CHAPTER

7

EXTERNAL FORCED CONVECTION

In Chapter 6 we considered the general and theoretical aspects of forced convection, with emphasis on differential formulation and analytical solutions. In this chapter we consider the practical aspects of forced convection to or from flat or curved surfaces subjected to *external flow*, characterized by the freely growing boundary layers surrounded by a free flow region that involves no velocity and temperature gradients.

We start this chapter with an overview of external flow, with emphasis on friction and pressure drag, flow separation, and the evaluation of average drag and convection coefficients. We continue with *parallel flow over flat plates.* In Chapter 6, we solved the boundary layer equations for steady, laminar, parallel flow over a flat plate, and obtained relations for the local friction coefficient and the Nusselt number. Using these relations as the starting point, we determine the average friction coefficient and Nusselt number. We then extend the analysis to turbulent flow over flat plates with and without an unheated starting length.

Next we consider *cross flow over cylinders and spheres*, and present graphs and empirical correlations for the drag coefficients and the Nusselt numbers, and discuss their significance. Finally, we consider *cross flow over tube banks* in aligned and staggered configurations, and present correlations for the pressure drop and the average Nusselt number for both configurations.

CONTENTS

7–1 ▪ DRAG AND HEAT TRANSFER IN EXTERNAL FLOW

Fluid flow over solid bodies frequently occurs in practice, and it is responsible for numerous physical phenomena such as the *drag force* acting on the automobiles, power lines, trees, and underwater pipelines; the *lift* developed by airplane wings; *upward draft* of rain, snow, hail, and dust particles in high winds; and the *cooling* of metal or plastic sheets, steam and hot water pipes, and extruded wires. Therefore, developing a good understanding of external flow and external forced convection is important in the mechanical and thermal design of many engineering systems such as aircraft, automobiles, buildings, electronic components, and turbine blades.

The flow fields and geometries for most external flow problems are too complicated to be solved analytically, and thus we have to rely on correlations based on experimental data. The availability of high-speed computers has made it possible to conduct series of "numerical experimentations" quickly by solving the governing equations numerically, and to resort to the expensive and time-consuming testing and experimentation only in the final stages of design. In this chapter we will mostly rely on relations developed experimentally.

The velocity of the fluid relative to an immersed solid body sufficiently far from the body (outside the boundary layer) is called the **free-stream velocity**, and is denoted by u_∞. It is usually taken to be equal to the **upstream velocity** $\mathcal{V}$ also called the **approach velocity**, which is the velocity of the approaching fluid far ahead of the body. This idealization is nearly exact for very thin bodies, such as a flat plate parallel to flow, but approximate for blunt bodies such as a large cylinder. The fluid velocity ranges from zero at the surface (the no-slip condition) to the free-stream value away from the surface, and the subscript "infinity" serves as a reminder that this is the value at a distance where the presence of the body is not felt. The upstream velocity, in general, may vary with location and time (e.g., the wind blowing past a building). But in the design and analysis, the upstream velocity is usually assumed to be *uniform* and *steady* for convenience, and this is what we will do in this chapter.

Friction and Pressure Drag

You may have seen high winds knocking down trees, power lines, and even trailers, and have felt the strong "push" the wind exerts on your body. You experience the same feeling when you extend your arm out of the window of a moving car. The force a flowing fluid exerts on a body in the flow direction is called **drag** (Fig. 7–1)

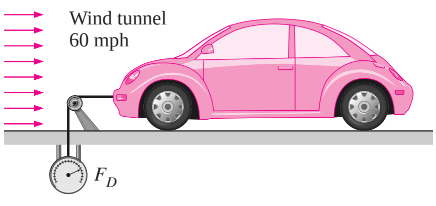


FIGURE 7–1
Schematic for measuring the drag force acting on a car in a wind tunnel.

A stationary fluid exerts only normal pressure forces on the surface of a body immersed in it. A moving fluid, however, also exerts tangential shear forces on the surface because of the no-slip condition caused by viscous effects. Both of these forces, in general, have components in the direction of flow, and thus the drag force is due to the combined effects of pressure and wall shear forces in the flow direction. The components of the pressure and wall shear forces in the normal direction to flow tend to move the body in that direction, and their sum is called **lift**.

In general, both the skin friction (wall shear) and pressure contribute to the drag and the lift. In the special case of a thin flat plate aligned parallel to the flow direction, the drag force depends on the wall shear only and is independent of pressure. When the flat plate is placed normal to the flow direction, however, the drag force depends on the pressure only and is independent of the wall shear since the shear stress in this case acts in the direction normal to flow (Fig. 7–2). For slender bodies such as wings, the shear force acts nearly parallel to the flow direction. The drag force for such slender bodies is mostly due to shear forces (the skin friction).

The drag force F_D depends on the density ρ of the fluid, the upstream velocity $\mathcal{V}$, and the size, shape, and orientation of the body, among other things. The drag characteristics of a body is represented by the dimensionless **drag coefficient** C_D defined as

Drag coefficient:
$$C_D = \frac{F_D}{\frac{1}{2}\rho\mathcal{V}^2 A} \qquad \textbf{(7-1)}$$

where A is the *frontal area* (the area projected on a plane normal to the direction of flow) for blunt bodies—bodies that tends to block the flow. The frontal area of a cylinder of diameter D and length L, for example, is $A = LD$. For parallel flow over flat plates or thin airfoils, A is the surface area. The drag coefficient is primarily a function of the shape of the body, but it may also depend on the Reynolds number and the surface roughness.

The drag force is the net force exerted by a fluid on a body in the direction of flow due to the combined effects of wall shear and pressure forces. The part of drag that is due directly to wall shear stress τ_w is called the **skin friction drag** (or just *friction drag*) since it is caused by frictional effects, and the part that is due directly to pressure P is called the **pressure drag** (also called the *form drag* because of its strong dependence on the form or shape of the body). When the friction and pressure drag coefficients are available, the total drag coefficient is determined by simply adding them,

$$C_D = C_{D,\text{ friction}} + C_{D,\text{ pressure}} \qquad \textbf{(7-2)}$$

The *friction drag* is the component of the wall shear force in the direction of flow, and thus it depends on the orientation of the body as well as the magnitude of the wall shear stress τ_w. The friction drag is *zero* for a surface normal to flow, and *maximum* for a surface parallel to flow since the friction drag in this case equals the total shear force on the surface. Therefore, for parallel flow over a flat plate, the drag coefficient is equal to the *friction drag coefficient,* or simply the *friction coefficient* (Fig. 7–3). That is,

Flat plate:
$$C_D = C_{D,\text{ friction}} = C_f \qquad \textbf{(7-3)}$$

Once the average friction coefficient C_f is available, the drag (or friction) force over the surface can be determined from Eq. 7-1. In this case A is the surface area of the plate exposed to fluid flow. When both sides of a thin plate are subjected to flow, A becomes the total area of the top and bottom surfaces. Note that the friction coefficient, in general, will vary with location along the surface.

Friction drag is a strong function of viscosity, and an "idealized" fluid with zero viscosity would produce zero friction drag since the wall shear stress would be zero (Fig. 7–4). The pressure drag would also be zero in this case during steady flow regardless of the shape of the body since there will be no pressure losses. For flow in the horizontal direction, for example, the pressure along a horizontal line will be constant (just like stationary fluids) since the

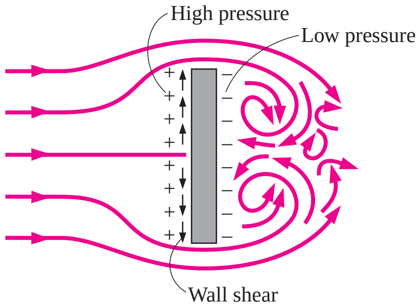


FIGURE 7–2
Drag force acting on a flat plate normal to flow depends on the pressure only and is independent of the wall shear, which acts normal to flow.

$C_{D,\text{ pressure}} = 0$

$C_D = C_{D,\text{ friction}} = C_f$

$F_{D,\text{ pressure}} = 0$

$F_D = F_{D,\text{ friction}} = F_f = C_f A \dfrac{\rho\mathcal{V}^2}{2}$

FIGURE 7–3
For parallel flow over a flat plate, the pressure drag is zero, and thus the drag coefficient is equal to the friction coefficient and the drag force is equal to the friction force.

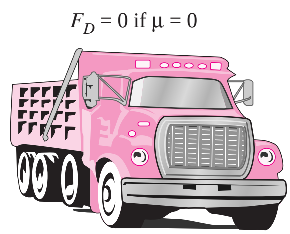


FIGURE 7–4
For the flow of an "idealized" fluid with zero viscosity past a body, both the friction drag and pressure drag are zero regardless of the shape of the body.

upstream velocity is constant, and thus there will be no net pressure force acting on the body in the horizontal direction. Therefore, the total drag is zero for the case of ideal inviscid fluid flow.

At low Reynolds numbers, most drag is due to friction drag. This is especially the case for highly streamlined bodies such as airfoils. The friction drag is also proportional to the surface area. Therefore, bodies with a larger surface area will experience a larger friction drag. Large commercial airplanes, for example, reduce their total surface area and thus drag by retracting their wing extensions when they reach the cruising altitudes to save fuel. The friction drag coefficient is independent of *surface roughness* in laminar flow, but is a strong function of surface roughness in turbulent flow due to surface roughness elements protruding further into the highly viscous laminar sublayer.

The pressure drag is proportional to the *difference* between the pressures acting on the front and back of the immersed body, and the frontal area. Therefore, the pressure drag is usually dominant for blunt bodies, negligible for streamlined bodies such as airfoils, and zero for thin flat plates parallel to the flow.

When a fluid is forced to flow over a curved surface at sufficiently high velocities, it will detach itself from the surface of the body. The low-pressure region behind the body where recirculating and back flows occur is called the *separation* region. The larger the separation area is, the larger the pressure drag will be. The effects of flow separation are felt far downstream in the form of reduced velocity (relative to the upstream velocity). The region of flow trailing the body where the effect of the body on velocity is felt is called the *wake* (Fig. 7–5). The separated region comes to an end when the two flow streams reattach, but the wake keeps growing behind the body until the fluid in the wake region regains its velocity. The viscous effects are the most significant in the boundary layer, the separated region, and the wake. The flow outside these regions can be considered to be inviscid.

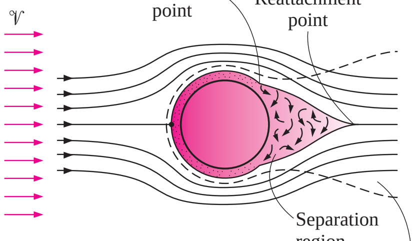


FIGURE 7–5
Separation and reattachment during flow over a cylinder, and the wake region.

Heat Transfer

The phenomena that affect drag force also affect heat transfer, and this effect appears in the Nusselt number. By nondimensionalizing the boundary layer equations, it was shown in Chapter 6 that the local and average Nusselt numbers have the functional form

$$\text{Nu}_x = f_1(x^*, \text{Re}_x, \text{Pr}) \quad \text{and} \quad \text{Nu} = f_2(\text{Re}_L, \text{Pr}) \qquad \textbf{(7-4a, b)}$$

The experimental data for heat transfer is often represented conveniently with reasonable accuracy by a simple power-law relation of the form

$$\text{Nu} = C\,\text{Re}_L^m \text{Pr}^n \qquad \textbf{(7-5)}$$

where m and n are constant exponents, and the value of the constant C depends on geometry and flow.

The fluid temperature in the thermal boundary layer varies from T_s at the surface to about T_∞ at the outer edge of the boundary. The fluid properties also vary with temperature, and thus with position across the boundary layer. In order to account for the variation of the properties with temperature, the fluid properties are usually evaluated at the so-called **film temperature**, defined as

$$T_f = \frac{T_s + T_\infty}{2} \tag{7-6}$$

which is the *arithmetic average* of the surface and the free-stream temperatures. The fluid properties are then assumed to remain constant at those values during the entire flow. An alternative way of accounting for the variation of properties with temperature is to evaluate all properties at the free stream temperature and to multiply the Nusselt number relation in Eq. 7-5 by $(\text{Pr}_\infty/\text{Pr}_s)^r$ or $(\mu_\infty/\mu_s)^r$.

The local drag and convection coefficients vary along the surface as a result of the changes in the velocity boundary layers in the flow direction. We are usually interested in the drag force and the heat transfer rate for the *entire* surface, which can be determined using the *average* friction and convection coefficient. Therefore, we present correlations for both local (identified with the subscript x) and average friction and convection coefficients. When relations for local friction and convection coefficients are available, the *average* friction and convection coefficients for the entire surface can be determined by integration from

$$C_D = \frac{1}{L}\int_0^L C_{D,x}\,dx \tag{7-7}$$

and

$$h = \frac{1}{L}\int_0^L h_x\,dx \tag{7-8}$$

When the average drag and convection coefficients are available, the drag force can be determined from Eq. 7-1 and the rate of heat transfer to or from an isothermal surface can be determined from

$$\dot{Q} = hA_s(T_s - T_\infty) \tag{7-9}$$

where A_s is the surface area.

7–2 ▪ PARALLEL FLOW OVER FLAT PLATES

Consider the parallel flow of a fluid over a flat plate of length L in the flow direction, as shown in Figure 7–6. The x-coordinate is measured along the plate surface from the leading edge in the direction of the flow. The fluid approaches the plate in the x-direction with uniform upstream velocity $\mathcal{V}$ and temperature T_∞. The flow in the velocity boundary layer starts out as laminar, but if the plate is sufficiently long, the flow will become turbulent at a distance x_{cr} from the leading edge where the Reynolds number reaches its critical value for transition.

The transition from laminar to turbulent flow depends on the *surface geometry, surface roughness, upstream velocity, surface temperature,* and the *type of fluid,* among other things, and is best characterized by the Reynolds number. The Reynolds number at a distance x from the leading edge of a flat plate is expressed as

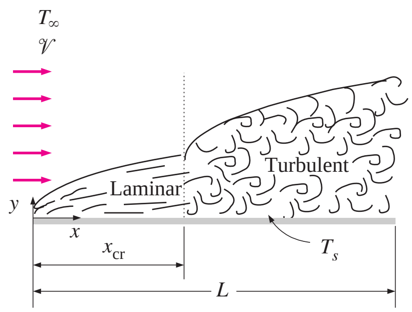


FIGURE 7–6
Laminar and turbulent regions of the boundary layer during flow over a flat plate.

$$\text{Re}_x = \frac{\rho \mathcal{V} x}{\mu} = \frac{\mathcal{V} x}{\nu} \quad \textbf{(7-10)}$$

Note that the value of the Reynolds number varies for a flat plate along the flow, reaching $\text{Re}_L = \mathcal{V}L/\nu$ at the end of the plate.

For flow over a flat plate, transition from laminar to turbulent is usually taken to occur at the *critical Reynolds number* of

$$\text{Re}_{cr} = \frac{\rho \mathcal{V} x_{cr}}{\mu} = 5 \times 10^5 \quad \textbf{(7-11)}$$

The value of the critical Reynolds number for a flat plate may vary from 10^5 to 3×10^6, depending on the surface roughness and the turbulence level of the free stream.

Friction Coefficient

Based on analysis, the boundary layer thickness and the local friction coefficient at location x for laminar flow over a flat plate were determined in Chapter 6 to be

$$\textit{Laminar:} \quad \delta_{v,x} = \frac{5x}{\text{Re}_x^{1/2}} \quad \text{and} \quad C_{f,x} = \frac{0.664}{\text{Re}_x^{1/2}}, \quad \text{Re}_x < 5 \times 10^5 \quad \textbf{(7-12a, b)}$$

The corresponding relations for turbulent flow are

$$\textit{Turbulent:} \quad \delta_{v,x} = \frac{0.382x}{\text{Re}_x^{1/5}} \quad \text{and} \quad C_{f,x} = \frac{0.0592}{\text{Re}_x^{1/5}}, \quad 5 \times 10^5 \le \text{Re}_x \le 10^7 \quad \textbf{(7-13a, b)}$$

where x is the distance from the leading edge of the plate and $\text{Re}_x = \mathcal{V}x/\nu$ is the Reynolds number at location x. Note that $C_{f,x}$ is proportional to $\text{Re}_x^{-1/2}$ and thus to $x^{-1/2}$ for laminar flow. Therefore, $C_{f,x}$ is supposedly *infinite* at the leading edge (x = 0) and decreases by a factor of $x^{-1/2}$ in the flow direction. The local friction coefficients are higher in turbulent flow than they are in laminar flow because of the intense mixing that occurs in the turbulent boundary layer. Note that $C_{f,x}$ reaches its highest values when the flow becomes fully turbulent, and then decreases by a factor of $x^{-1/5}$ in the flow direction.

