/4}$ [Ayyaswamy and Catton (1973), Ref. 3],

$$\mathrm{Nu} = \mathrm{Nu}_{\theta=90°}(\sin\theta)^{1/4} \qquad \theta_{cr} < \theta < 90°, \text{ any } H/L \tag{9-50}$$

For enclosures tilted more than 90°, the recommended relation is [Arnold et al., (1974), Ref. 2]

$$\mathrm{Nu} = 1 + (\mathrm{Nu}_{\theta=90°} - 1)\sin\theta \qquad 90° < \theta < 180°, \text{ any } H/L \tag{9-51}$$

More recent but more complex correlations are also available in the literature [e.g., and ElSherbiny et al. (1982), Ref. 17].

TABLE 9–2

Critical angles for inclined rectangular enclosures

Aspect ratio, H/L	Critical angle, θ_{cr}
1	25°
3	53°
6	60°
12	67°
> 12	70°

Vertical Rectangular Enclosures

For vertical enclosures (Fig. 9–26), Catton (1978, Ref. 7) recommends these two correlations due to Berkovsky and Polevikov (1977, Ref. 6),

$$\mathrm{Nu} = 0.18\left(\frac{\mathrm{Pr}}{0.2+\mathrm{Pr}}\mathrm{Ra}_L\right)^{0.29} \qquad \begin{array}{c} 1 < H/L < 2 \\ \text{any Prandtl number} \\ \mathrm{Ra}_L\,\mathrm{Pr}/(0.2+\mathrm{Pr}) > 10^3 \end{array} \tag{9-52}$$

$$\mathrm{Nu} = 0.22\left(\frac{\mathrm{Pr}}{0.2+\mathrm{Pr}}\mathrm{Ra}_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} \qquad \begin{array}{c} 2 < H/L < 10 \\ \text{any Prandtl number} \\ \mathrm{Ra}_L < 10^{10} \end{array} \tag{9-53}$$

For vertical enclosures with larger aspect ratios, the following correlations can be used [MacGregor and Emery (1969), Ref. 26]

$$\mathrm{Nu} = 0.42\,\mathrm{Ra}_L^{1/4}\,\mathrm{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} \qquad \begin{array}{c} 10 < H/L < 40 \\ 1 < \mathrm{Pr} < 2\times10^4 \\ 10^4 < \mathrm{Ra}_L < 10^7 \end{array} \tag{9-54}$$

$$\mathrm{Nu} = 0.46\mathrm{Ra}_L^{1/3} \qquad \begin{array}{c} 1 < H/L < 40 \\ 1 < \mathrm{Pr} < 20 \\ 10^6 < \mathrm{Ra}_L < 10^9 \end{array} \tag{9-55}$$

Again all fluid properties are to be evaluated at the average temperature $(T_1 + T_2)/2$.

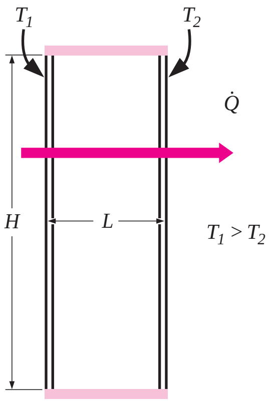


FIGURE 9–26
A vertical rectangular enclosure with isothermal surfaces.

Concentric Cylinders

Consider two long concentric horizontal cylinders maintained at uniform but different temperatures of T_i and T_o, as shown in Figure 9–27. The diameters of the inner and outer cylinders are D_i and D_o, respectively, and the characteristic length is the spacing between the cylinders, $L_c = (D_o - D_i)/2$. The rate of heat transfer through the annular space between the natural convection unit is expressed as

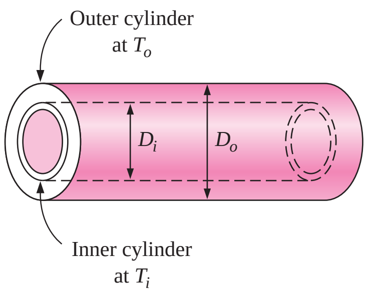


FIGURE 9–27
Two concentric horizontal isothermal cylinders.

