

STEADY HEAT CONDUCTION

In heat transfer analysis, we are often interested in the rate of heat transfer through a medium under steady conditions and surface temperatures. Such problems can be solved easily without involving any differential equations by the introduction of *thermal resistance concepts* in an analogous manner to electrical circuit problems. In this case, the thermal resistance corresponds to electrical resistance, temperature difference corresponds to voltage, and the heat transfer rate corresponds to electric current.

We start this chapter with *one-dimensional steady heat conduction* in a plane wall, a cylinder, and a sphere, and develop relations for *thermal resistances* in these geometries. We also develop thermal resistance relations for convection and radiation conditions at the boundaries. We apply this concept to heat conduction problems in *multilayer* plane walls, cylinders, and spheres and generalize it to systems that involve heat transfer in two or three dimensions. We also discuss the *thermal contact resistance* and the *overall heat transfer coefficient* and develop relations for the critical radius of insulation for a cylinder and a sphere. Finally, we discuss steady heat transfer from *finned surfaces* and some complex geometries commonly encountered in practice through the use of *conduction shape factors*.



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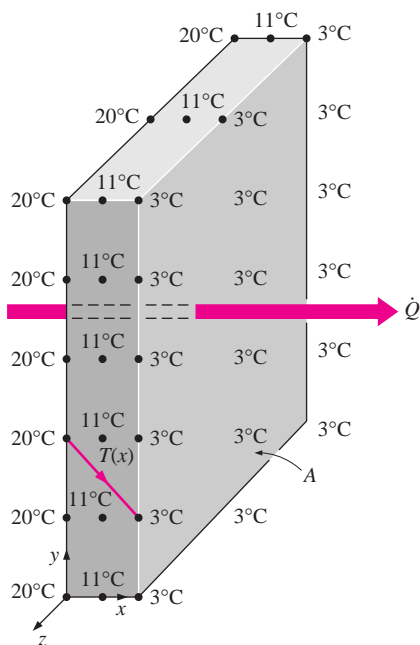


FIGURE 3-1
Heat flow through a wall is one-dimensional when the temperature of the wall varies in one direction only.

3-1 ■ STEADY HEAT CONDUCTION IN PLANE WALLS

Consider steady heat conduction through the walls of a house during a winter day. We know that heat is continuously lost to the outdoors through the wall. We intuitively feel that heat transfer through the wall is in the *normal direction* to the wall surface, and no significant heat transfer takes place in the wall in other directions (Fig. 3–1).

Recall that heat transfer in a certain direction is driven by the *temperature gradient* in that direction. There will be no heat transfer in a direction in which there is no change in temperature. Temperature measurements at several locations on the inner or outer wall surface will confirm that a wall surface is nearly *isothermal*. That is, the temperatures at the top and bottom of a wall surface as well as at the right or left ends are almost the same. Therefore, there will be no heat transfer through the wall from the top to the bottom, or from left to right, but there will be considerable temperature difference between the inner and the outer surfaces of the wall, and thus significant heat transfer in the direction from the inner surface to the outer one.

The small thickness of the wall causes the temperature gradient in that direction to be large. Further, if the air temperatures in and outside the house remain constant, then heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional*. The temperature of the wall in this case will depend on one direction only (say the *x*-direction) and can be expressed as $T(x)$.

Noting that heat transfer is the only energy interaction involved in this case and there is no heat generation, the *energy balance* for the wall can be expressed as

$$\left(\begin{array}{c} \text{Rate of} \\ \text{heat transfer} \\ \text{into the wall} \end{array} \right) - \left(\begin{array}{c} \text{Rate of} \\ \text{heat transfer} \\ \text{out of the wall} \end{array} \right) = \left(\begin{array}{c} \text{Rate of change} \\ \text{of the energy} \\ \text{of the wall} \end{array} \right)$$

or

$$\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \frac{dE_{\text{wall}}}{dt} \quad (3-1)$$

But $dE_{\text{wall}}/dt = 0$ for *steady* operation, since there is no change in the temperature of the wall with time at any point. Therefore, the rate of heat transfer into the wall must be equal to the rate of heat transfer out of it. In other words, *the rate of heat transfer through the wall must be constant*, $\dot{Q}_{\text{cond, wall}} = \text{constant}$.

Consider a plane wall of thickness L and average thermal conductivity k . The two surfaces of the wall are maintained at constant temperatures of T_1 and T_2 . For one-dimensional steady heat conduction through the wall, we have $T(x)$. Then Fourier's law of heat conduction for the wall can be expressed as

$$\dot{Q}_{\text{cond, wall}} = -kA \frac{dT}{dx} \quad (\text{W}) \quad (3-2)$$

where the rate of conduction heat transfer $\dot{Q}_{\text{cond, wall}}$ and the wall area A are constant. Thus we have $dT/dx = \text{constant}$, which means that *the temperature*

through the wall varies linearly with x . That is, the temperature distribution in the wall under steady conditions is a *straight line* (Fig. 3–2).