The *average* friction coefficient over the entire plate is detennined by substituting the relations above into Eq. 7-7 and performing the integrations (Fig.7–7). We get

$$\textit{Laminar:} \quad C_f = \frac{1.328}{\text{Re}_L^{1/2}} \quad \text{Re}_L < 5 \times 10^5 \quad \textbf{(7-14)}$$

$$\textit{Turbulent:} \quad C_f = \frac{0.074}{\text{Re}_L^{1/5}} \quad 5 \times 10^5 \le \text{Re}_L \le 10^7 \quad \textbf{(7-15)}$$

The first relation gives the average friction coefficient for the entire plate when the flow is *laminar* over the *entire* plate. The second relation gives the average friction coefficient for the entire plate only when the flow is *turbulent* over the *entire* plate, or when the laminar flow region of the plate is too small relative to the turbulent flow region (that is, $x_{cr} \ll L$ where the length of the plate x_{cr} over which the flow is laminar can be determined from $\text{Re}_{cr} = 5 \times 10^5 = \mathcal{V}x_{cr}/\nu$).

$$\begin{aligned} C_f &= \frac{1}{L}\int_0^L C_{f,x}\,dx \\ &= \frac{1}{L}\int_0^L \frac{0.664}{\text{Re}_x^{1/2}}\,dx \\ &= \frac{0.664}{L}\int_0^L \left(\frac{\mathcal{V}x}{\nu}\right)^{-1/2} dx \\ &= \frac{0.664}{L}\left(\frac{\mathcal{V}}{\nu}\right)^{-1/2} \left.\frac{x^{1/2}}{\frac{1}{2}}\right|_0^L \\ &= \frac{2 \times 0.664}{L}\left(\frac{\mathcal{V}L}{\nu}\right)^{-1/2} \\ &= \frac{1.328}{\text{Re}_L^{1/2}} \end{aligned}$$

FIGURE 7–7
The average friction coefficient over a surface is determined by integrating the local friction coefficient over the entire surface.

In some cases, a flat plate is sufficiently long for the flow to become turbulent, but not long enough to disregard the laminar flow region. In such cases, the *average* friction coefficient over the entire plate is determined by performing the integration in Eq. 7-7 over two parts: the laminar region $0 \leq x \leq x_{cr}$ and the turbulent region $x_{cr} < x \leq L$ as

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,\,x\ \text{laminar}}\, dx + \int_{x_{cr}}^{L} C_{f,\,x,\ \text{turbulent}}\, dx\right) \qquad \textbf{(7-16)}$$

Note that we included the transition region with the turbulent region. Again taking the critical Reynolds number to be $\text{Re}_{cr} = 5 \times 10^5$ and performing the integrations of Eq. 7-16 after substituting the indicated expressions, the *average* friction coefficient over the entire plate is determined to be

$$C_f = \frac{0.074}{\text{Re}_L^{1/5}} - \frac{1742}{\text{Re}_L} \qquad 5 \times 10^5 \leq \text{Re}_L \leq 10^7 \qquad \textbf{(7-17)}$$

The constants in this relation will be different for different critical Reynolds numbers. Also, the surfaces are assumed to be *smooth*, and the free stream to be *turbulent free*. For laminar flow, the friction coefficient depends on only the Reynolds number, and the surface roughness has no effect. For turbulent flow, however, surface roughness causes the friction coefficient to increase severalfold, to the point that in fully turbulent regime the friction coefficient is a function of surface roughness alone, and independent of the Reynolds number (Fig. 7–8).

A curve fit of experimental data for the average friction coefficient in this regime is given by Schlichting as

Rough surface, turbulent:
$$C_f = \left(1.89 - 1.62 \log \frac{\varepsilon}{L}\right)^{-2.5} \qquad \textbf{(7-18)}$$

were ε is the surface roughness, and L is the length of the plate in the flow direction. In the absence of a better relation, the relation above can be used for turbulent flow on rough surfaces for $\text{Re} > 10^6$, especially when $\varepsilon/L > 10^{-4}$.

Relative roughness, ε/L	Friction coefficient C_f
0.0*	0.0029
1×10^{-5}	0.0032
1×10^{-4}	0.0049
1×10^{-3}	0.0084

*Smooth surface for Re = 10^7. Others calculated from Eq. 7-18.

FIGURE 7–8
For turbulent flow, surface roughness may cause the friction coefficient to increase severalfold.

Heat Transfer Coefficient

The local Nusselt number at a location x for laminar flow over a flat plate was determined in Chapter 6 by solving the differential energy equation to be

Laminar:
$$\text{Nu}_x = \frac{h_x x}{k} = 0.332\ \text{Re}_x^{0.5}\ \text{Pr}^{1/3} \qquad \text{Pr} > 0.60 \qquad \textbf{(7-19)}$$

The corresponding relation for turbulent flow is

Turbulent:
$$\text{Nu}_x = \frac{h_x x}{k} = 0.0296\ \text{Re}_x^{0.8}\ \text{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \leq \text{Pr} \leq 60 \\ 5 \times 10^5 \leq \text{Re}_x \leq 10^7 \end{array} \qquad \textbf{(7-20)}$$

Note that h_x is proportional to $\text{Re}_x^{0.5}$ and thus to $x^{-0.5}$ for laminar flow. Therefore, h_x is *infinite* at the leading edge ($x = 0$) and decreases by a factor of $x^{-0.5}$ in the flow direction. The variation of the boundary layer thickness δ and the friction and heat transfer coefficients along an isothermal flat plate are shown in Figure 7–9. The local friction and heat transfer coefficients are higher in

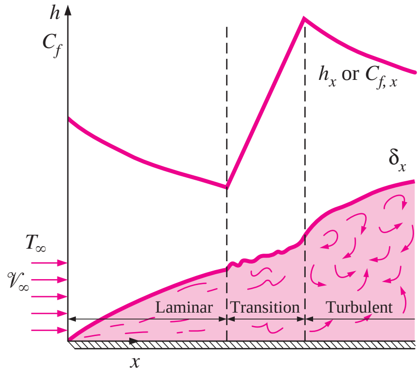


FIGURE 7–9
The variation of the local friction and heat transfer coefficients for flow over a flat plate.

turbulent flow than they are in laminar flow. Also, h_x reaches its highest values when the flow becomes fully turbulent, and then decreases by a factor of $x^{-0.2}$ in the flow direction, as shown in the figure.

The *average* Nusselt number over the entire plate is determined by substituting the relations above into Eq. 7-8 and performing the integrations. We get

Laminar: $$\text{Nu} = \frac{hL}{k} = 0.664\ \text{Re}_L^{0.5}\ \text{Pr}^{1/3} \qquad \text{Re}_L < 5 \times 10^5 \tag{7-21}$$

Turbulent: $$\text{Nu} = \frac{hL}{k} = 0.037\ \text{Re}_L^{0.8}\ \text{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \leq \text{Pr} \leq 60 \\ 5 \times 10^5 \leq \text{Re}_L \leq 10^7 \end{array} \tag{7-22}$$

The first relation gives the average heat transfer coefficient for the entire plate when the flow is *laminar* over the *entire* plate. The second relation gives the average heat transfer coefficient for the entire plate only when the flow is *turbulent* over the *entire* plate, or when the laminar flow region of the plate is too small relative to the turbulent flow region.

In some cases, a flat plate is sufficiently long for the flow to become turbulent, but not long enough to disregard the laminar flow region. In such cases, the *average* heat transfer coefficient over the entire plate is determined by performing the integration in Eq. 7-8 over two parts as

$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,\text{ laminar}}\, dx + \int_{x_{cr}}^{L} h_{x,\text{ trubulent}}\, dx\right) \tag{7-23}$$

Again taking the critical Reynolds number to be $\text{Re}_{cr} = 5 \times 10^5$ and performing the integrations in Eq. 7-23 after substituting the indicated expressions, the *average* Nusselt number over the *entire* plate is determined to be (Fig. 7–10)

$$\text{Nu} = \frac{hL}{k} = (0.037\ \text{Re}_L^{0.8} - 871)\text{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \leq \text{Pr} \leq 60 \\ 5 \times 10^5 \leq \text{Re}_L \leq 10^7 \end{array} \tag{7-24}$$

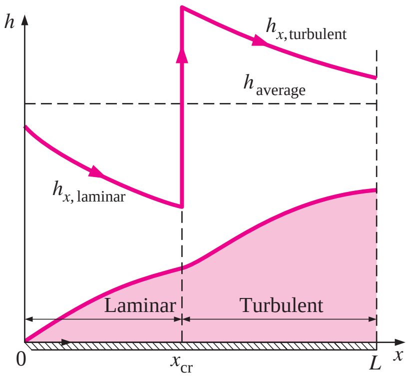


FIGURE 7–10
Graphical representation of the average heat transfer coefficient for a flat plate with combined laminar and turbulent flow.

The constants in this relation will be different for different critical Reynolds numbers.

Liquid metals such as mercury have high thermal conductivities, and are commonly used in applications that require high heat transfer rates. However, they have very small Prandtl numbers, and thus the thermal boundary layer develops much faster than the velocity boundary layer. Then we can assume the velocity in the thermal boundary layer to be constant at the free stream value and solve the energy equation. It gives

$$\text{Nu}_x = 0.565(\text{Re}_x\ \text{Pr})^{1/2} \qquad \text{Pr} < 0.05 \tag{7-25}$$

It is desirable to have a single correlation that applies to *all fluids*, including liquid metals. By curve-fitting existing data, Churchill and Ozoe (Ref. 3) proposed the following relation which is applicable for *all Prandtl numbers* and is claimed to be accurate to ±1%,

$$\text{Nu}_x = \frac{h_x x}{k} = \frac{0.3387\ \text{Pr}^{1/3}\ \text{Re}_x^{1/2}}{[1 + (0.0468/\text{Pr})^{2/3}]^{1/4}} \tag{7-26}$$

These relations have been obtained for the case of *isothermal* surfaces but could also be used approximately for the case of nonisothermal surfaces by assuming the surface temperature to be constant at some average value.

Also, the surfaces are assumed to be *smooth*, and the free stream to be *turbulent free*. The effect of variable properties can be accounted for by evaluating all properties at the film temperature.

Flat Plate with Unheated Starting Length

So far we have limited our consideration to situations for which the entire plate is heated from the leading edge. But many practical applications involve surfaces with an unheated starting section of length ξ, shown in Figure 7–11, and thus there is no heat transfer for $0 < x < \xi$. In such cases, the velocity boundary layer starts to develop at the leading edge ($x = 0$), but the thermal boundary layer starts to develop where heating starts ($x = \xi$).

Consider a flat plate whose heated section is maintained at a constant temperature ($T = T_s$ constant for $x > \xi$). Using integral solution methods (see Kays and Crawford, 1994), the local Nusselt numbers for both laminar and turbulent flows are determined to be

Laminar:
$$\text{Nu}_x = \frac{\text{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \frac{0.332\ \text{Re}_x^{0.5}\ \text{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}} \quad \textbf{(7-27)}$$

Turbulent:
$$\text{Nu}_x = \frac{\text{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \frac{0.0296\ \text{Re}_x^{0.8}\ \text{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}} \quad \textbf{(7-28)}$$

for $x > \xi$. Note that for $\xi = 0$, these Nu_x relations reduce to $\text{Nu}_{x\,(\text{for } \xi = 0)}$, which is the Nusselt number relation for a flat plate without an unheated starting length. Therefore, the terms in brackets in the denominator serve as correction factors for plates with unheated starting lengths.

The determination of the average Nusselt number for the heated section of a plate requires the integration of the local Nusselt number relations above, which cannot be done analytically. Therefore, integrations must be done numerically. The results of numerical integrations have been correlated for the average convection coefficients [Thomas, (1977) Ref. 11] as

Laminar:
$$h = \frac{2[1-(\xi/x)^{3/4}]}{1-\xi/L}\, h_{x=L} \quad \textbf{(7-29)}$$

Turbulent:
$$h = \frac{5[1-(\xi/x)^{9/10}]}{4(1-\xi/L)}\, h_{x=L} \quad \textbf{(7-30)}$$

The first relation gives the average convection coefficient for the entire heated section of the plate when the flow is laminar over the entire plate. Note that for $\xi = 0$ it reduces to $h_L = 2h_{x=L}$, as expected. The second relation gives the average convection coefficient for the case of turbulent flow over the entire plate or when the laminar flow region is small relative to the turbulent region.

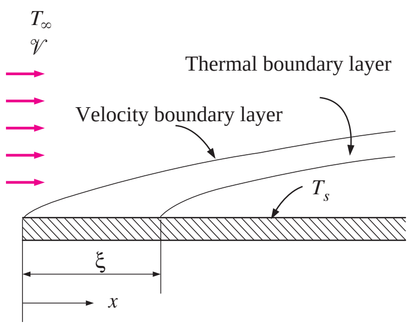


FIGURE 7–11
Flow over a flat plate with an unheated starting length.

Uniform Heat Flux

When a flat plate is subjected to *uniform heat flux* instead of uniform temperature, the local Nusselt number is given by

Laminar:
$$\text{Nu}_x = 0.453\ \text{Re}_x^{0.5}\ \text{Pr}^{1/3} \quad \textbf{(7-31)}$$

Turbulent:
$$\text{Nu}_x = 0.0308\ \text{Re}_x^{0.8}\ \text{Pr}^{1/3} \quad \textbf{(7-32)}$$

These relations give values that are 36 percent higher for laminar flow and 4 percent higher for turbulent flow relative to the isothermal plate case. When the plate involves an unheated starting length, the relations developed for the uniform surface temperature case can still be used provided that Eqs. 7-31 and 7-32 are used for $\text{Nu}_{x(\text{for } \xi = 0)}$ in Eqs. 7-27 and 7-28, respectively.

When heat flux $\dot{q}_s$ is prescribed, the rate of heat transfer to or from the plate and the surface temperature at a distance x are determined from

$$\dot{Q} = \dot{q}_s A_s \tag{7-33}$$

and

$$\dot{q}_s = h_x[T_s(x) - T_\infty] \quad \rightarrow \quad T_s(x) = T_\infty + \frac{\dot{q}_s}{h_x} \tag{7-34}$$

where A_s is the heat transfer surface area.

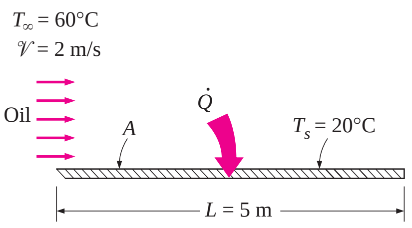


FIGURE 7–12
Schematic for Example 7-1.

EXAMPLE 7–1 Flow of Hot Oil over a Flat Plate

Engine oil at 60°C flows over the upper surface of a 5-m-long flat plate whose temperature is 20°C with a velocity of 2 m/s (Fig. 7–12). Determine the total drag force and the rate of heat transfer per unit width of the entire plate.

SOLUTION Engine oil flows over a flat plate. The total drag force and the rate of heat transfer per unit width of the plate are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\text{Re}_{cr} = 5 \times 10^5$.

Properties The properties of engine oil at the film temperature of $T_f = (T_s + T_\infty)/2 = (20 + 60)/2 = 40$°C are (Table A–14).

$$\rho = 876 \text{ kg/m}^3 \qquad \text{Pr} = 2870$$
$$k = 0.144 \text{ W/m} \cdot \text{°C} \qquad \nu = 242 \times 10^{-6} \text{ m}^2\text{/s}$$

Analysis Noting that $L = 5$ m, the Reynolds number at the end of the plate is

$$\text{Re}_L = \frac{\mathscr{V}L}{\nu} = \frac{(2 \text{ m/s})(5 \text{ m})}{0.242 \times 10^{-5} \text{ m}^2\text{/s}} = 4.13 \times 10^4$$

which is less than the critical Reynolds number. Thus we have *laminar flow* over the entire plate, and the average friction coefficient is

$$C_f = 1.328 \text{ Re}_L^{-0.5} = 1.328 \times (4.13 \times 10^3)^{-0.5} = 0.0207$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the plate per unit width becomes

$$F_D = C_f A_s \frac{\rho \mathscr{V}^2}{2} = 0.0207 \times (5 \times 1 \text{ m}^2) \frac{(876 \text{ kg/m}^3)(2 \text{ m/s})^2}{2} \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)$$
$$= \mathbf{181 \text{ N}}$$

The total drag force acting on the entire plate can be determined by multiplying the value obtained above by the width of the plate.

This force per unit width corresponds to the weight of a mass of about 18 kg. Therefore, a person who applies an equal and opposite force to the plate to keep it from moving will feel like he or she is using as much force as is necessary to hold a 18-kg mass from dropping.

Similarly, the Nusselt number is determined using the laminar flow relations for a flat plate,

$$\text{Nu} = \frac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} = 0.664 \times (4.13 \times 10^4)^{0.5} \times 2870^{1/3} = 1918$$

Then,

$$h = \frac{k}{L}\text{Nu} = \frac{0.144 \text{ W/m}\cdot{}^\circ\text{C}}{5 \text{ m}}(1918) = 55.2 \text{ W/m}^2\cdot{}^\circ\text{C}$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (55.2 \text{ W/m}^2\cdot{}^\circ\text{C})(5 \times 1 \text{ m}^2)(60 - 20)^\circ\text{C} = \mathbf{11{,}040 \text{ W}}$$

Discussion Note that heat transfer is always from the higher-temperature medium to the lower-temperature one. In this case, it is from the oil to the plate. The heat transfer rate is per m width of the plate. The heat transfer for the entire plate can be obtained by multiplying the value obtained by the actual width of the plate.