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) \qquad \text{(W/m)} \tag{9-56}$$

The recommended relation for effective thermal conductivity is [Raithby and Hollands (1975), Ref. 28]

$$\frac{k_{\text{eff}}}{k} = 0.386\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra}_L)^{1/4} \tag{9-57}$$

where the geometric factor for concentric cylinders F_{cyl} is

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} \tag{9-58}$$

The k_{eff} relation in Eq. 9-57 is applicable for $0.70 \leq \text{Pr} \leq 6000$ and $10^2 \leq F_{\text{cyl}}\text{Ra}_L \leq 10^7$. For $F_{\text{cyl}}\text{Ra}_L < 100$, natural convection currents are negligible and thus $k_{\text{eff}} = k$. Note that k_{eff} cannot be less than k, and thus we should set $k_{\text{eff}} = k$ if $k_{\text{eff}}/k < 1$. The fluid properties are evaluated at the average temperature of $(T_i + T_o)/2$.

Concentric Spheres

For concentric isothermal spheres, the rate of heat transfer through the gap between the spheres by natural convection is expressed as (Fig. 9–28)

$$\dot{Q} = k_{\text{eff}}\pi\left(\frac{D_iD_o}{L_c}\right)(T_i - T_o) \qquad \text{(W)} \tag{9-59}$$

where $L_c = (D_o - D_i)/2$ is the characteristic length. The recommended relation for effective thermal conductivity is [Raithby and Hollands (1975), Ref. 28]

$$\frac{k_{\text{eff}}}{k} = 0.74\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{sph}}\text{Ra}_L)^{1/4} \tag{9-60}$$

where the geometric factor for concentric spheres F_{sph} is

$$F_{\text{sph}} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5} \tag{9-61}$$

The k_{eff} relation in Eq. 9-60 is applicable for $0.70 \leq \text{Pr} \leq 4200$ and $10^2 \leq F_{\text{sph}}\text{Ra}_L \leq 10^4$. If $k_{\text{eff}}/k < 1$, we should set $k_{\text{eff}} = k$.

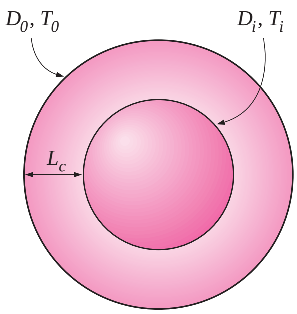


FIGURE 9–28
Two concentric isothermal spheres.

Combined Natural Convection and Radiation

Gases are nearly transparent to radiation, and thus heat transfer through a gas layer is by simultaneous convection (or conduction, if the gas is quiescent) and radiation. Natural convection heat transfer coefficients are typically very low compared to those for forced convection. Therefore, radiation is usually disregarded in forced convection problems, but it must be considered in natural convection problems that involve a gas. This is especially the case for surfaces with high emissivities. For example, about half of the heat transfer through the air space of a double pane window is by radiation. The total rate of heat transfer is determined by adding the convection and radiation components,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} \tag{9-62}$$

Radiation heat transfer from a surface at temperature T_s surrounded by surfaces at a temperature T_{surr} (both in absolute temperature unit K) is determined from

$$\dot{Q}_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) \qquad (W) \tag{9-63}$$

where ε is the emissivity of the surface, A_s is the surface area, and $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot$ K^4 is the Stefan–Boltzmann constant.

When the end effects are negligible, radiation heat transfer between two large parallel plates at absolute temperatures T_1 and T_2 is expressed as (see Chapter 12 for details)

$$\dot{Q}_{rad} = \frac{\sigma A_s(T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \varepsilon_{effective}\, \sigma A_s(T_1^4 - T_2^4) \qquad (W) \tag{9-64}$$

where ε_1 and ε_2 are the emissivities of the plates, and $\varepsilon_{effective}$ is the *effective emissivity* defined as

$$\varepsilon_{effective} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} \tag{9-65}$$

The emissivity of an ordinary glass surface, for example, is 0.84. Therefore, the effective emissivity of two parallel glass surfaces facing each other is 0.72. Radiation heat transfer between concentric cylinders and spheres is discussed in Chapter 12.

Note that in some cases the temperature of the surrounding medium may be below the surface temperature ($T_\infty < T_s$), while the temperature of the surrounding surfaces is above the surface temperature ($T_{surr} > T_s$). In such cases, convection and radiation heat transfers are subtracted from each other instead of being added since they are in opposite directions. Also, for a metal surface, the radiation effect can be reduced to negligible levels by polishing the surface and thus lowering the surface emissivity to a value near zero.

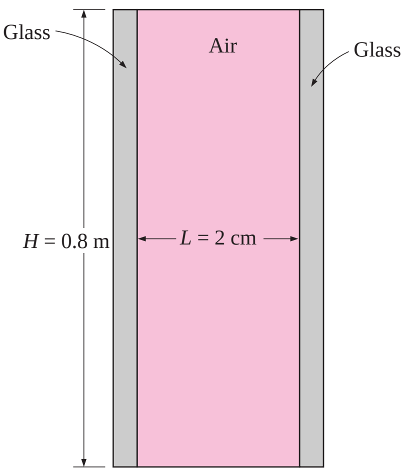


FIGURE 9–29
Schematic for Example 9–4.

EXAMPLE 9–4 Heat Loss through a Double-Pane Window

The vertical 0.8-m-high, 2-m-wide double-pane window shown in Fig. 9–29 consists of two sheets of glass separated by a 2-cm air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 12°C and 2°C, determine the rate of heat transfer through the window.

SOLUTION Two glasses of a double-pane window are maintained at specified temperatures. The rate of heat transfer through the window is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Radiation heat transfer is not considered.

Properties The properties of air at the average temperature of $T_{ave} = (T_1 + T_2)/2 = (12 + 2)/2 = 7$°C and 1 atm pressure are (Table A-15)

$$k = 0.02416 \text{ W/m} \cdot °\text{C} \qquad \text{Pr} = 0.7344$$

$$\nu = 1.399 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = \frac{1}{T_{ave}} = \frac{1}{280 \text{ K}}$$

Analysis We have a rectangular enclosure filled with air. The characteristic length in this case is the distance between the two glasses, $L = 0.02$ m. Then the Rayleigh number becomes

$$\text{Ra}_L = \frac{g\beta(T_1 - T_2)L^3}{\nu^2}$$
$$= \frac{(9.81 \text{ m/s}^2)[1/(280 \text{ K})](12 - 2 \text{ K})(0.02 \text{ m})^3}{(1.399 \times 10^{-5} \text{m}^2/\text{s})^2}(0.7344) = 1.051 \times 10^4$$

The aspect ratio of the geometry is $H/L = 0.8/0.02 = 40$. Then the Nusselt number in this case can be determined from Eq. 9-54 to be

$$\text{Nu} = 0.42\text{Ra}_L^{1/4}\,\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3}$$
$$= 0.42(1.051 \times 10^4)^{1/4}(0.7344)^{0.012}\left(\frac{0.8}{0.02}\right)^{-0.3} = 1.401$$

Then,

$$A_s = H \times W = (0.8 \text{ m})(2 \text{ m}) = 1.6 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_1 - T_2) = k\text{Nu}A_s\frac{T_1 - T_2}{L}$$
$$= (0.02416 \text{ W/m}\cdot{}^\circ\text{C})(1.401)(1.6 \text{ m}^2)\frac{(12 - 2)^\circ\text{C}}{0.02 \text{ m}} = \mathbf{27.1\ W}$$

Therefore, heat will be lost through the window at a rate of 27.1 W.

Discussion Recall that a Nusselt number of Nu = 1 for an enclosure corresponds to pure conduction heat transfer through the enclosure. The air in the enclosure in this case remains still, and no natural convection currents occur in the enclosure. The Nusselt number in our case is 1.32, which indicates that heat transfer through the enclosure is 1.32 times that by pure conduction. The increase in heat transfer is due to the natural convection currents that develop in the enclosure.