EXAMPLE 7–2 Cooling of a Hot Block by Forced Air at High Elevation

The local atmospheric pressure in Denver, Colorado (elevation 1610 m), is 83.4 kPa. Air at this pressure and 20°C flows with a velocity of 8 m/s over a 1.5 m × 6 m flat plate whose temperature is 140°C (Fig. 7-13). Determine the rate of heat transfer from the plate if the air flows parallel to the (*a*) 6-m-long side and (*b*) the 1.5-m side.

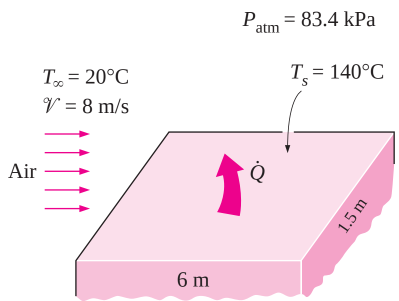


FIGURE 7–13
Schematic for Example 7-2.

SOLUTION The top surface of a hot block is to be cooled by forced air. The rate of heat transfer is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $\text{Re}_{cr} = 5 \times 10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas.

Properties The properties k, μ, C_p, and Pr of ideal gases are independent of pressure, while the properties ν and α are inversely proportional to density and thus pressure. The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (140 + 20)/2 = 80°\text{C}$ and 1 atm pressure are (Table A–15)

$$k = 0.02953 \text{ W/m}\cdot{}^\circ\text{C} \qquad \text{Pr} = 0.7154$$
$$\nu_{@\,1\text{ atm}} = 2.097 \times 10^{-5} \text{ m}^2\text{/s}$$

The atmospheric pressure in Denver is $P = (83.4 \text{ kPa})/(101.325 \text{ kPa/atm}) = 0.823$ atm. Then the kinematic viscosity of air in Denver becomes

$$\nu = \nu_{@\,1\text{ atm}}/P = (2.097 \times 10^{-5} \text{ m}^2\text{/s})/0.823 = 2.548 \times 10^{-5} \text{ m}^2\text{/s}$$

Analysis (*a*) When air flow is parallel to the long side, we have $L = 6$ m, and the Reynolds number at the end of the plate becomes

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(8 \text{ m/s})(6 \text{ m})}{2.548 \times 10^{-5} \text{ m}^2\text{/s}} = 1.884 \times 10^6$$

which is greater than the critical Reynolds number. Thus, we have combined laminar and turbulent flow, and the average Nusselt number for the entire plate is determined to be

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\mathrm{Pr}^{1/3}$$
$$= [0.037(1.884 \times 10^6)^{0.8} - 871]0.7154^{1/3}$$
$$= 2687$$

Then

$$h = \frac{k}{L}\mathrm{Nu} = \frac{0.02953\ \mathrm{W/m \cdot {}^\circ C}}{6\ \mathrm{m}}(2687) = 13.2\ \mathrm{W/m^2 \cdot {}^\circ C}$$
$$A_s = wL = (1.5\ \mathrm{m})(6\ \mathrm{m}) = 9\ \mathrm{m^2}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (13.2\ \mathrm{W/m^2 \cdot {}^\circ C})(9\ \mathrm{m^2})(140 - 20)^\circ\mathrm{C} = \mathbf{1.43 \times 10^4\ W}$$

Note that if we disregarded the laminar region and assumed turbulent flow over the entire plate, we would get Nu = 3466 from Eq. 7–22, which is 29 percent higher than the value calculated above.

(*b*) When air flow is along the short side, we have $L = 1.5$ m, and the Reynolds number at the end of the plate becomes

$$\mathrm{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(8\ \mathrm{m/s})(1.5\ \mathrm{m})}{2.548 \times 10^{-5}\ \mathrm{m^2/s}} = 4.71 \times 10^5$$

which is less than the critical Reynolds number. Thus we have laminar flow over the entire plate, and the average Nusselt number is

$$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} = 0.664 \times (4.71 \times 10^5)^{0.5} \times 0.7154^{1/3} = 408$$

Then

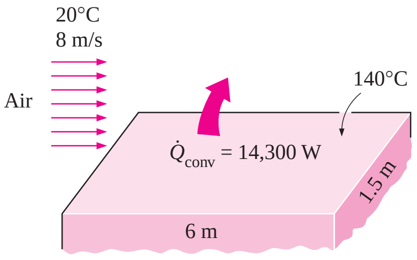
$$h = \frac{k}{L}\mathrm{Nu} = \frac{0.02953\ \mathrm{W/m \cdot {}^\circ C}}{1.5\ \mathrm{m}}(408) = 8.03\ \mathrm{W/m^2 \cdot {}^\circ C}$$

and

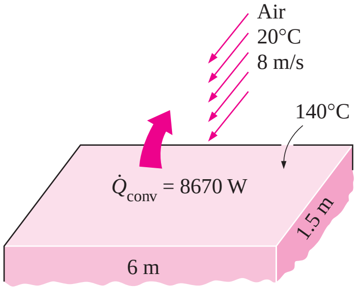
$$\dot{Q} = hA_s(T_s - T_\infty) = (8.03\ \mathrm{W/m^2 \cdot {}^\circ C})(9\ \mathrm{m^2})(140 - 20)^\circ\mathrm{C} = \mathbf{8670\ W}$$

which is considerably less than the heat transfer rate determined in case (*a*).

Discussion Note that the *direction* of fluid flow can have a significant effect on convection heat transfer to or from a surface (Fig. 7-14). In this case, we can increase the heat transfer rate by 65 percent by simply blowing the air along the long side of the rectangular plate instead of the short side.



(*a*) Flow along the long side



(*b*) Flow along the short side

FIGURE 7–14
The direction of fluid flow can have a significant effect on convection heat transfer.

EXAMPLE 7–3 Cooling of Plastic Sheets by Forced Air

The forming section of a plastics plant puts out a continuous sheet of plastic that is 4 ft wide and 0.04 in. thick at a velocity of 30 ft/min. The temperature of the plastic sheet is 200°F when it is exposed to the surrounding air, and a 2-ft-long section of the plastic sheet is subjected to air flow at 80°F at a velocity of 10 ft/s on both sides along its surfaces normal to the direction of motion of the sheet, as shown in Figure 7–15. Determine (*a*) the rate of heat transfer from the plastic sheet to air by forced convection and radiation and (*b*) the temperature of the plastic sheet at the end of the cooling section. Take the density, specific heat, and emissivity of the plastic sheet to be $\rho = 75$ lbm/ft^3, $C_p = 0.4$ Btu/lbm · °F, and $\varepsilon = 0.9$.

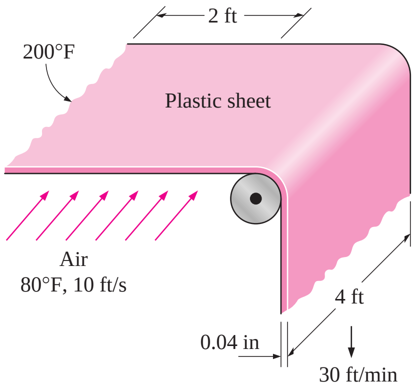


FIGURE 7–15
Schematic for Example 7–3.

SOLUTION Plastic sheets are cooled as they leave the forming section of a plastics plant. The rate of heat loss from the plastic sheet by convection and radiation and the exit temperature of the plastic sheet are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5 \times 10^5$. **3** Air is an ideal gas. **4** The local atmospheric pressure is 1 atm. **5** The surrounding surfaces are at the temperature of the room air.

Properties The properties of the plastic sheet are given in the problem statement. The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (200 + 80)/2 = 140$°F and 1 atm pressure are (Table A–15E)

$$k = 0.01623 \text{ Btu/h} \cdot \text{ft} \cdot \text{°F} \qquad \text{Pr} = 0.7202$$
$$\nu = 0.7344 \text{ ft}^2/\text{h} = 0.204 \times 10^{-3} \text{ ft}^2/\text{s}$$

Analysis (*a*) We expect the temperature of the plastic sheet to drop somewhat as it flows through the 2-ft-long cooling section, but at this point we do not know the magnitude of that drop. Therefore, we assume the plastic sheet to be isothermal at 200°F to get started. We will repeat the calculations if necessary to account for the temperature drop of the plastic sheet.

Noting that $L = 4$ ft, the Reynolds number at the end of the air flow across the plastic sheet is

$$\text{Re}_L = \frac{\mathcal{V}L}{\nu} = \frac{(10 \text{ ft/s})(4 \text{ ft})}{0.204 \times 10^{-3} \text{ ft}^2/\text{s}} = 1.961 \times 10^5$$

which is less than the critical Reynolds number. Thus, we have *laminar flow* over the entire sheet, and the Nusselt number is determined from the laminar flow relations for a flat plate to be

$$\text{Nu} = \frac{hL}{k} = 0.664 \text{Re}_L^{0.5} \text{Pr}^{1/3} = 0.664 \times (1.961 \times 10^5)^{0.5} \times (0.7202)^{1/3} = 263.6$$

Then,

$$h = \frac{k}{L}\text{Nu} = \frac{0.01623 \text{ Btu/h} \cdot \text{ft} \cdot \text{°F}}{4 \text{ ft}}(263.6) = 1.07 \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{°F}$$
$$A_s = (2 \text{ ft})(4 \text{ ft})(2 \text{ sides}) = 16 \text{ ft}^2$$

and

$$\begin{aligned}\dot{Q}_{\text{conv}} &= hA_s(T_s - T_\infty) \\ &= (1.07 \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{°F})(16 \text{ ft}^2)(200 - 80)\text{°F} \\ &= 2054 \text{ Btu/h}\end{aligned}$$

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4) \\ &= (0.9)(0.1714 \times 10^{-8} \text{ Btu/h} \cdot \text{ft}^2 \cdot \text{R}^4)(16 \text{ ft}^2)[(660 \text{ R})^4 - (540 \text{ R})^4] \\ &= 2584 \text{ Btu/h}\end{aligned}$$

Therefore, the rate of cooling of the plastic sheet by combined convection and radiation is

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 2054 + 2584 = \mathbf{4638\ Btu/h}$$

(*b*) To find the temperature of the plastic sheet at the end of the cooling section, we need to know the mass of the plastic rolling out per unit time (or the mass flow rate), which is determined from

$$\dot{m} = \rho A_c \mathcal{V}_{plastic} = (75\ \text{lbm/ft}^3)\left(\frac{4 \times 0.04}{12}\ \text{ft}^3\right)\left(\frac{30}{60}\ \text{ft/s}\right) = 0.5\ \text{lbm/s}$$

Then, an energy balance on the cooled section of the plastic sheet yields

$$\dot{Q} = \dot{m} C_p (T_2 - T_1) \quad \rightarrow \quad T_2 = T_1 + \frac{\dot{Q}}{\dot{m} C_p}$$

Noting that $\dot{Q}$ is a negative quantity (heat loss) for the plastic sheet and substituting, the temperature of the plastic sheet as it leaves the cooling section is determined to be

$$T_2 = 200°\text{F} + \frac{-4638\ \text{Btu/h}}{(0.5\ \text{lbm/s})(0.4\ \text{Btu/lbm} \cdot °\text{F})}\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = \mathbf{193.6°F}$$

Discussion The average temperature of the plastic sheet drops by about 6.4°F as it passes through the cooling section. The calculations now can be repeated by taking the average temperature of the plastic sheet to be 196.8°F instead of 200°F for better accuracy, but the change in the results will be insignificant because of the small change in temperature.

7–3 ▪ FLOW ACROSS CYLINDERS AND SPHERES

Flow across cylinders and spheres is frequently encountered in practice. For example, the tubes in a shell-and-tube heat exchanger involve both *internal flow* through the tubes and *external flow* over the tubes, and both flows must be considered in the analysis of the heat exchanger. Also, many sports such as soccer, tennis, and golf involve flow over spherical balls.

The characteristic length for a circular cylinder or sphere is taken to be the *external diameter D*. Thus, the Reynolds number is defined as $\text{Re} = \mathcal{V}D/\nu$ where $\mathcal{V}$ is the uniform velocity of the fluid as it approaches the cylinder or sphere. The critical Reynolds number for flow across a circular cylinder or sphere is about $\text{Re}_{cr} \approx 2 \times 10^5$. That is, the boundary layer remains laminar for about $\text{Re} \lesssim 2 \times 10^5$ and becomes turbulent for $\text{Re} \gtrsim 2 \times 10^5$.

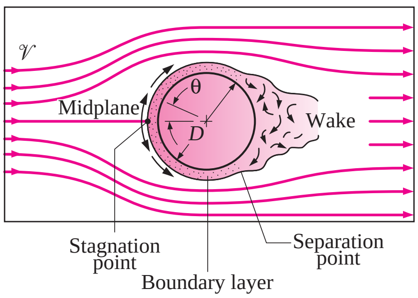


FIGURE 7–16
Typical flow patterns in cross flow over a cylinder.

Cross flow over a cylinder exhibits complex flow patterns, as shown in Figure 7–16. The fluid approaching the cylinder branches out and encircles the cylinder, forming a boundary layer that wraps around the cylinder. The fluid particles on the midplane strike the cylinder at the stagnation point, bringing the fluid to a complete stop and thus raising the pressure at that point. The pressure decreases in the flow direction while the fluid velocity increases.

At very low upstream velocities ($\text{Re} \lesssim 1$), the fluid completely wraps around the cylinder and the two arms of the fluid meet on the rear side of the cylinder

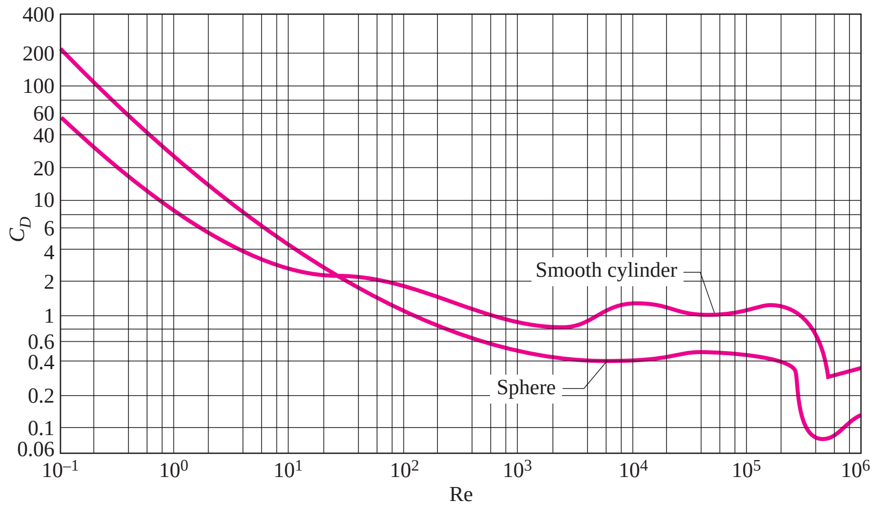


FIGURE 7–17
Average drag coefficient for cross flow over a smooth circular cylinder and a smooth sphere (from Schlichting, Ref. 10).

in an orderly manner. Thus, the fluid follows the curvature of the cylinder. At higher velocities, the fluid still hugs the cylinder on the frontal side, but it is too fast to remain attached to the surface as it approaches the top of the cylinder. As a result, the boundary layer detaches from the surface, forming a separation region behind the cylinder. Flow in the wake region is characterized by random vortex formation and pressures much lower than the stagnation point pressure.

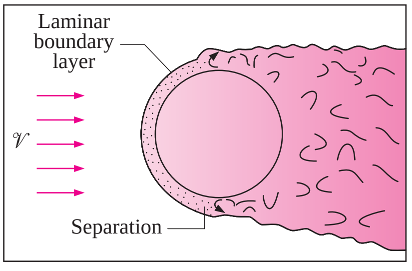
The nature of the flow across a cylinder or sphere strongly affects the total drag coefficient C_D. Both the *friction drag* and the *pressure drag* can be significant. The high pressure in the vicinity of the stagnation point and the low pressure on the opposite side in the wake produce a net force on the body in the direction of flow. The drag force is primarily due to friction drag at low Reynolds numbers (Re < 10) and to pressure drag at high Reynolds numbers (Re > 5000). Both effects are significant at intermediate Reynolds numbers.

The average drag coefficients C_D for cross flow over a smooth single circular cylinder and a sphere are given in Figure 7–17. The curves exhibit different behaviors in different ranges of Reynolds numbers:

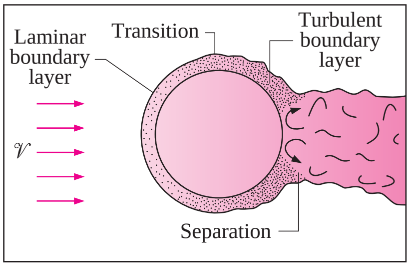
- For Re $\lesssim 1$, we have creeping flow, and the drag coefficient decreases with increasing Reynolds number. For a sphere, it is $C_D = 24/\text{Re}$. There is no flow separation in this regime.
- At about Re $= 10$, separation starts occurring on the rear of the body with vortex shedding starting at about Re ≈ 90. The region of separation increases with increasing Reynolds number up to about Re $= 10^3$. At this point, the drag is mostly (about 95 percent) due to pressure drag. The drag coefficient continues to decrease with increasing Reynolds number in this range of $10 < \text{Re} < 10^3$. (A decrease in the drag coefficient does not necessarily indicate a decrease in drag. The drag force is proportional to the square of the velocity, and the increase in velocity at higher Reynolds numbers usually more than offsets the decrease in the drag coefficient.)