EXAMPLE 9–5 Heat Transfer through a Spherical Enclosure

The two concentric spheres of diameters $D_i = 20$ cm and $D_o = 30$ cm shown in Fig. 9–30 are separated by air at 1 atm pressure. The surface temperatures of the two spheres enclosing the air are $T_i = 320$ K and $T_o = 280$ K, respectively. Determine the rate of heat transfer from the inner sphere to the outer sphere by natural convection.

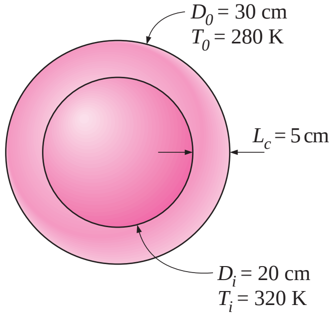


FIGURE 9–30
Schematic for Example 9–5.

SOLUTION Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Radiation heat transfer is not considered.

Properties The properties of air at the average temperature of $T_{ave} = (T_i + T_o)/2 = (320 + 280)/2 = 300 \text{ K} = 27^\circ\text{C}$ and 1 atm pressure are (Table A-15)

$$k = 0.02566 \text{ W/m}\cdot{}^\circ\text{C} \qquad \text{Pr} = 0.7290$$
$$\nu = 1.580 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = \frac{1}{T_{ave}} = \frac{1}{300 \text{ K}}$$

Analysis We have a spherical enclosure filled with air. The characteristic length in this case is the distance between the two spheres,

$$L_c = (D_o - D_i)/2 = (0.3 - 0.2)/2 = 0.05 \text{ m}$$

The Rayleigh number is

$$\begin{aligned} Ra_L &= \frac{g\beta(T_i - T_o)L^3}{\nu^2} Pr \\ &= \frac{(9.81 \text{ m/s}^2)[1/(300 \text{ K})](320 - 280 \text{ K})(0.05 \text{ m})^3}{(1.58 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.729) = 4.776 \times 10^5 \end{aligned}$$

The effective thermal conductivity is

$$\begin{aligned} F_{sph} &= \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5} \\ &= \frac{0.05 \text{ m}}{[(0.2 \text{ m})(0.3 \text{ m})]^4 [(0.2 \text{ m}^{-7/5} + (0.3 \text{ m})^{-7/5}]^5} = 0.005229 \end{aligned}$$

$$\begin{aligned} k_{eff} &= 0.74k\left(\frac{Pr}{0.861 + Pr}\right)^{1/4} (F_{sph}Ra_L)^{1/4} \\ &= 0.74(0.02566 \text{ W/m} \cdot {}^\circ\text{C})\left(\frac{0.729}{0.861 + 0.729}\right)(0.005229 \times 4.776 \times 10^5)^{1/4} \\ &= 0.1104 \text{ W/m} \cdot {}^\circ\text{C} \end{aligned}$$

Then the rate of heat transfer between the spheres becomes

$$\begin{aligned} \dot{Q} &= k_{eff}\pi\left(\frac{D_i D_o}{L_c}\right)(T_i - T_o) \\ &= (0.1104 \text{ W/m} \cdot {}^\circ\text{C})\pi\left(\frac{(0.2 \text{ m})(0.3 \text{ m})}{0.05 \text{ m}}\right)(320 - 280)\text{K} = \mathbf{16.7 \text{ W}} \end{aligned}$$

Therefore, heat will be lost from the inner sphere to the outer one at a rate of 16.7 W.

Discussion Note that the air in the spherical enclosure will act like a stationary fluid whose thermal conductivity is $k_{eff}/k = 0.1104/0.02566 = 4.3$ times that of air as a result of natural convection currents. Also, radiation heat transfer between spheres is usually very significant, and should be considered in a complete analysis.

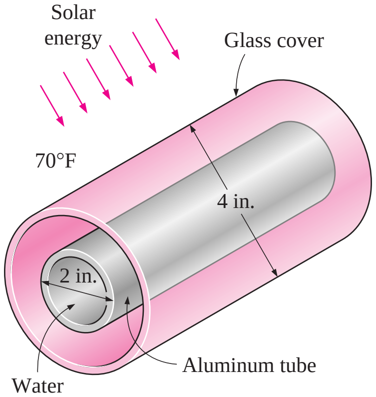


FIGURE 9–31
Schematic for Example 9–6.

EXAMPLE 9–6 Heating Water in a Tube by Solar Energy

A solar collector consists of a horizontal aluminum tube having an outer diameter of 2 in. enclosed in a concentric thin glass tube of 4-in.-diameter (Fig. 9–31). Water is heated as it flows through the tube, and the annular space between the aluminum and the glass tubes is filled with air at 1 atm pressure. The pump circulating the water fails during a clear day, and the water temperature in the tube starts rising. The aluminum tube absorbs solar radiation at a rate of 30 Btu/h per foot length, and the temperature of the ambient air outside is 70°F. Disregarding any heat loss by radiation, determine the temperature of the aluminum tube when steady operation is established (i.e., when the rate of heat loss from the tube equals the amount of solar energy gained by the tube).

SOLUTION The circulating pump of a solar collector that consists of a horizontal tube and its glass cover fails. The equilibrium temperature of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube and its cover are isothermal. **3** Air is an ideal gas. **4** Heat loss by radiation is negligible.

Properties The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and thus we cannot evaluate the average temperatures. Therefore, we will assume the glass temperature to be 110°F, and use properties at an anticipated average temperature of (70 + 110)/2 = 90°F (Table A-15E),

$$k = 0.01505 \text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F} \qquad \text{Pr} = 0.7275$$

$$\nu = 0.6310 \text{ ft}^2/\text{h} = 1.753\times10^{-4}\text{ ft}^2/\text{s} \qquad \beta = \frac{1}{T_{\text{ave}}} = \frac{1}{550\text{ K}}$$

Analysis We have a horizontal cylindrical enclosure filled with air at 1 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{\text{tube-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q}_{\text{so1ar gain}} = 30\text{ Btu/h} \qquad \text{(per foot of tube)}$$

The heat transfer surface area of the glass cover is

$$A_o = A_{\text{glass}} = (\pi D_o L) = \pi(4/12\text{ ft})(1\text{ ft}) = 1.047\text{ ft}^2 \qquad \text{(per foot of tube)}$$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, it is clear that the solution will require a trial-and-error approach. Assuming the glass cover temperature to be 100°F, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$\text{Ra}_{D_o} = \frac{g\beta(T_s - T_\infty)D_o^3}{\nu^2}\text{Pr}$$
$$= \frac{(32.2\text{ ft/s}^2)[1/(550\text{ R})](110-70\text{ R})(4/12\text{ ft})^3}{(1.753\times10^{-4}\text{ ft}^2/\text{s})^2}(0.7275) = 2.054\times10^6$$

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{[1+(0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.054\times10^6)^{1/6}}{[1+(0.559/0.7275)^{9/16}]^{8/27}}\right\}^2$$
$$= 17.89$$

$$h_o = \frac{k}{D_0}\text{Nu} = \frac{0.0150\text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}}{4/12\text{ ft}}(17.89) = 0.8075\text{ Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

$$\dot{Q}_o = h_o A_o(T_o - T_\infty) = (0.8075\text{ Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(1.047\text{ ft}^2)(110-70){}^\circ\text{F}$$
$$= 33.8\text{ Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 110°F for the glass cover is high. Repeating the calculations with lower temperatures, the glass cover temperature corresponding to 30 Btu/h is determined to be 106°F.

The temperature of the aluminum tube is determined in a similar manner using the natural convection relations for two horizontal concentric cylinders. The characteristic length in this case is the distance between the two cylinders, which is

$$L_c = (D_o - D_i)/2 = (4 - 2)/2 = 1 \text{ in.} = 1/12 \text{ ft}$$

We start the calculations by assuming the tube temperature to be 200°F, and thus an average temperature of (106 + 200)/2 = 154°F = 614 R. This gives