- In the moderate range of $10^3 < \text{Re} < 10^5$, the drag coefficient remains relatively constant. This behavior is characteristic of blunt bodies. The flow in the boundary layer is laminar in this range, but the flow in the separated region past the cylinder or sphere is highly turbulent with a wide turbulent wake.
- There is a sudden drop in the drag coefficient somewhere in the range of $10^5 < \text{Re} < 10^6$ (usually, at about 2×10^5). This large reduction in C_D is due to the flow in the boundary layer becoming *turbulent,* which moves the separation point further on the rear of the body, reducing the size of the wake and thus the magnitude of the pressure drag. This is in contrast to streamlined bodies, which experience an increase in the drag coefficient (mostly due to friction drag) when the boundary layer becomes turbulent.

Flow separation occurs at about $\theta \approx 80°$ (measured from the stagnation point) when the boundary layer is *laminar* and at about $\theta \approx 140°$ when it is *turbulent* (Fig. 7–18). The delay of separation in turbulent flow is caused by the rapid fluctuations of the fluid in the transverse direction, which enables the turbulent boundary layer to travel further along the surface before separation occurs, resulting in a narrower wake and a smaller pressure drag. In the range of Reynolds numbers where the flow changes from laminar to turbulent, even the drag force F_D decreases as the velocity (and thus Reynolds number) increases. This results in a sudden decrease in drag of a flying body and instabilities in flight.



(a) Laminar flow ($\text{Re} < 2 \times 10^5$)



(b) Turbulence occurs ($\text{Re} > 2 \times 10^5$)

FIGURE 7–18
Turbulence delays flow separation.

Effect of Surface Roughness

We mentioned earlier that *surface roughness,* in general, increases the drag coefficient in turbulent flow. This is especially the case for streamlined bodies. For blunt bodies such as a circular cylinder or sphere, however, an increase in the surface roughness may actually *decrease* the drag coefficient, as shown in Figure 7–19 for a sphere. This is done by tripping the flow into turbulence at a lower Reynolds number, and thus causing the fluid to close in behind the body, narrowing the wake and reducing pressure drag considerably. This results in a much smaller drag coefficient and thus drag force for a rough-surfaced cylinder or sphere in a certain range of Reynolds number compared to a smooth one of identical size at the same velocity. At $\text{Re} = 10^5$, for example, $C_D = 0.1$ for a rough sphere with $\varepsilon/D = 0.0015$, whereas $C_D = 0.5$ for a smooth one. Therefore, the drag coefficient in this case is reduced by a factor of 5 by simply roughening the surface. Note, however, that at $\text{Re} = 10^6$, $C_D = 0.4$ for the rough sphere while $C_D = 0.1$ for the smooth one. Obviously, roughening the sphere in this case will increase the drag by a factor of 4 (Fig. 7–20).

The discussion above shows that roughening the surface can be used to great advantage in reducing drag, but it can also backfire on us if we are not careful—specifically, if we do not operate in the right range of Reynolds number. With this consideration, golf balls are intentionally roughened to induce *turbulence* at a lower Reynolds number to take advantage of the sharp *drop* in the drag coefficient at the onset of turbulence in the boundary layer (the typical velocity range of golf balls is 15 to 150 m/s, and the Reynolds number is less than 4×10^5). The critical Reynolds number of dimpled golf balls is

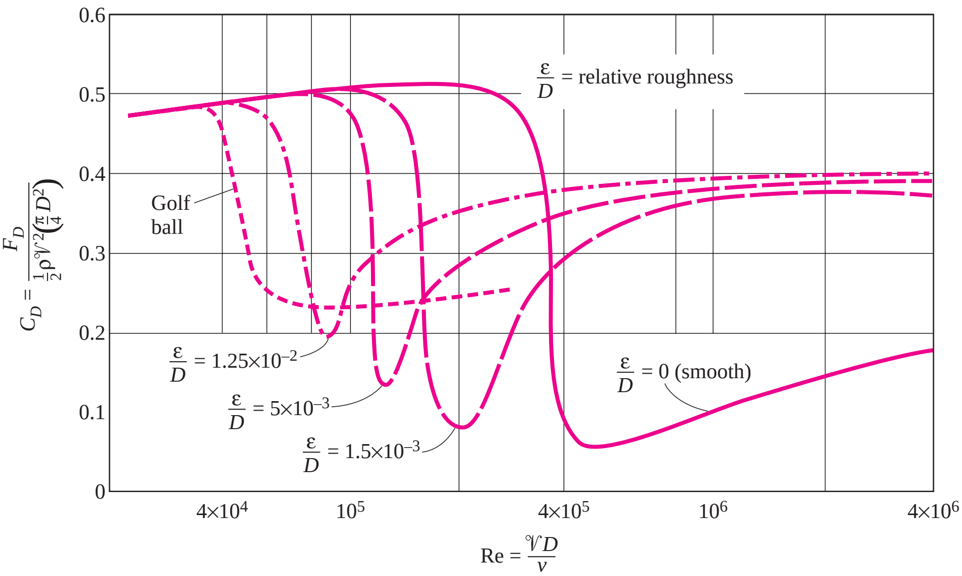


FIGURE 7–19
The effect of surface roughness on the drag coefficient of a sphere (from Blevins, Ref. 1).

about 4 × 10^4. The occurrence of turbulent flow at this Reynolds number reduces the drag coefficient of a golf ball by half, as shown in Figure 7–19. For a given hit, this means a longer distance for the ball. Experienced golfers also give the ball a spin during the hit, which helps the rough ball develop a lift and thus travel higher and further. A similar argument can be given for a tennis ball. For a table tennis ball, however, the distances are very short, and the balls never reach the speeds in the turbulent range. Therefore, the surfaces of table tennis balls are made smooth.

Once the drag coefficient is available, the drag force acting on a body in cross flow can be determined from Eq. 7-1 where A is the *frontal area* ($A = LD$ for a cylinder of length L and $A = \pi D^2/4$ for a sphere). It should be kept in mind that the free-stream turbulence and disturbances by other bodies in flow (such as flow over tube bundles) may affect the drag coefficients significantly.

	C_D	
Re	Smooth surface	Rough surface, $\varepsilon/D = 0.0015$
10^5	0.5	0.1
10^6	0.1	0.4

FIGURE 7–20
Surface roughness may increase or decrease the drag coefficient of a spherical object, depending on the value of the Reynolds number.

EXAMPLE 7–4 **Drag Force Acting on a Pipe in a River**

A 2.2-cm-outer-diameter pipe is to cross a river at a 30-m-wide section while being completely immersed in water (Fig. 7–21). The average flow velocity of water is 4 m/s and the water temperature is 15°C. Determine the drag force exerted on the pipe by the river.

SOLUTION A pipe is crossing a river. The drag force that acts on the pipe is to be determined.

Assumptions **1** The outer surface of the pipe is smooth so that Figure 7–17 can be used to determine the drag coefficient. **2** Water flow in the river is steady. **3** The direction of water flow is normal to the pipe. **4** Turbulence in river flow is not considered.

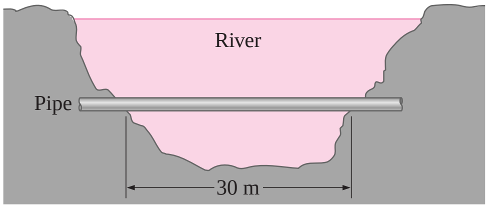


FIGURE 7–21
Schematic for Example 7–4.

Properties The density and dynamic viscosity of water at 15°C are ρ = 999.1 kg/m^3 and μ = 1.138 × 10^{-3} kg/m · s (Table A-9).

Analysis Noting that D = 0.022 m, the Reynolds number for flow over the pipe is

$$\text{Re} = \frac{\mathcal{V}D}{\nu} = \frac{\rho \mathcal{V} D}{\mu} = \frac{(999.1\ \text{kg/m}^3)(4\ \text{m/s})(0.022\ \text{m})}{1.138 \times 10^{-3}\ \text{kg/m} \cdot \text{s}} = 7.73 \times 10^4$$

The drag coefficient corresponding to this value is, from Figure 7-17, C_D = 1.0. Also, the frontal area for flow past a cylinder is $A = LD$. Then the drag force acting on the pipe becomes

$$F_D = C_D A \frac{\rho \mathcal{V}^2}{2} = 1.0(30 \times 0.022\ \text{m}^2)\frac{(999.1\ \text{kg/m}^3)(4\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg} \cdot \text{m/s}^2}\right)$$
$$= \mathbf{5275\ N}$$

Discussion Note that this force is equivalent to the weight of a mass over 500 kg. Therefore, the drag force the river exerts on the pipe is equivalent to hanging a total of over 500 kg in mass on the pipe supported at its ends 30 m apart. The necessary precautions should be taken if the pipe cannot support this force.

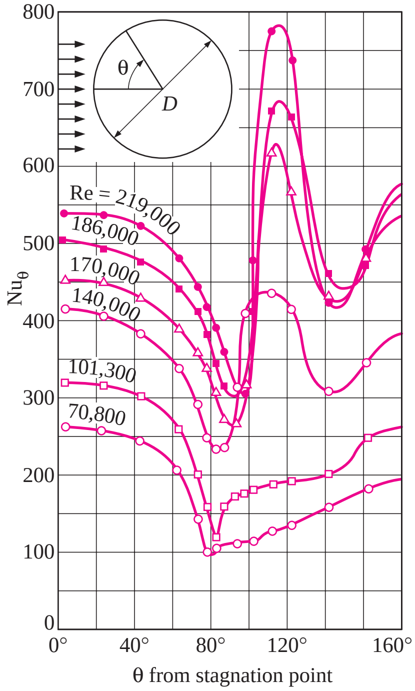


FIGURE 7–22
Variation of the local heat transfer coefficient along the circumference of a circular cylinder in cross flow of air (from Giedt, Ref. 5).

Heat Transfer Coefficient

Flows across cylinders and spheres, in general, involve *flow separation*, which is difficult to handle analytically. Therefore, such flows must be studied experimentally or numerically. Indeed, flow across cylinders and spheres has been studied experimentally by numerous investigators, and several empirical correlations have been developed for the heat transfer coefficient.

The complicated flow pattern across a cylinder greatly influences heat transfer. The variation of the local Nusselt number Nu_θ around the periphery of a cylinder subjected to cross flow of air is given in Figure 7–22. Note that, for all cases, the value of Nu_θ starts out relatively high at the stagnation point ($\theta = 0°$) but decreases with increasing θ as a result of the thickening of the laminar boundary layer. On the two curves at the bottom corresponding to Re = 70,800 and 101,300, Nu_θ reaches a minimum at $\theta \approx 80°$, which is the separation point in laminar flow. Then Nu_θ increases with increasing θ as a result of the intense mixing in the separated flow region (the wake). The curves at the top corresponding to Re = 140,000 to 219,000 differ from the first two curves in that they have *two* minima for Nu_θ. The sharp increase in Nu_θ at about $\theta \approx 90°$ is due to the transition from laminar to turbulent flow. The later decrease in Nu_θ is again due to the thickening of the boundary layer. Nu_θ reaches its second minimum at about $\theta \approx 140°$, which is the flow separation point in turbulent flow, and increases with θ as a result of the intense mixing in the turbulent wake region.

The discussions above on the local heat transfer coefficients are insightful; however, they are of little value in heat transfer calculations since the calculation of heat transfer requires the *average* heat transfer coefficient over the entire surface. Of the several such relations available in the literature for the average Nusselt number for cross flow over a cylinder, we present the one proposed by Churchill and Bernstein:

$$Nu_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62\, Re^{1/2}\, Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}\left[1 + \left(\frac{Re}{282{,}000}\right)^{5/8}\right]^{4/5} \quad \textbf{(7-35)}$$

This relation is quite comprehensive in that it correlates available data well for Re Pr > 0.2. The fluid properties are evaluated at the *film temperature* $T_f = \frac{1}{2}(T_\infty + T_s)$, which is the average of the free-stream and surface temperatures.

For flow over a *sphere*, Whitaker recommends the following comprehensive correlation:

$$Nu_{sph} = \frac{hD}{k} = 2 + [0.4\, Re^{1/2} + 0.06\, Re^{2/3}]\, Pr^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \quad \textbf{(7-36)}$$

which is valid for $3.5 \le Re \le 80{,}000$ and $0.7 \le Pr \le 380$. The fluid properties in this case are evaluated at the free-stream temperature T_∞, except for μ_s, which is evaluated at the surface temperature T_s. Although the two relations above are considered to be quite accurate, the results obtained from them can be off by as much as 30 percent.

The average Nusselt number for flow across cylinders can be expressed compactly as

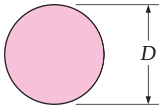
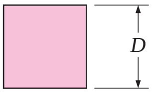
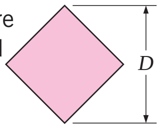
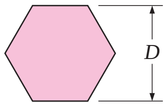
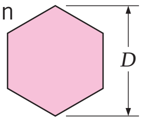
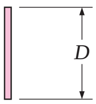
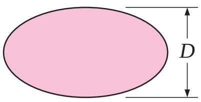
$$Nu_{cyl} = \frac{hD}{k} = C\, Re^m\, Pr^n \quad \textbf{(7-37)}$$

where $n = \frac{1}{3}$ and the experimentally determined constants C and m are given in Table 7–1 for circular as well as various noncircular cylinders. The characteristic length D for use in the calculation of the Reynolds and the Nusselt numbers for different geometries is as indicated on the figure. All fluid properties are evaluated at the film temperature.

The relations for cylinders above are for *single* cylinders or cylinders oriented such that the flow over them is not affected by the presence of others. Also, they are applicable to *smooth* surfaces. *Surface roughness* and the *free-stream turbulence* may affect the drag and heat transfer coefficients significantly. Eq. 7-37 provides a simpler alternative to Eq. 7-35 for flow over cylinders. However, Eq. 7-35 is more accurate, and thus should be preferred in calculations whenever possible.

TABLE 7-1

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, Ref. 14, and Jakob, Ref. 6)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\ Pr^{1/3}$ $Nu = 0.911Re^{0.385}\ Pr^{1/3}$ $Nu = 0.683Re^{0.466}\ Pr^{1/3}$ $Nu = 0.193Re^{0.618}\ Pr^{1/3}$ $Nu = 0.027Re^{0.805}\ Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\ Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\ Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\ Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\ Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\ Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\ Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\ Pr^{1/3}$

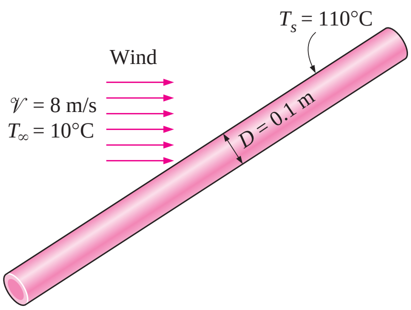


FIGURE 7–23
Schematic for Example 7–5.

EXAMPLE 7–5 **Heat Loss from a Steam Pipe in Windy Air**

A long 10-cm-diameter steam pipe whose external surface temperature is 110°C passes through some open area that is not protected against the winds (Fig. 7–23). Determine the rate of heat loss from the pipe per unit of its length when the air is at 1 atm pressure and 10°C and the wind is blowing across the pipe at a velocity of 8 m/s.

SOLUTION A steam pipe is exposed to windy air. The rate of heat loss from the steam is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas.

Properties The properties of air at the average film temperature of $T_f = (T_s + T_\infty)/2 = (110 + 10)/2 = 60°C$ and 1 atm pressure are (Table A-15)

$$k = 0.02808 \text{ W/m}\cdot°\text{C} \qquad \text{Pr} = 0.7202$$
$$\nu = 1.896 \times 10^{-5} \text{ m}^2/\text{s}$$

Analysis The Reynolds number is

$$\text{Re} = \frac{\mathcal{V}D}{\nu} = \frac{(8 \text{ m/s})(0.1 \text{ m})}{1.896 \times 10^{-5} \text{ m}^2/\text{s}} = 4.219 \times 10^4$$

The Nusselt number can be determined from

$$\text{Nu} = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{1/2}\,\text{Pr}^{1/3}}{[1 + (0.4/\text{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$
$$= 0.3 + \frac{0.62(4.219 \times 10^4)^{1/2}(0.7202)^{1/3}}{[1 + (0.4/0.7202)^{2/3}]^{1/4}}\left[1 + \left(\frac{4.219 \times 10^4}{282{,}000}\right)^{5/8}\right]^{4/5}$$
$$= 124$$

and

$$h = \frac{k}{D}\text{Nu} = \frac{0.02808 \text{ W/m}\cdot°\text{C}}{0.1 \text{ m}}(124) = 34.8 \text{ W/m}^2\cdot°\text{C}$$

Then the rate of heat transfer from the pipe per unit of its length becomes

$$A_s = pL = \pi DL = \pi(0.1 \text{ m})(1 \text{ m}) = 0.314 \text{ m}^2$$
$$\dot{Q} = hA_s(T_s - T_\infty) = (34.8 \text{ W/m}^2\cdot\text{C})(0.314 \text{ m}^2)(110 - 10)°\text{C} = \mathbf{1093\ W}$$

The rate of heat loss from the entire pipe can be obtained by multiplying the value above by the length of the pipe in m.