$$\text{Ra}_L = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr}$$
$$= \frac{(32.2 \text{ ft/s}^2)[1/614 \text{ R})](200 - 106 \text{ R})(1/12 \text{ ft})^3}{(2.117 \times 10^{-4} \text{ ft}^2/\text{s})^2}(0.7184) = 4.579 \times 10^4$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5}$$
$$= \frac{[\ln(4/2)]^4}{(1/12 \text{ ft})^3[(2/12 \text{ ft})^{-3/5} + (4/12 \text{ ft})^{-3/5}]^5} = 0.1466$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra}_L)^{1/4}$$
$$= 0.386(0.01653 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F})\left(\frac{0.7184}{0.861 + 0.7184}\right)(0.1466 \times 4.579 \times 10^4)^{1/4}$$
$$= 0.04743 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o)$$
$$= \frac{2\pi(0.04743 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F})}{\ln(4/2)}(200 - 106)^\circ\text{F} = 40.4 \text{ Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 200°F for the tube is high. By trying other values, the tube temperature corresponding to 30 Btu/h is determined to be **180°F.** Therefore, the tube will reach an equilibrium temperature of 180°F when the pump fails.

Discussion Note that we have not considered heat loss by radiation in the calculations, and thus the tube temperature determined above is probably too high. This problem is considered again in Chapter 12 by accounting for the effect of radiation heat transfer.

9–6 ▪ COMBINED NATURAL AND FORCED CONVECTION

The presence of a temperature gradient in a fluid in a gravity field always gives rise to natural convection currents, and thus heat transfer by natural convection. Therefore, forced convection is always accompanied by natural convection.

We mentioned earlier that the convection heat transfer coefficient, natural or forced, is a strong function of the fluid velocity. Heat transfer coefficients encountered in forced convection are typically much higher than those encountered in natural convection because of the higher fluid velocities associated with forced convection. As a result, we tend to ignore natural convection in heat transfer analyses that involve forced convection, although we recognize that natural convection always accompanies forced convection. The error involved in ignoring natural convection is negligible at high velocities but may be considerable at low velocities associated with forced convection. Therefore, it is desirable to have a criterion to assess the relative magnitude of natural convection in the presence of forced convection.

For a given fluid, it is observed that the parameter Gr/Re^2 represents the importance of natural convection relative to forced convection. This is not surprising since the convection heat transfer coefficient is a strong function of the Reynolds number Re in forced convection and the Grashof number Gr in natural convection.

A plot of the nondimensionalized heat transfer coefficient for combined natural and forced convection on a vertical plate is given in Fig. 9–32 for different fluids. We note from this figure that natural convection is negligible when Gr/Re$^2 < 0.1$, forced convection is negligible when Gr/Re$^2 > 10$, and neither is negligible when $0.1 <$ Gr/Re$^2 < 10$. Therefore, both natural and forced convection must be considered in heat transfer calculations when the Gr and Re^2 are of the same order of magnitude (one is within a factor of 10 times the other). Note that forced convection is small relative to natural convection only in the rare case of extremely low forced flow velocities.

Natural convection may *help* or *hurt* forced convection heat transfer, depending on the relative directions of *buoyancy-induced* and the *forced convection* motions (Fig. 9–33):

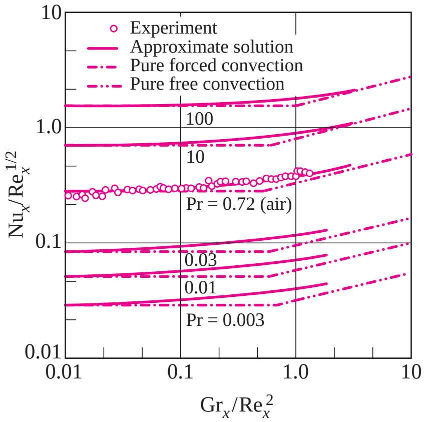


FIGURE 9–32
Variation of the local Nusselt number NU_x for combined natural and forced convection from a hot isothermal vertical plate (from Lloyd and Sparrow, Ref. 25).