Discussion The simpler Nusselt number relation in Table 7–1 in this case would give Nu = 128, which is 3 percent higher than the value obtained above using Eq. 7-35.

EXAMPLE 7–6 Cooling of a Steel Ball by Forced Air

A 25-cm-diameter stainless steel ball (ρ = 8055 kg/m^3, C_p = 480 J/kg · °C) is removed from the oven at a uniform temperature of 300°C (Fig. 7–24). The ball is then subjected to the flow of air at 1 atm pressure and 25°C with a velocity of 3 m/s. The surface temperature of the ball eventually drops to 200°C. Determine the average convection heat transfer coefficient during this cooling process and estimate how long the process will take.

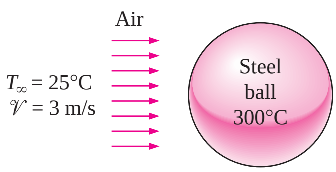


FIGURE 7–24
Schematic for Example 7–6.

SOLUTION A hot stainless steel ball is cooled by forced air. The average convection heat transfer coefficient and the cooling time are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas. **4** The outer surface temperature of the ball is uniform at all times. **5** The surface temperature of the ball during cooling is changing. Therefore, the convection heat transfer coefficient between the ball and the air will also change. To avoid this complexity, we take the surface temperature of the ball to be constant at the average temperature of (300 + 200)/2 = 250°C in the evaluation of the heat transfer coefficient and use the value obtained for the entire cooling process.

Properties The dynamic viscosity of air at the average surface temperature is $\mu_s = \mu_{@\ 250°C} = 2.76 \times 10^{-5}$ kg/m · s. The properties of air at the free-stream temperature of 25°C and 1 atm are (Table A-15)

$$k = 0.02551 \text{ W/m} \cdot °\text{C} \qquad \nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\mu = 1.849 \times 10^{-5} \text{ kg/m} \cdot \text{s} \qquad \text{Pr} = 0.7296$$

Analysis The Reynolds number is determined from

$$\text{Re} = \frac{\mathcal{V}D}{\nu} = \frac{(3 \text{ m/s})(0.25 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 4.802 \times 10^4$$

The Nusselt number is

$$\begin{aligned}\text{Nu} = \frac{hD}{k} &= 2 + [0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}]\,\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \\ &= 2 + [0.4(4.802 \times 10^4)^{1/2} + 0.06(4.802 \times 10^4)^{2/3}](0.7296)^{0.4} \\ &\quad \times \left(\frac{1.849 \times 10^{-5}}{2.76 \times 10^{-5}}\right)^{1/4} \\ &= 135\end{aligned}$$

Then the average convection heat transfer coefficient becomes

$$h = \frac{k}{D}\text{Nu} = \frac{0.02551 \text{ W/m} \cdot °\text{C}}{0.25 \text{ m}}(135) = \mathbf{13.8 \text{ W/m}^2 \cdot °\text{C}}$$

In order to estimate the time of cooling of the ball from 300°C to 200°C, we determine the *average* rate of heat transfer from Newton's law of cooling by using the *average* surface temperature. That is,

$$A_s = \pi D^2 = \pi(0.25 \text{ m})^2 = 0.1963 \text{ m}^2$$
$$\dot{Q}_{\text{ave}} = hA_s(T_{s,\text{ ave}} - T_\infty) = (13.8 \text{ W/m}^2 \cdot °\text{C})(0.1963 \text{ m}^2)(250 - 25)°\text{C} = 610 \text{ W}$$

Next we determine the *total* heat transferred from the ball, which is simply the change in the energy of the ball as it cools from 300°C to 200°C:

$$m = \rho V = \rho\tfrac{1}{6}\pi D^3 = (8055 \text{ kg/m}^3)\,\tfrac{1}{6}\pi(0.25 \text{ m})^3 = 65.9 \text{ kg}$$
$$Q_{\text{total}} = mC_p(T_2 - T_1) = (65.9 \text{ kg})(480 \text{ J/kg} \cdot °\text{C})(300 - 200)°\text{C} = 3{,}163{,}000 \text{ J}$$

In this calculation, we assumed that the entire ball is at 200°C, which is not necessarily true. The inner region of the ball will probably be at a higher temperature than its surface. With this assumption, the time of cooling is determined to be

$$\Delta t \approx \frac{Q}{\dot{Q}_{\text{ave}}} = \frac{3{,}163{,}000 \text{ J}}{610 \text{ J/s}} = 5185 \text{ s} = \mathbf{1 \text{ h } 26 \text{ min}}$$

Discussion The time of cooling could also be determined more accurately using the transient temperature charts or relations introduced in Chapter 4. But the simplifying assumptions we made above can be justified if all we need is a ballpark value. It will be naive to expect the time of cooling to be exactly 1 h 26 min, but, using our engineering judgment, it is realistic to expect the time of cooling to be somewhere between one and two hours.

7–4 ▪ FLOW ACROSS TUBE BANKS

Cross-flow over tube banks is commonly encountered in practice in heat transfer equipment such as the condensers and evaporators of power plants, refrigerators, and air conditioners. In such equipment, one fluid moves through the tubes while the other moves over the tubes in a perpendicular direction.

In a heat exchanger that involves a tube bank, the tubes are usually placed in a *shell* (and thus the *name shell-and-tube heat exchanger*), especially when the fluid is a liquid, and the fluid flows through the space between the tubes and the shell. There are numerous types of shell-and-tube heat exchangers, some of which are considered in Chap. 13. In this section we will consider the general aspects of flow over a tube bank, and try to develop a better and more intuitive understanding of the performance of heat exchangers involving a tube bank.

Flow *through* the tubes can be analyzed by considering flow through a single tube, and multiplying the results by the number of tubes. This is not the case for flow *over* the tubes, however, since the tubes affect the flow pattern and turbulence level downstream, and thus heat transfer to or from them, as shown in Figure 7–25. Therefore, when analyzing heat transfer from a tube bank in cross flow, we must consider all the tubes in the bundle at once.

The tubes in a tube bank are usually arranged either *in-line* or *staggered* in the direction of flow, as shown in Figure 7–26. The outer tube diameter D is taken as the characteristic length. The arrangement of the tubes in the tube bank is characterized by the *transverse pitch* S_T, *longitudinal pitch* S_L, and the *diagonal pitch* S_D between tube centers. The diagonal pitch is determined from

$$S_D = \sqrt{S_L^2 + (S_T/2)^2} \qquad \textbf{(7-38)}$$

As the fluid enters the tube bank, the flow area decreases from $A_1 = S_T L$ to $A_T = (S_T - D)L$ between the tubes, and thus flow velocity increases. In staggered arrangement, the velocity may increase further in the diagonal region if the tube rows are very close to each other. In tube banks, the flow characteristics are dominated by the maximum velocitiy $\mathcal{V}_{\max}$ that occurs within the tube bank rather than the approach velocity $\mathcal{V}$. Therefore, the Reynolds number is defined on the basis of maximum velocity as

$$\text{Re}_D = \frac{\rho \mathcal{V}_{\max} D}{\mu} = \frac{\mathcal{V}_{\max} D}{\nu} \qquad \textbf{(7-39)}$$

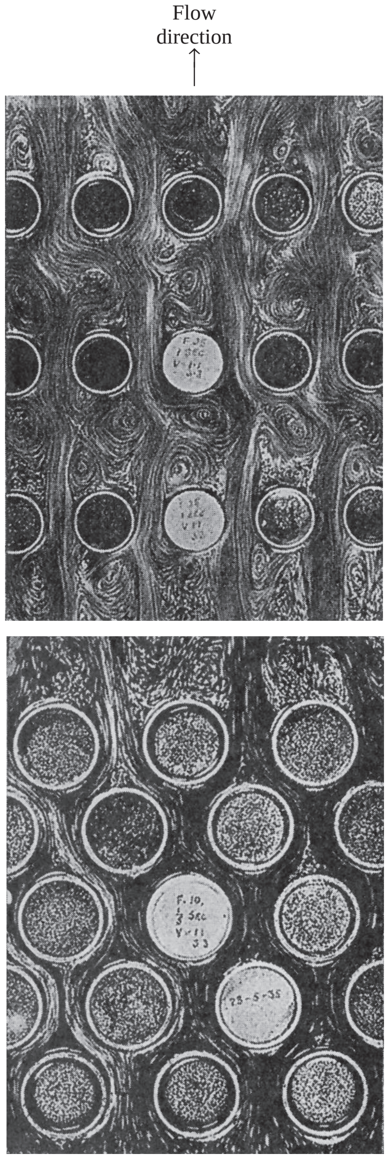
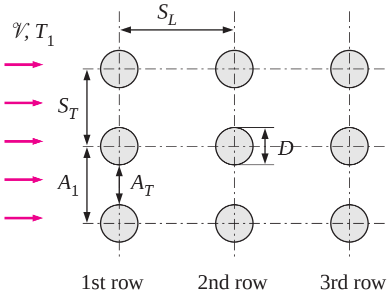
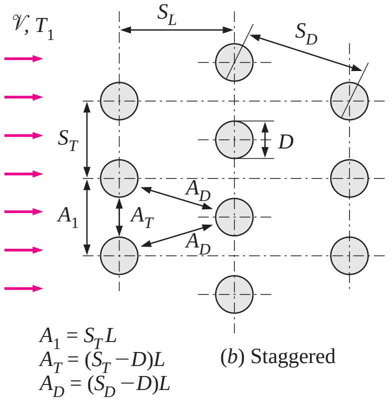


FIGURE 7-25
Flow patterns for staggered and in-line tube banks (photos by R. D. Willis, Ref 12).



(*a*) In-line



(*b*) Staggered

FIGURE 7–26
Arrangement of the tubes in in-line and staggered tube banks (A_1, A_T, and A_D are flow areas at indicated locations, and L is the length of the tubes).

The maximum velocity is determined from the conservation of mass requirement for steady incompressible flow. For *in-line* arrangement, the maximum velocity occurs at the minimum flow area between the tubes, and the conservation of mass can be expressed as (see Fig. 7-26*a*) $\rho \mathcal{V} A_1 = \rho \mathcal{V}_{max} A_T$ or $\mathcal{V} S_T = \mathcal{V}_{max}(S_T - D)$. Then the maximum velocity becomes

$$\mathcal{V}_{max} = \frac{S_T}{S_T - D}\mathcal{V} \qquad \textbf{(7-40)}$$

In *staggered* arrangement, the fluid approaching through area A_1 in Figure 7–26*b* passes through area A_T and then through area $2A_D$ as it wraps around the pipe in the next row. If $2A_D > A_T$, maximum velocity will still occur at A_T between the tubes, and thus the $\mathcal{V}_{max}$ relation Eq. 7-40 can also be used for staggered tube banks. But if $2A_D < A_T$ [or, if $2(S_D - D) < (S_T - D)$], maximum velocity will occur at the diagonal cross sections, and the maximum velocity in this case becomes

Staggered and $S_D < (S_T + D)/2$:
$$\mathcal{V}_{max} = \frac{S_T}{2(S_D - D)}\mathcal{V} \qquad \textbf{(7-41)}$$

since $\rho \mathcal{V} A_1 = \rho \mathcal{V}_{max}(2A_D)$ or $\mathcal{V} S_T = 2\mathcal{V}_{max}(S_D - D)$.

The nature of flow around a tube in the first row resembles flow over a single tube discussed in section 7–3, especially when the tubes are not too close to each other. Therefore, each tube in a tube bank that consists of a single transverse row can be treated as a single tube in cross-flow. The nature of flow around a tube in the second and subsequent rows is very different, however, because of wakes formed and the turbulence caused by the tubes upstream. The level of turbulence, and thus the heat transfer coefficient, increases with row number because of the combined effects of upstream rows. But there is no significant change in turbulence level after the first few rows, and thus the heat transfer coefficient remains constant.

Flow through tube banks is studied experimentally since it is too complex to be treated analytically. We are primarily interested in the average heat transfer coefficient for the entire tube bank, which depends on the number of tube rows along the flow as well as the arrangement and the size of the tubes.

Several correlations, all based on experimental data, have been proposed for the average Nusselt number for cross flow over tube banks. More recently, Zukauskas has proposed correlations whose general form is

$$\text{Nu}_D = \frac{hD}{k} = C\,\text{Re}_D^m\,\text{Pr}^n(\text{Pr}/\text{Pr}_s)^{0.25} \qquad \textbf{(7-42)}$$

where the values of the constants C, m, and n depend on value Reynolds number. Such correlations are given in Table 7–2 explicitly for $0.7 < \text{Pr} < 500$ and $0 < \text{Re}_D < 2 \times 10^6$. The uncertainty in the values of Nusselt number obtained from these relations is ±15 percent. Note that all properties except Pr_s are to be evaluated at the arithmetic mean temperature of the fluid determined from

$$T_m = \frac{T_i + T_e}{2} \qquad \textbf{(7-43)}$$

where T_i and T_e are the fluid temperatures at the inlet and the exit of the tube bank, respectively.

TABLE 7–2

Nusselt number correlations for cross flow over tube banks for N > 16 and 0.7 < Pr < 500 (from Zukauskas, Ref. 15, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$\text{Nu}_D = 0.9\,\text{Re}_D^{0.4}\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}$
	100–1000	$\text{Nu}_D = 0.52\,\text{Re}_D^{0.5}\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}$
	$1000–2 \times 10^5$	$\text{Nu}_D = 0.27\,\text{Re}_D^{0.63}\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$\text{Nu}_D = 0.033\,\text{Re}_D^{0.8}\text{Pr}^{0.4}(\text{Pr}/\text{Pr}_s)^{0.25}$
Staggered	0–500	$\text{Nu}_D = 1.04\,\text{Re}_D^{0.4}\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}$
	500–1000	$\text{Nu}_D = 0.71\,\text{Re}_D^{0.5}\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}$
	$1000–2 \times 10^5$	$\text{Nu}_D = 0.35(S_T/S_L)^{0.2}\,\text{Re}_D^{0.6}\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$\text{Nu}_D = 0.031(S_T/S_L)^{0.2}\,\text{Re}_D^{0.8}\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}$

*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

The average Nusselt number relations in Table 7–2 are for tube banks with 16 or more rows. Those relations can also be used for tube banks with N_L provided that they are modified as

$$\text{Nu}_{D,\,N_L} = F\,\text{Nu}_D \tag{7-44}$$

where F is a *correction factor F* whose values are given in Table 7–3. For $\text{Re}_D > 1000$, the correction factor is independent of Reynolds number.

Once the Nusselt number and thus the average heat transfer coefficient for the entire tube bank is known, the heat transfer rate can be determined from Newton's law of cooling using a suitable temperature difference ΔT. The first thought that comes to mind is to use $\Delta T = T_s - T_m = T_s - (T_i + T_e)/2$. But this will, in general, over predict the heat transfer rate. We will show in the next chapter that the proper temperature difference for internal flow (flow over tube banks is still internal flow through the shell) is the *1ogarithmic mean temperature difference* $\Delta T_{\ln}$ defined as

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} \tag{7-45}$$

We will also show that the exit temperature of the fluid T_e can be determined from

TABLE 7–3

Correction factor F to be used in $\text{Nu}_{D,\,N_L} = F\text{Nu}_D$ for $N_L < 16$ and $\text{Re}_D > 1000$ (from Zukauskas, Ref 15, 1987).

N_L	1	2	3	4	5	7	10	13
In-line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}C_p}\right) \tag{7-46}$$

where $A_s = N\pi DL$ is the heat transfer surface area and $\dot{m} = \rho\mathcal{V}(N_T S_T L)$ is the mass flow rate of the fluid. Here N is the total number of tubes in the bank, N_T is the number of tubes in a transverse plane, L is the length of the tubes, and $\mathcal{V}$ is the velocity of the fluid just before entering the tube bank. Then the heat transfer rate can be determined from

$$\dot{Q} = hA_s\Delta T_{\ln} = \dot{m}C_p(T_e - T_i) \tag{7-47}$$

The second relation is usually more convenient to use since it does not require the calculation of $\Delta T_{\ln}$.

Pressure Drop

Another quantity of interest associated with tube banks is the *pressure drop* ΔP, which is the difference between the pressures at the inlet and the exit of the tube bank. It is a measure of the resistance the tubes offer to flow over them, and is expressed as

$$\Delta P = N_L f\chi\frac{\rho\mathcal{V}_{\max}^2}{2} \tag{7-48}$$

where f is the friction factor and χ is the correction factor, both plotted in Figures 7–27*a* and 7–27*b* against the Reynolds number based on the maximum velocity $\mathcal{V}_{\max}$. The friction factor in Figure 7–27*a* is for a *square* in-line tube bank ($S_T = S_L$), and the correction factor given in the insert is used to account for the effects of deviation of rectangular in-line arrangements from square arrangement. Similarly, the friction factor in Figure 7–27*b* is for an *equilateral* staggered tube bank ($S_T = S_D$), and the correction factor is to account for the effects of deviation from equilateral arrangement. Note that $\chi = 1$ for both square and equilateral triangle arrangements. Also, pressure drop occurs in the flow direction, and thus we used N_L (the number of rows) in the ΔP relation.

The power required to move a fluid through a tube bank is proportional to the pressure drop, and when the pressure drop is available, the pumping power required can be determined from

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = \frac{\dot{m}\Delta P}{\rho} \tag{7-49}$$

where $\dot{V} = \mathcal{V}(N_T S_T L)$ is the volume flow rate and $\dot{m} = \rho\dot{V} = \rho\mathcal{V}(N_T S_T L)$ is the mass flow rate of the fluid through the tube bank. Note that the power required to keep a fluid flowing through the tube bank (and thus the operating cost) is proportional to the pressure drop. Therefore, the benefits of enhancing heat transfer in a tube bank via rearrangement should be weighed against the cost of additional power requirements.

In this section we limited our consideration to tube banks with base surfaces (no fins). Tube banks with finned surfaces are also commonly used in practice, especially when the fluid is a gas, and heat transfer and pressure drop correlations can be found in the literature for tube banks with pin fins, plate fins, strip fins, etc.

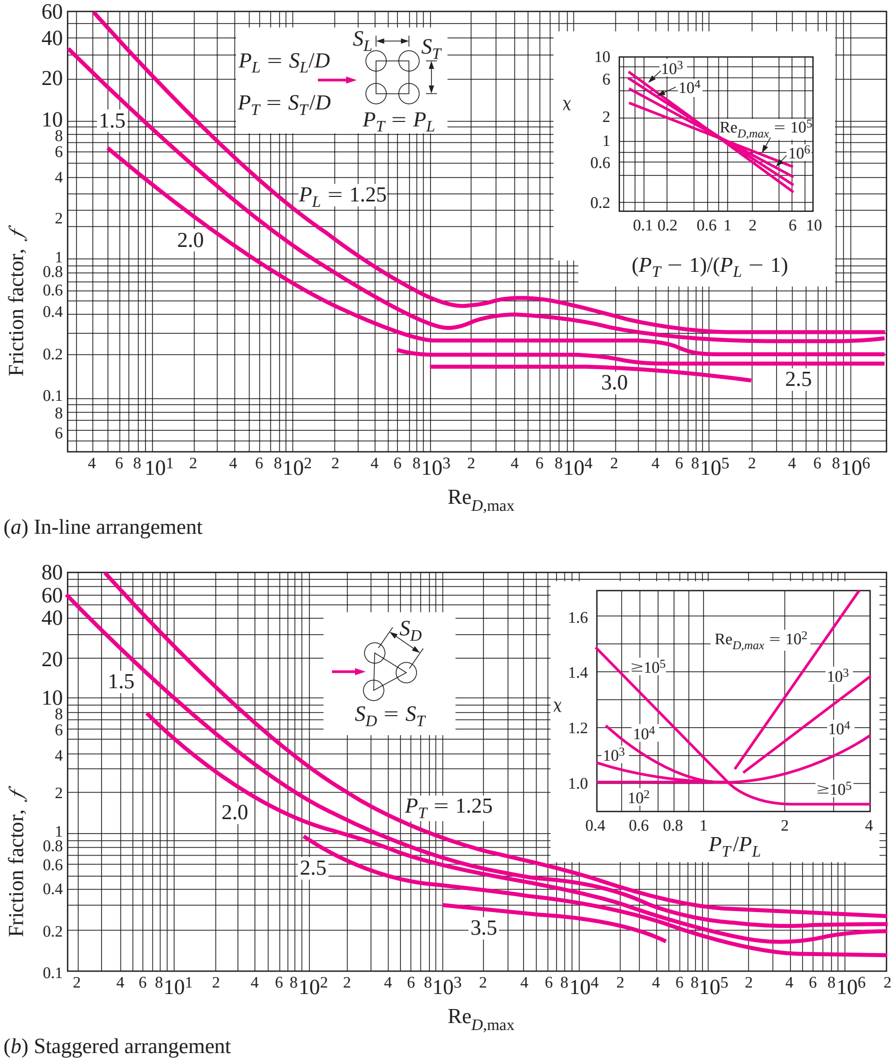


FIGURE 7–27
Friction factor f and correction factor χ for tube banks (from Zukauskas, Ref. 16, 1985).

EXAMPLE 7–7 Preheating Air by Geothermal Water in a Tube Bank

In an industrial facility, air is to be preheated before entering a furnace by geothermal water at 120°C flowing through the tubes of a tube bank located in a duct. Air enters the duct at 20°C and 1 atm with a mean velocity of 4.5 m/s, and flows over the tubes in normal direction. The outer diameter of the tubes is 1.5 cm, and the tubes are arranged in-line with longitudinal and transverse pitches of $S_L = S_T = 5$ cm. There are 6 rows in the flow direction with 10 tubes in each row, as shown in Figure 7–28. Determine the rate of heat transfer per unit length of the tubes, and the pressure drop across the tube bank.

SOLUTION Air is heated by geothermal water in a tube bank. The rate of heat transfer to air and the pressure drop of air are to be determined.

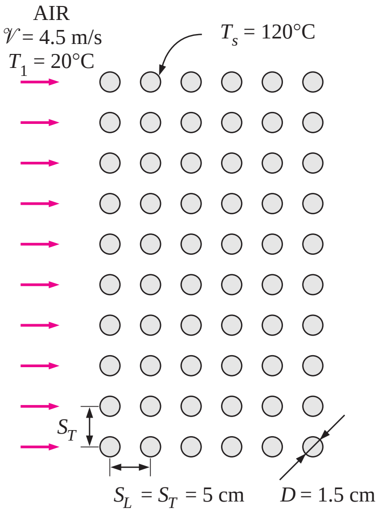


FIGURE 7–28
Schematic for Example 7–7.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the tubes is equal to the temperature of geothermal water.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of 60°C (will be checked later) and 1 atm are Table A–15):

$$k = 0.02808 \text{ W/m}\cdot\text{K}, \qquad \rho = 1.06 \text{ kg/m}^3$$
$$C_p = 1.007 \text{ kJ/kg}\cdot\text{K}, \qquad \text{Pr} = 0.7202$$
$$\mu = 2.008 \times 10^{-5} \text{ kg/m}\cdot\text{s} \qquad \text{Pr}_s = \text{Pr}_{@Ts} = 0.7073$$

Also, the density of air at the inlet temperature of 20°C (for use in the mass flow rate calculation at the inlet) is $\rho_1 = 1.204$ kg/m^3

Analysis It is given that $D = 0.015$ m, $S_L = S_T = 0.05$ m, and $\mathscr{V} = 4.5$ m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become

$$\mathscr{V}_{max} = \frac{S_T}{S_T - D}\mathscr{V} = \frac{0.05}{0.05 - 0.015}(4.5 \text{ m/s}) = 6.43 \text{ m/s}$$
$$\text{Re}_D = \frac{\rho \mathscr{V}_{max} D}{\mu} = \frac{(1.06 \text{ kg/m}^3)(6.43 \text{ m/s})(0.015 \text{ m})}{2.008 \times 10^{-5} \text{ kg/m}\cdot\text{s}} = 5091$$

The average Nusselt number is determined using the proper relation from Table 7–2 to be

$$\text{Nu}_D = 0.27\,\text{Re}_D^{0.63}\,\text{Pr}^{0.36}(\text{Pr/Pr}_s)^{0.25}$$
$$= 0.27(5091)^{0.63}(0.7202)^{0.36}(0.7202/0.7073)^{0.25} = 52.2$$

This Nusselt number is applicable to tube banks with $N_L > 16$. In our case, the number of rows is $N_L = 6$, and the corresponding correction factor from Table 7–3 is $F = 0.945$. Then the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\text{Nu}_{D,N_L} = F\text{Nu}_D = (0.945)(52.2) = 49.3$$
$$h = \frac{\text{Nu}_{D,N_L}k}{D} = \frac{49.3(0.02808 \text{ W/m}\cdot{}^\circ\text{C})}{0.015 \text{ m}} = 92.2 \text{ W/m}^2\cdot{}^\circ\text{C}$$

The total number of tubes is $N = N_L \times N_T = 6 \times 10 = 60$. For a unit tube length ($L = 1$ m), the heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 60\pi(0.015 \text{ m})(1 \text{ m}) = 2.827 \text{ m}^2$$
$$\dot{m} = \dot{m}_1 = \rho_1 \mathscr{V}(N_T S_T L)$$
$$= (1.204 \text{ kg/m}^3)(4.5 \text{ m/s})(10)(0.05 \text{ m})(1 \text{ m}) = 2.709 \text{ kg/s}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer become

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}C_p}\right)$$
$$= 120 - (120 - 20)\exp\left(-\frac{(2.827 \text{ m}^2)(92.2 \text{ W/m}^2\cdot{}^\circ\text{C})}{(2.709 \text{ kg/s})(1007 \text{ J/kg}\cdot{}^\circ\text{C})}\right) = 29.11^\circ\text{C}$$

$$\Delta T_{ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{(120 - 29.11) - (120 - 20)}{\ln[(120 - 29.11)/(120 - 20)]} = 95.4°C$$

$$\dot{Q} = hA_s\Delta T_{ln} = (92.2 \text{ W/m}^2 \cdot °C)(2.827 \text{ m}^2)(95.4°C) = \mathbf{2.49 \times 10^4 \text{ W}}$$

The rate of heat transfer can also be determined in a simpler way from

$$\dot{Q} = hA_s\Delta T_{in} = \dot{m}C_p(T_e - T_i)$$
$$= (2.709 \text{ kg/s})(1007 \text{ J/kg} \cdot °C)(29.11 - 20)°C = 2.49 \times 10^4 \text{ W}$$

For this square in-line tube bank, the friction coefficient corresponding to $Re_D = 5088$ and $S_L/D = 5/1.5 = 3.33$ is, from Fig. 7–27*a*, $f = 0.16$. Also, $\chi = 1$ for the square arrangements. Then the pressure drop across the tube bank becomes

$$\Delta P = N_L f\chi \frac{\rho V_{max}^2}{2}$$
$$= 6(0.16)(1)\frac{(1.06 \text{ kg/m}^3)(6.43 \text{ m/s})^3}{2}\left(\frac{1\text{N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = \mathbf{21 \text{ Pa}}$$

Discussion The arithmetic mean fluid temperature is $(T_i + T_e)/2 = (20 + 110.9)/2 = 65.4°C$, which is fairly close to the assumed value of 60°C. Therefore, there is no need to repeat calculations by reevaluating the properties at 65.4°C (it can be shown that doing so would change the results by less than 1 percent, which is much less than the uncertainty in the equations and the charts used).

TOPIC OF SPECIAL INTEREST

Reducing Heat Transfer through Surfaces: Thermal Insulation

Thermal insulations are materials or combinations of materials that are used primarily to provide resistance to heat flow (Fig. 7–29). You are probably familiar with several kinds of insulation available in the market. Most insulations are heterogeneous materials made of low thermal conductivity materials, and they involve air pockets. This is not surprising since air has one of the lowest thermal conductivities and is readily available. The *Styrofoam* commonly used as a packaging material for TVs, VCRs, computers, and just about anything because of its light weight is also an excellent insulator.

Temperature difference is the driving force for heat flow, and the greater the temperature difference, the larger the rate of heat transfer. We can slow down the heat flow between two mediums at different temperatures by putting "barriers" on the path of heat flow. Thermal insulations serve as such barriers, and they play a major role in the design and manufacture of all energy-efficient devices or systems, and they are usually the cornerstone of energy conservation projects. A 1991 Drexel University study of the energy-intensive U.S. industries revealed that insulation saves the U.S.

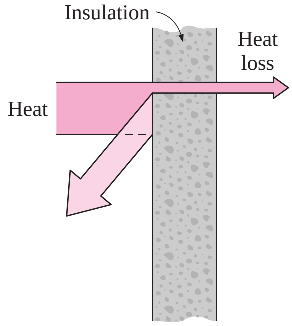


FIGURE 7–29
Thermal insulation retards heat transfer by acting as a barrier in the path of heat flow.

*This section can be skipped without a loss in continuity.

industry nearly 2 billion barrels of oil per year, valued at $60 billion a year in energy costs, and more can be saved by practicing better insulation techniques and retrofitting the older industrial facilities.

Heat is generated in furnaces or heaters by burning a fuel such as coal, oil, or natural gas or by passing electric current through a *resistance heater.* Electricity is rarely used for heating purposes since its unit cost is much higher. The heat generated is absorbed by the medium in the furnace and its surfaces, causing a temperature rise above the ambient temperature. This temperature difference drives heat transfer from the hot medium to the ambient, and insulation reduces the amount of heat loss and thus saves fuel and money. Therefore, insulation *pays for itself* from the energy it saves. Insulating properly requires a one-time capital investment, but its effects are dramatic and long term. The payback period of insulation is often less than one year. That is, the money insulation saves during the first year is usually greater than its initial material and installation costs. On a broader perspective, insulation also helps the environment and fights air pollution and the greenhouse effect by reducing the amount of fuel burned and thus the amount of CO_2 and other gases released into the atmosphere (Fig. 7–30).

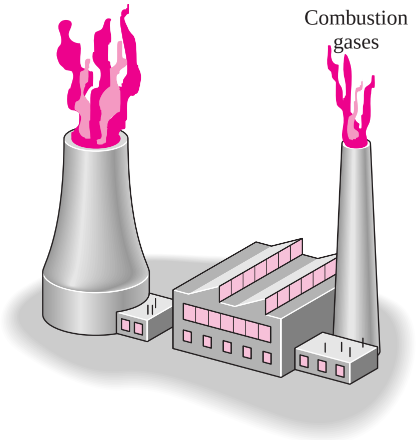


FIGURE 7–30
Insulation also helps the environment by reducing the amount of fuel burned and the air pollutants released.

Saving energy with insulation is not limited to hot surfaces. We can also save energy and money by insulating *cold surfaces* (surfaces whose temperature is below the ambient temperature) such as chilled water lines, cryogenic storage tanks, refrigerated trucks, and air-conditioning ducts. The source of "coldness" is *refrigeration,* which requires energy input, usually electricity. In this case, heat is transferred from the surroundings to the cold surfaces, and the refrigeration unit must now work harder and longer to make up for this heat gain and thus it must consume more electrical energy. A cold canned drink can be kept cold much longer by wrapping it in a blanket. A refrigerator with well-insulated walls will consume much less electricity than a similar refrigerator with little or no insulation. Insulating a house will result in reduced cooling load, and thus reduced electricity consumption for air-conditioning.

Whether we realize it or not, we have an *intuitive* understanding and appreciation of thermal insulation. As babies we feel much better in our blankies, and as children we know we should wear a sweater or coat when going outside in cold weather (Fig. 7–31). When getting out of a pool after swimming on a windy day, we quickly wrap in a towel to stop shivering. Similarly, early man used animal furs to keep warm and built shelters using mud bricks and wood. Cork was used as a roof covering for centuries. The need for effective thermal insulation became evident with the development of mechanical refrigeration later in the nineteenth century, and a great deal of work was done at universities and government and private laboratories in the 1910s and 1920s to identify and characterize thermal insulation.

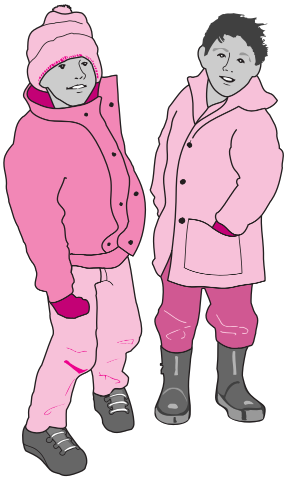


FIGURE 7–31
In cold weather, we minimize heat loss from our bodies by putting on thick layers of insulation (coats or furs).

Thermal insulation in the form of *mud, clay, straw, rags,* and *wood strips* was first used in the eighteenth century on steam engines to keep workmen from being burned by hot surfaces. As a result, boiler room temperatures dropped and it was noticed that fuel consumption was also reduced. The realization of improved engine efficiency and energy savings prompted the search for materials with improved thermal efficiency. One of the first such materials was *mineral wool* insulation, which, like many materials, was discovered by accident. About 1840, an iron producer in Wales aimed a stream of high-pressure steam at the slag flowing from a blast furnace, and manufactured mineral wool was born. In the early 1860s, this slag wool was a by-product of manufacturing cannons for the Civil War and quickly found its way into many industrial uses. By 1880, builders began installing mineral wool in houses, with one of the most notable applications being General Grant's house. The insulation of this house was described in an article: "it keeps the house cool in summer and warm in winter; it prevents the spread of fire; and it deadens the sound between floors" [Edmunds (1989), Ref. 4]. An article published in 1887 in *Scientific American* detailing the benefits of insulating the entire house gave a major boost to the use of insulation in residential buildings.

The energy crisis of the 1970s had a tremendous impact on the public awareness of energy and limited energy reserves and brought an emphasis on *energy conservation.* We have also seen the development of new and more effective insulation materials since then, and a considerable increase in the use of insulation. Thermal insulation is used in more places than you may be aware of. The walls of your house are probably filled with some kind of insulation, and the roof is likely to have a thick layer of insulation. The "thickness" of the walls of your refrigerator is due to the insulation layer sandwiched between two layers of sheet metal (Fig. 7–32). The walls of your range are also insulated to conserve energy, and your hot water tank contains less water than you think because of the 2- to 4-cm-thick insulation in the walls of the tank. Also, your hot water pipe may look much thicker than the cold water pipe because of insulation.