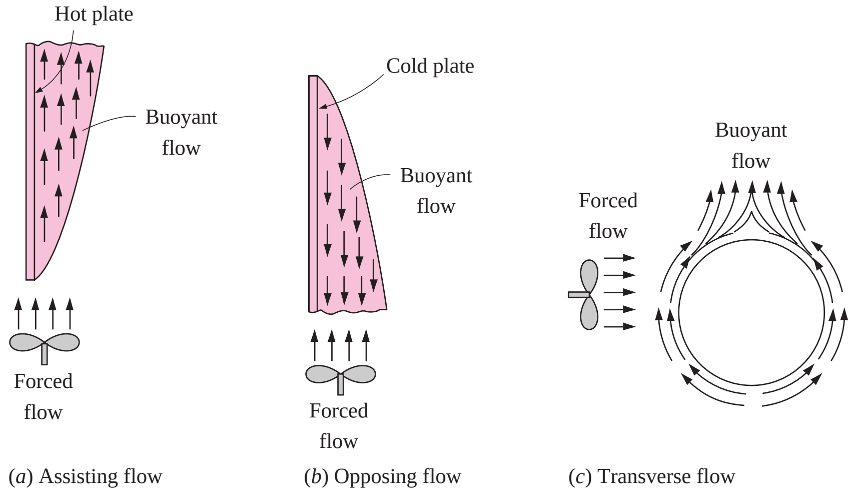


FIGURE 9–33
Natural convection can *enhance* or *inhibit* heat transfer, depending on the relative directions of *buoyancy-induced motion* and the *forced convection motion.*

1. In *assisting flow*, the buoyant motion is in the *same* direction as the forced motion. Therefore, natural convection assists forced convection and *enhances* heat transfer. An example is upward forced flow over a hot surface.
2. In *opposing flow*, the buoyant motion is in the *opposite* direction to the forced motion. Therefore, natural convection resists forced convection and *decreases* heat transfer. An example is upward forced flow over a cold surface.
3. In *transverse flow*, the buoyant motion is *perpendicular* to the forced motion. Transverse flow enhances fluid mixing and thus *enhances* heat transfer. An example is horizontal forced flow over a hot or cold cylinder or sphere.

When determining heat transfer under combined natural and forced convection conditions, it is tempting to add the contributions of natural and forced convection in assisting flows and to subtract them in opposing flows. However, the evidence indicates differently. A review of experimental data suggests a correlation of the form

$$\text{Nu}_{\text{combined}} = (\text{Nu}^n_{\text{forced}} \pm \text{Nu}^n_{\text{natural}})^{1/n} \qquad \textbf{(9–41)}$$

where $\text{Nu}_{\text{forced}}$ and $\text{Nu}_{\text{natural}}$ are determined from the correlations for *pure forced* and *pure natural convection*, respectively. The plus sign is for *assisting* and *transverse* flows and the minus sign is for *opposing* flows. The value of the exponent n varies between 3 and 4, depending on the geometry involved. It is observed that $n = 3$ correlates experimental data for vertical surfaces well. Larger values of n are better suited for horizontal surfaces.

A question that frequently arises in the cooling of heat-generating equipment such as electronic components is whether to use a fan (or a pump if the cooling medium is a liquid)—that is, whether to utilize *natural* or *forced* convection in the cooling of the equipment. The answer depends on the maximum allowable operating temperature. Recall that the convection heat transfer rate from a surface at temperature T_s in a medium at T_∞ is given by

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

where h is the convection heat transfer coefficient and A_s is the surface area. Note that for a fixed value of power dissipation and surface area, h and T_s are *inversely proportional*. Therefore, the device will operate at a *higher* temperature when h is low (typical of natural convection) and at a *lower* temperature when h is high (typical of forced convection).

Natural convection is the preferred mode of heat transfer since no blowers or pumps are needed and thus all the problems associated with these, such as noise, vibration, power consumption, and malfunctioning, are avoided. Natural convection is adequate for cooling *low-power-output* devices, especially when they are attached to extended surfaces such as heat sinks. For *high-power-output* devices, however, we have no choice but to use a blower or a pump to keep the operating temperature below the maximum allowable level. For *very-high-power-output* devices, even forced convection may not be sufficient to keep the surface temperature at the desirable levels. In such cases, we may have to use *boiling* and *condensation* to take advantage of the very high heat transfer coefficients associated with phase change processes.