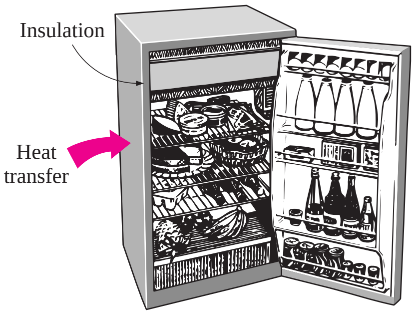


FIGURE 7–32
The insulation layers in the walls of a refrigerator reduce the amount of heat flow into the refrigerator and thus the running time of the refrigerator, saving electricity.

Reasons for Insulating

If you examine the engine compartment of your car, you will notice that the firewall between the engine and the passenger compartment as well as the inner surface of the hood are insulated. The reason for insulating the hood is not to conserve the waste heat from the engine but to protect people from burning themselves by touching the hood surface, which will be too hot if not insulated. As this example shows, the use of insulation is not limited to energy conservation. Various reasons for using insulation can be summarized as follows:

- **Energy Conservation** Conserving energy by reducing the rate of heat flow is the primary reason for insulating surfaces. Insulation materials that will perform satisfactorily in the temperature range of −268°C to 1000°C (−450°F to 1800°F) are widely available.
- **Personnel Protection and Comfort** A surface that is too hot poses a danger to people who are working in that area of accidentally touching the hot surface and burning themselves (Fig. 7–33). To prevent this danger and to comply with the OSHA (Occupational Safety and Health Administration) standards, the temperatures of hot surfaces should be reduced to below 60°C (140°F) by insulating them. Also, the excessive heat coming off the hot surfaces creates an unpleasant environment in which to work, which adversely affects the performance or productivity of the workers, especially in summer months.

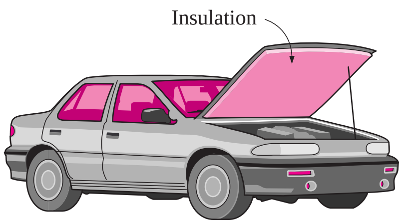


FIGURE 7–33
The hood of the engine compartment of a car is insulated to reduce its temperature and to protect people from burning themselves.

- **Maintaining Process Temperature** Some processes in the chemical industry are temperature-sensitive, and it may become necessary to insulate the process tanks and flow sections heavily to maintain the same temperature throughout.
- **Reducing Temperature Variation and Fluctuations** The temperature in an enclosure may vary greatly between the midsection and the edges if the enclosure is not insulated. For example, the temperature near the walls of a poorly insulated house is much lower than the temperature at the midsections. Also, the temperature in an uninsulated enclosure will follow the temperature changes in the environment closely and fluctuate. Insulation minimizes temperature nonuniformity in an enclosure and slows down fluctuations.
- **Condensation and Corrosion Prevention** Water vapor in the air condenses on surfaces whose temperature is below the dew point, and the outer surfaces of the tanks or pipes that contain a cold fluid frequently fall below the dew-point temperature unless they have adequate insulation. The liquid water on exposed surfaces of the metal tanks or pipes may promote corrosion as well as algae growth.
- **Fire Protection** Damage during a fire may be minimized by keeping valuable combustibles in a safety box that is well insulated. Insulation may lower the rate of heat flow to such levels that the temperature in the box never rises to unsafe levels during fire.
- **Freezing Protection** Prolonged exposure to subfreezing temperatures may cause water in pipes or storage vessels to freeze and burst as a result of heat transfer from the water to the cold ambient. The bursting of pipes as a result of freezing can cause considerable damage. Adequate insulation will slow down the heat loss from the water and prevent freezing during limited exposure to subfreezing temperatures. For example, covering vegetables during a cold night will protect them from freezing, and burying water pipes in the ground at a sufficient depth will keep them from freezing during the entire winter. Wearing thick gloves will protect the fingers from possible frostbite. Also, a molten metal or plastic in a container will solidify on the inner surface if the container is not properly insulated.
- **Reducing Noise and Vibration** An added benefit of thermal insulation is its ability to dampen noise and vibrations (Fig. 7–34). The insulation materials differ in their ability to reduce noise and vibration, and the proper kind can be selected if noise reduction is an important consideration.

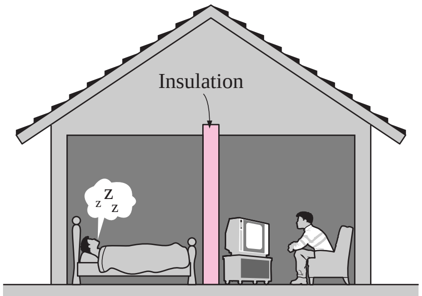


FIGURE 7–34
Insulation materials absorb vibration and sound waves, and are used to minimize sound transmission.

There are a wide variety of insulation materials available in the market, but most are primarily made of fiberglass, mineral wool, polyethylene, foam, or calcium silicate. They come in various trade names such as Ethafoam Polyethylene Foam Sheeting, Solimide Polimide Foam Sheets, FPC Fiberglass Reinforced Silicone Foam Sheeting, Silicone Sponge Rubber Sheets, fiberglass/mineral wool insulation blankets, wire-reinforced mineral wool insulation, Reflect-All Insulation, granulated bulk mineral wool insulation, cork insulation sheets, foil-faced fiberglass insulation, blended sponge rubber sheeting, and numerous others.

Today various forms of *fiberglass insulation* are widely used in process industries and heating and air-conditioning applications because of their low cost, light weight, resiliency, and versatility. But they are not suitable for some applications because of their low resistance to moisture and fire and their limited maximum service temperature. Fiberglass insulations come in various forms such as unfaced fiberglass insulation, vinyl-faced fiberglass insulation, foil-faced fiberglass insulation, and fiberglass insulation sheets. The reflective foil-faced fiberglass insulation resists vapor penetration and retards radiation because of the aluminum foil on it and is suitable for use on pipes, ducts, and other surfaces.

Mineral wool is resilient, lightweight, fibrous, wool-like, thermally efficient, fire resistant up to 1100°C (2000°F), and forms a sound barrier. Mineral wool insulation comes in the form of blankets, rolls, or blocks. *Calcium silicate* is a solid material that is suitable for use at high temperatures, but it is more expensive. Also, it needs to be cut with a saw during installation, and thus it takes longer to install and there is more waste.

Superinsulators

You may be tempted to think that the most effective way to reduce heat transfer is to use insulating materials that are known to have very low thermal conductivities such as urethane or rigid foam ($k = 0.026$ W/m · °C) or fiberglass ($k = 0.035$ W/m · °C). After all, they are widely available, inexpensive, and easy to install. Looking at the thermal conductivities of materials, you may also notice that the thermal conductivity of air at room temperature is 0.026 W/m · °C, which is lower than the conductivities of practically all of the ordinary insulating materials. Thus you may think that a layer of enclosed air space is as effective as any of the common insulating materials of the same thickness. Of course, heat transfer through the air will probably be higher than what a pure conduction analysis alone would indicate because of the natural convection currents that are likely to occur in the air layer. Besides, air is transparent to radiation, and thus heat will also be transferred by radiation. The thermal conductivity of air is practically independent of pressure unless the pressure is extremely high or extremely low. Therefore, we can reduce the thermal conductivity of air and thus the conduction heat transfer through the air by evacuating the air space. In the limiting case of absolute vacuum, the thermal conductivity will be zero since there will be no particles in this case to "conduct" heat from one surface to the other, and thus the conduction heat transfer will be zero. Noting that the thermal conductivity cannot be negative, an absolute vacuum must be the ultimate insulator, right? Well, not quite.

The purpose of insulation is to reduce "total" heat transfer from a surface, not just conduction. A vacuum totally eliminates conduction but offers zero resistance to radiation, whose magnitude can be comparable to conduction or natural convection in gases (Fig. 7–35). Thus, a vacuum is

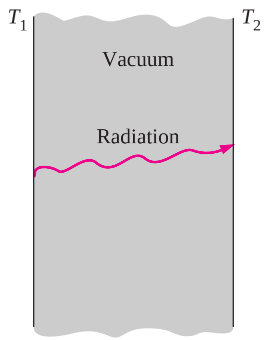


FIGURE 7–35
Evacuating the space between two surfaces completely eliminates heat transfer by conduction or convection but leaves the door wide open for radiation.

no more effective in reducing heat transfer than sealing off one of the lanes of a two-lane road is in reducing the flow of traffic on a one-way road.

Insulation against radiation heat transfer between two surfaces is achieved by placing "barriers" between the two surfaces, which are highly reflective thin metal sheets. Radiation heat transfer between two surfaces is inversely proportional to the number of such sheets placed between the surfaces. Very effective insulations are obtained by using closely packed layers of highly reflective thin metal sheets such as aluminum foil (usually 25 sheets per cm) separated by fibers made of insulating material such as glass fiber (Fig. 7–36). Further, the space between the layers is evacuated to form a vacuum under 0.000001 atm pressure to minimize conduction or convection heat transfer through the air space between the layers. The result is an insulating material whose apparent thermal conductivity is below 2×10^{-5} W/m · °C, which is one thousand times less than the conductivity of air or any common insulating material. These specially built insulators are called **superinsulators,** and they are commonly used in space applications and cryogenics, which is the branch of heat transfer dealing with temperatures below 100 K (−173°C) such as those encountered in the liquefaction, storage, and transportation of gases, with helium, hydrogen, nitrogen, and oxygen being the most common ones.

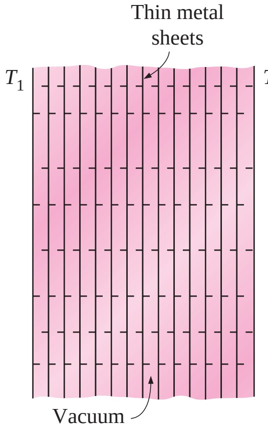


FIGURE 7–36
Superinsulators are built by closely packing layers of highly reflective thin metal sheets and evacuating the space between them.

The R-value of Insulation

The effectiveness of insulation materials is given by some manufacturers in terms of their ***R*-value,** which is the *thermal resistance* of the material *per unit surface area*. For *flat insulation* the *R*-value is obtained by simply dividing the thickness of the insulation by its thermal conductivity. That is,

$$R\text{-value} = \frac{L}{k} \qquad \text{(flat insulation)} \tag{7-50}$$

where L is the thickness and k is the thermal conductivity of the material. Note that doubling the thickness L doubles the *R*-value of flat insulation. For *pipe insulation,* the *R*-value is determined using the thermal resistance relation from

$$R\text{-value} = \frac{r_2}{k} \ln \frac{r_2}{r_1} \qquad \text{(pipe insulation)} \tag{7-51}$$

where r_1 is the inside radius of insulation and r_2 is the outside radius of insulation. Once the *R*-value is available, the rate of heat transfer through the insulation can be determined from

$$\dot{Q} = \frac{\Delta T}{R\text{-value}} \times \text{Area} \tag{7-52}$$

where ΔT is the temperature difference across the insulation and Area is the outer surface area for a cylinder.

In the United States, the *R*-values of insulation are expressed without any units, such as *R*-19 and *R*-30. These *R*-values are obtained by dividing the thickness of the material in *feet* by its thermal conductivity in the unit Btu/h · ft · °F so that the *R*-values actually have the unit h · ft^2 · °F/Btu. For example, the *R*-value of 6-in.-thick glass fiber insulation whose thermal conductivity is 0.025 Btu/h · ft · °F is (Fig. 7–37)

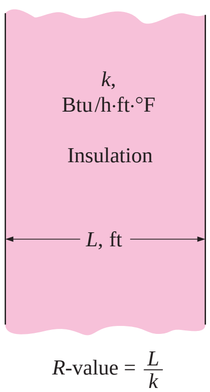


FIGURE 7–37
The *R*-value of an insulating material is simply the ratio of the thickness of the material to its thermal conductivity in proper units.

$$R\text{-value} = \frac{L}{k} = \frac{0.5 \text{ ft}}{0.025 \text{ Btu/h} \cdot \text{ft} \cdot {}^\circ\text{F}} = 20 \text{ h} \cdot \text{ft}^2 \cdot {}^\circ\text{F/Btu}$$

Thus, this 6-in.-thick glass fiber insulation would be referred to as *R*-20 insulation by the builders. The unit of *R*-value is $m^2 \cdot °C/W$ in SI units, with the conversion relation $1\ m^2 \cdot °C/W = 5.678\ h \cdot ft^2 \cdot °F/Btu$. Therefore, a small *R*-value in SI corresponds to a large *R*-value in English units.

Optimum Thickness of Insulation

It should be realized that insulation does not eliminate heat transfer; it merely reduces it. The thicker the insulation, the lower the rate of heat transfer but also the higher the cost of insulation. Therefore, there should be an *optimum* thickness of insulation that corresponds to a minimum combined cost of insulation and heat lost. The determination of the optimum thickness of insulation is illustrated in Figure 7–38. Notice that the cost of insulation increases roughly linearly with thickness while the cost of heat loss decreases exponentially. The total cost, which is the sum of the insulation cost and the lost heat cost, decreases first, reaches a minimum, and then increases. The thickness corresponding to the minimum total cost is the optimum thickness of insulation, and this is the recommended thickness of insulation to be installed.

If you are mathematically inclined, you can determine the *optimum thickness* by obtaining an expression for the *total cost*, which is the sum of the expressions for the lost heat cost and insulation cost as a function of thickness; *differentiating* the total cost expression with respect to the thickness; and *setting* it equal to zero. The thickness value satisfying the resulting equation is the optimum thickness. The cost values can be determined from an annualized lifetime analysis or simply from the requirement that the insulation pay for itself within two or three years. Note that the optimum thickness of insulation depends on the fuel cost, and the higher the fuel cost, the larger the optimum thickness of insulation. Considering that insulation will be in service for many years and the fuel prices are likely to escalate, a reasonable increase in fuel prices must be assumed in calculations. Otherwise, what is optimum insulation today will be inadequate insulation in the years to come, and we may have to face the possibility of costly retrofitting projects. This is what happened in the 1970s and 1980s to insulations installed in the 1960s.

The discussion above on optimum thickness is valid when the type and manufacturer of insulation are already selected, and the only thing to be determined is the most economical thickness. But often there are several suitable insulations for a job, and the selection process can be rather confusing since each insulation can have a different thermal conductivity, different installation cost, and different service life. In such cases, a selection can be made by preparing an annualized cost versus thickness chart like Figure 7–39 for each insulation, and determining the one having the *lowest* minimum cost. The insulation with the lowest annual cost is obviously the most economical insulation, and the insulation thickness corresponding to the *minimum total cost* is the *optimum thickness*. When the optimum thickness falls between two commercially available thicknesses, it is a good practice to be conservative and choose the thicker insulation. The

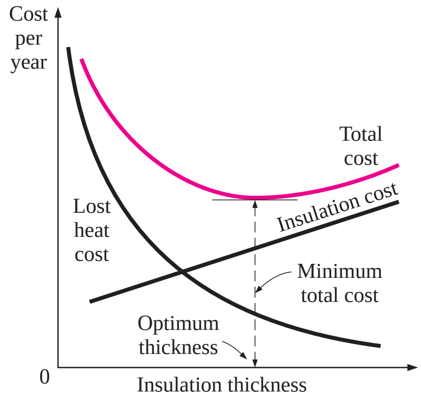


FIGURE 7–38
Determination of the optimum thickness of insulation on the basis of minimum total cost.

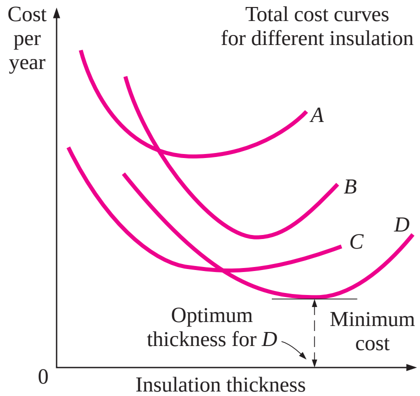


FIGURE 7–39
Determination of the most economical type of insulation and its optimum thickness.

extra thickness will provide a little safety cushion for any possible decline in performance over time and will help the environment by reducing the production of greenhouse gases such as CO_2.

The determination of the optimum thickness of insulation requires a heat transfer and economic analysis, which can be tedious and time-consuming. But a selection can be made in a few minutes using the tables and charts prepared by TIMA (Thermal Insulation Manufacturers Association) and member companies. The primary inputs required for using these tables or charts are the operating and ambient temperatures, pipe diameter (in the case of pipe insulation), and the unit fuel cost. Recommended insulation thicknesses for hot surfaces at specified temperatures are given in Table 7–4. Recommended thicknesses of *pipe insulations* as a function of service temperatures are 0.5 to 1 in. for 150°F, 1 to 2 in. for 250°F, 1.5 to 3 in. for 350°F, 2 to 4.5 in. for 450°F, 2.5 to 5.5 in. for 550°F, and 3 to 6 in. for 650°F for nominal pipe diameters of 0.5 to 36 in. The lower recommended insulation thicknesses are for pipes with small diameters, and the larger ones are for pipes with large diameters.

TABLE 7–4

Recommended insulation thicknesses for flat hot surfaces as a function of surface temperature (from TIMA *Energy Savings Guide*)

Surface temperature	Insulation thickness
150°F (66°C)	2″ (5.1 cm)
250°F (121°C)	3″ (7.6 cm)
350°F (177°C)	4″ (10.2 cm)
550°F (288°C)	6″ (15.2 cm)
750°F (400°C)	9″ (22.9 cm)
950°F (510°C)	10″ (25.44 cm)

EXAMPLE 7–8 Effect of Insulation on Surface Temperature

Hot water at $T_i = 120°C$ flows in a stainless steel pipe ($k = 15$ W/m · °C) whose inner diameter is 1.6 cm and thickness is 0.2 cm. The pipe is to be covered with adequate insulation so that the temperature of the outer surface of the insulation does not exceed 40°C when the ambient temperature is $T_o = 25°C$. Taking the heat transfer coefficients inside and outside the pipe to be $h_i = 70$ W/m² · °C and $h_o = 20$ W/m² · °C, respectively, determine the thickness of fiberglass insulation ($k = 0.038$ W/m · °C) that needs to be installed on the pipe.

SOLUTION A steam pipe is to be covered with enough insulation to reduce the exposed surface temperature. The thickness of insulation that needs to be installed is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be $k = 15$ W/m · °C for the steel pipe and $k = 0.038$ W/m · °C for fiberglass insulation.

Analysis The thermal resistance network for this problem involves four resistances in series and is given in Figure 7–40. The inner radius of the pipe is $r_1 = 0.8$ cm and the outer radius of the pipe and thus the inner radius of the insulation is $r_2 = 1.0$ cm. Letting r_3 represent the outer radius of the insulation, the areas of the surfaces exposed to convection for an $L = 1$-m-long section of the pipe become

$$A_1 = 2\pi r_1 L = 2\pi(0.008 \text{ m})(1 \text{ m}) = 0.0503 \text{ m}^2$$

$$A_3 = 2\pi r_3 L = 2\pi r_3 (1 \text{ m}) = 6.28 r_3 \text{ m}^2$$

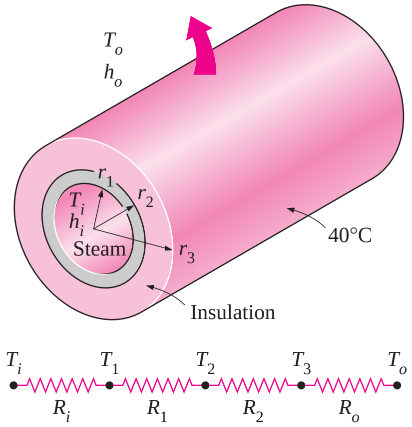


FIGURE 7–40
Schematic for Example 7–8.

Then the individual thermal resistances are determined to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_i A_1} = \frac{1}{(70\ \text{W/m}^2 \cdot {}^\circ\text{C})(0.0503\ \text{m}^2)} = 0.284^\circ\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(0.01/0.008)}{2\pi(15\ \text{W/m} \cdot {}^\circ\text{C})(1\ \text{m})} = 0.0024^\circ\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(r_3/0.01)}{2\pi(0.038\ \text{W/m} \cdot {}^\circ\text{C})(1\ \text{m})}$$

$$= 4.188 \ln(r_3/0.01)^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_o A_3} = \frac{1}{(20\ \text{W/m}^2 \cdot {}^\circ\text{C})(6.28 r_3\ \text{m}^2)} = \frac{1}{125.6 r_3}\ {}^\circ\text{C/W}$$

Noting that all resistances are in series, the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_2 + R_0$$
$$= [0.284 + 0.0024 + 4.188 \ln(r_3/0.01) + 1/125.6 r_3]^\circ\text{C/W}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_i - T_o}{R_{\text{total}}} = \frac{(120 - 125)^\circ\text{C}}{[0.284 + 0.0024 + 4.188 \ln(r_3/0.01) + 1/125.6 r_3]^\circ\text{C/W}}$$

Noting that the outer surface temperature of insulation is specified to be 40°C, the rate of heat loss can also be expressed as

$$\dot{Q} = \frac{T_3 - T_o}{R_o} = \frac{(40 - 25)^\circ\text{C}}{(1/125.6 r_3)^\circ\text{C/W}} = 1884 r_3$$

Setting the two relations above equal to each other and solving for r_3 gives $r_3 = 0.0170$ m. Then the minimum thickness of fiberglass insulation required is

$$t = r_3 - r_2 = 0.0170 - 0.0100 = 0.0070\ \text{m} = \mathbf{0.70\ cm}$$

Discussion Insulating the pipe with at least 0.70-cm-thick fiberglass insulation will ensure that the outer surface temperature of the pipe will be at 40°C or below.

EXAMPLE 7–9 Optimum Thickness of Insulation

During a plant visit, you notice that the outer surface of a cylindrical curing oven is very hot, and your measurements indicate that the average temperature of the exposed surface of the oven is 180°F when the surrounding air temperature is 75°F. You suggest to the plant manager that the oven should be insulated, but the manager does not think it is worth the expense. Then you propose to the manager to pay for the insulation yourself if he lets you keep the savings from the fuel bill for one year. That is, if the fuel bill is $5000/yr before insulation and drops to $2000/yr after insulation, you will get paid $3000. The manager agrees since he has nothing to lose, and a lot to gain. Is this a smart bet on your part?

The oven is 12 ft long and 8 ft in diameter, as shown in Figure 7–41. The plant operates 16 h a day 365 days a year, and thus 5840 h/yr. The insulation

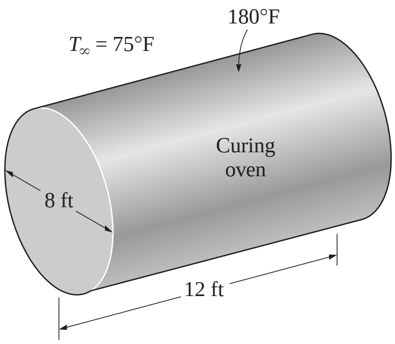


FIGURE 7–41
Schematic for Example 7–9.

to be used is fiberglass (k_{ins} = 0.024 Btu/h · ft · °F), whose cost is \$0.70/ft^2 per inch of thickness for materials, plus \$2.00/ft^2 for labor regardless of thickness. The combined heat transfer coefficient on the outer surface is estimated to be h_o = 3.5 Btu/h · ft^2 · °F. The oven uses natural gas, whose unit cost is \$0.75/therm input (1 therm = 100,000 Btu), and the efficiency of the oven is 80 percent. Disregarding any inflation or interest, determine how much money you will make out of this venture, if any, and the thickness of insulation (in whole inches) that will maximize your earnings.

SOLUTION A cylindrical oven is to be insulated to reduce heat losses. The optimum thickness of insulation and the potential earnings are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the insulation is one-dimensional. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** The surfaces of the cylindrical oven can be treated as plain surfaces since its diameter is greater than 3 ft.

Properties The thermal conductivity of insulation is given to be k = 0.024 Btu/h · ft · °F.

Analysis The exposed surface area of the oven is

$$A_s = 2A_{\text{base}} + A_{\text{side}} = 2\pi r^2 + 2\pi rL = 2\pi(4\text{ ft})^2 + 2\pi(4\text{ ft})(12\text{ ft}) = 402\text{ ft}^2$$

The rate of heat loss from the oven before the insulation is installed is determined from

$$\dot{Q} = h_o A_s(T_s - T_\infty) = (3.5\text{ Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(402\text{ ft}^2)(180 - 75)^\circ\text{F} = 147{,}700\text{ Btu/h}$$

Noting that the plant operates 5840 h/yr, the total amount of heat loss from the oven per year is

$$Q = \dot{Q}\Delta t = (147{,}700\text{ Btu/h})(5840\text{ h/yr}) = 0.863 \times 10^9\text{ Btu/yr}$$

The efficiency of the oven is given to be 80 percent. Therefore, to generate this much heat, the oven must consume energy (in the form of natural gas) at a rate of

$$\begin{aligned} Q_{\text{in}} &= Q/\eta_{\text{oven}} = (0.863 \times 10^9\text{ Btu/yr})/0.80 = 1.079 \times 10^9\text{ Btu/yr} \\ &= 10{,}790\text{ therms} \end{aligned}$$

since 1 therm = 100,000 Btu. Then the annual fuel cost of this oven before insulation becomes

$$\begin{aligned} \text{Annual cost} &= Q_{\text{in}} \times \text{Unit cost} \\ &= (10{,}790\text{ therm/yr})(\$0.75\text{/therm}) = \$8093\text{/yr} \end{aligned}$$

That is, the heat losses from the exposed surfaces of the oven are currently costing the plant over \$8000/yr.

When insulation is installed, the rate of heat transfer from the oven can be determined from

$$\dot{Q}_{\text{ins}} = \frac{T_s - T_\infty}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = A_s \frac{T_s - T_\infty}{\dfrac{t_{\text{ins}}}{k_{\text{ins}}} + \dfrac{1}{h_o}}$$

We expect the surface temperature of the oven to increase and the heat transfer coefficient to decrease somewhat when insulation is installed. We assume these two effects to counteract each other. Then the relation above for 1-in.-thick insulation gives the rate of heat loss to be

$$\dot{Q}_{ins} = \frac{A_s(T_s - T_\infty)}{\frac{t_{ins}}{k_{ins}} + \frac{1}{h_o}} = \frac{(402\text{ ft}^2)(180 - 75)°\text{F}}{\frac{1/12\text{ ft}}{0.024\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}} + \frac{1}{3.5\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F}}}$$
$$= 11{,}230\text{ Btu/h}$$

Also, the total amount of heat loss from the oven per year and the amount and cost of energy consumption of the oven become

$$Q_{ins} = \dot{Q}_{ins}\,\Delta t = (11{,}230\text{ Btu/h})(5840\text{ h/yr}) = 0.6558 \times 10^8\text{ Btu/yr}$$

$$Q_{in,\,ins} = Q_{ins}/\eta_{oven} = (0.6558 \times 10^8\text{ Btu/yr})/0.80 = 0.820 \times 10^8\text{ Btu/yr}$$
$$= 820\text{ therms}$$

$$\text{Annual cost} = Q_{in,\,ins} \times \text{Unit cost}$$
$$= (820\text{ therm/yr})(\$0.75\text{/therm}) = \$615\text{/yr}$$

Therefore, insulating the oven by 1-in.-thick fiberglass insulation will reduce the fuel bill by \$8093 − \$615 = \$7362 per year. The unit cost of insulation is given to be \$2.70/ft^2. Then the installation cost of insulation becomes

$$\text{Insulation cost} = (\text{Unit cost})(\text{Surface area}) = (\$2.70\text{/ft}^2)(402\text{ ft}^2) = \$1085$$

The sum of the insulation and heat loss costs is

$$\text{Total cost} = \text{Insulation cost} + \text{Heat loss cost} = \$1085 + \$615 = \$1700$$

Then the net earnings will be

$$\text{Earnings} = \text{Income} - \text{Expenses} = \$8093 - \$1700 = \$6393$$

To determine the thickness of insulation that maximizes your earnings, we repeat the calculations above for 2-, 3-, 4-, and 5-in.-thick insulations, and list the results in Table 7–5. Note that the total cost of insulation decreases first with increasing insulation thickness, reaches a minimum, and then starts to increase.

TABLE 7–5

The variation of total insulation cost with insulation thickness

Insulation thickness	Heat loss, Btu/h	Lost fuel, therms/yr	Lost fuel cost, \$/yr	Insulation cost, \$	Total cost, \$
1 in.	11,230	820	615	1085	1700
2 in.	5838	426	320	1367	1687
3 in.	3944	288	216	1648	1864
4 in.	2978	217	163	1930	2093
5 in.	2392	175	131	2211	2342

We observe that the total insulation cost is a minimum at \$1687 for the case of **2-in.-thick** insulation. The earnings in this case are

$$\text{Maximum earnings} = \text{Income} - \text{Minimum expenses}$$
$$= \$8093 - \$1687 = \mathbf{\$6406}$$

which is not bad for a day's worth of work. The plant manager is also a big winner in this venture since the heat losses will cost him only \$320/yr during the second and consequent years instead of \$8093/yr. A thicker insulation could probably be justified in this case if the cost of insulation is annualized over the lifetime of insulation, say 20 years. Several energy conservation measures are being marketed as explained above by several power companies and private firms.

SUMMARY

The force a flowing fluid exerts on a body in the flow direction is called *drag*. The part of drag that is due directly to wall shear stress τ_w is called the *skin friction drag* since it is caused by frictional effects, and the part that is due directly to pressure is called the *pressure drag* or *form drag* because of its strong dependence on the form or shape of the body.

The *drag coefficient* C_D is a dimensionless number that represents the drag characteristics of a body, and is defined as

$$C_D = \frac{F_D}{\frac{1}{2}\rho \mathcal{V}^2 A}$$

where A is the *frontal area* for blunt bodies, and surface area for parallel flow over flat plates or thin airfoils. For flow over a flat plate, the Reynolds number is

$$\text{Re}_x = \frac{\rho \mathcal{V} x}{\mu} = \frac{\mathcal{V} x}{\nu}$$

Transition from laminar to turbulent occurs at the *critical Reynolds* number of

$$\text{Re}_{x,\,\text{cr}} = \frac{\rho \mathcal{V} x_{\text{cr}}}{\mu} = 5 \times 10^5$$

For parallel flow over a flat plate, the local friction and convection coefficients are

Laminar: $C_{f,x} = \dfrac{0.664}{\text{Re}_x^{1/2}}$ $\quad \text{Re}_x < 5 \times 10^5$

$$\text{Nu}_x = \frac{h_x x}{k} = 0.332\, \text{Re}_x^{0.5}\, \text{Pr}^{1/3} \qquad \text{Pr} > 0.6$$

Turbulent: $C_{f,x} = \dfrac{0.0592}{\text{Re}_x^{1/5}}, \quad 5 \times 10^5 \le \text{Re}_x \le 10^7$

$$\text{Nu}_x = \frac{h_x x}{k} = 0.0296\, \text{Re}_x^{0.8}\, \text{Pr}^{1/3} \qquad \begin{matrix} 0.6 \le \text{Pr} \le 60 \\ 5 \times 10^5 \le \text{Re}_x \le 10^7 \end{matrix}$$

The *average* friction coefficient relations for flow over a flat plate are:

Laminar: $C_f = \dfrac{1.328}{\text{Re}_L^{1/2}} \quad \text{Re}_L < 5 \times 10^5$

Turbulent: $C_f = \dfrac{0.074}{\text{Re}_L^{1/5}} \quad 5 \times 10^5 \le \text{Re}_L \le 10^7$

Combined: $C_f = \dfrac{0.074}{\text{Re}_L^{1/5}} - \dfrac{1742}{\text{Re}_L} \quad 5 \times 10^5 \le \text{Re}_L \le 10^7$

Rough surface, turbulent: $C_f = \left(1.89 - 1.62 \log \dfrac{\varepsilon}{L}\right)^{-2.5}$

The average Nusselt number relations for flow over a flat plate are:

Laminar: $\text{Nu} = \dfrac{hL}{k} = 0.664\, \text{Re}_L^{0.5}\, \text{Pr}^{1/3} \quad \text{Re}_L < 5 \times 10^5$

Turbulent:

$$\text{Nu} = \frac{hL}{k} = 0.037\, \text{Re}_L^{0.8}\, \text{Pr}^{1/3} \qquad \begin{matrix} 0.6 \le \text{Pr} \le 60 \\ 5 \times 10^5 \le \text{Re}_L \le 10^7 \end{matrix}$$

Combined:

$$\text{Nu} = \frac{hL}{k} = (0.037\, \text{Re}_L^{0.8} - 871)\, \text{Pr}^{1/3}, \qquad \begin{matrix} 0.6 \le \text{Pr} \le 60 \\ 5 \times 10^5 \le \text{Re}_L \le 10^7 \end{matrix}$$

For isothermal surfaces with an unheated starting section of length ξ, the local Nusselt number and the average convection coefficient relations are

Laminar: $\text{Nu}_x = \dfrac{\text{Nu}_{x\,(\text{for } \xi = 0)}}{[1 - (\xi/x)^{3/4}]^{1/3}} = \dfrac{0.332\, \text{Re}_x^{0.5}\, \text{Pr}^{1/3}}{[1 - (\xi/x)^{3/4}]^{1/3}}$

Turbulent: $\text{Nu}_x = \dfrac{\text{Nu}_{x\,(\text{for } \xi = 0)}}{[1 - (\xi/x)^{9/10}]^{1/9}} = \dfrac{0.0296\, \text{Re}_x^{0.8}\, \text{Pr}^{1/3}}{[1 - (\xi/x)^{9/10}]^{1/9}}$

Laminar: $h = \dfrac{2[1 - (\xi/x)^{3/4}]}{1 - \xi/L}\, h_{x=L}$

Turbulent: $h = \dfrac{5[1 - (\xi/x)^{9/10}]}{(1 - \xi/L)}\, h_{x=L}$