Separating the variables in the above equation and integrating from $x = 0$, where $T(0) = T_1$, to $x = L$, where $T(L) = T_2$, we get

$$\int_{x=0}^L \dot{Q}_{\text{cond, wall}} dx = - \int_{T=T_1}^{T_2} kA dT$$

Performing the integrations and rearranging gives

$$\dot{Q}_{\text{cond, wall}} = kA \frac{T_1 - T_2}{L} \quad (\text{W}) \quad (3-3)$$

which is identical to Eq. 3–1. Again, *the rate of heat conduction through a plane wall is proportional to the average thermal conductivity, the wall area, and the temperature difference, but is inversely proportional to the wall thickness*. Also, once the rate of heat conduction is available, the temperature $T(x)$ at any location x can be determined by replacing T_2 in Eq. 3–3 by T , and L by x .

The Thermal Resistance Concept

Equation 3–3 for heat conduction through a plane wall can be rearranged as

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \quad (\text{W}) \quad (3-4)$$

where

$$R_{\text{wall}} = \frac{L}{kA} \quad (^\circ\text{C}/\text{W}) \quad (3-5)$$

is the *thermal resistance* of the wall against heat conduction or simply the **conduction resistance** of the wall. Note that the thermal resistance of a medium depends on the *geometry* and the *thermal properties* of the medium.

The equation above for heat flow is analogous to the relation for *electric current flow* I , expressed as

$$I = \frac{V_1 - V_2}{R_e} \quad (3-6)$$

where $R_e = L/\sigma_e A$ is the *electric resistance* and $V_1 - V_2$ is the *voltage difference* across the resistance (σ_e is the electrical conductivity). Thus, the *rate of heat transfer* through a layer corresponds to the *electric current*, the *thermal resistance* corresponds to *electrical resistance*, and the *temperature difference* corresponds to *voltage difference* across the layer (Fig. 3–3).

Consider convection heat transfer from a solid surface of area A_s and temperature T_s to a fluid whose temperature sufficiently far from the surface is T_∞ , with a convection heat transfer coefficient h . Newton's law of cooling for convection heat transfer rate $\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$ can be rearranged as

$$\dot{Q}_{\text{conv}} = \frac{T_s - T_\infty}{R_{\text{conv}}} \quad (\text{W}) \quad (3-7)$$

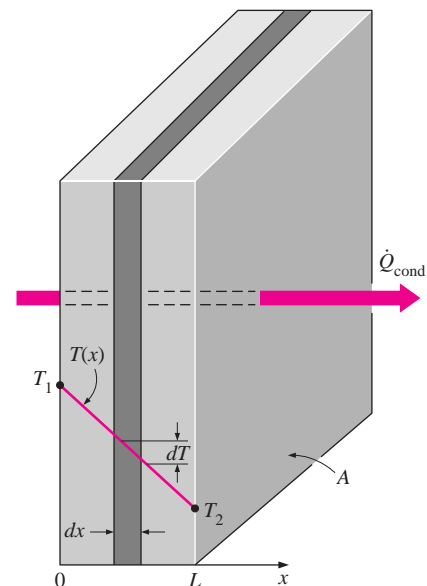


FIGURE 3–2

Under steady conditions, the temperature distribution in a plane wall is a straight line.

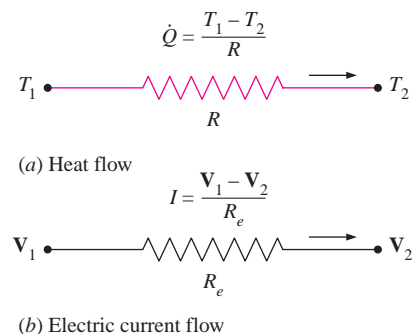


FIGURE 3–3

Analogy between thermal and electrical resistance concepts.

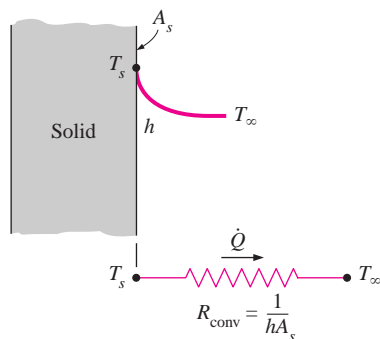


FIGURE 3-4
Schematic for convection
resistance at a surface.

where

$$R_{\text{conv}} = \frac{1}{hA_s} \quad (^\circ\text{C}/\text{W}) \quad (3-8)$$

is the *thermal resistance* of the surface against heat convection, or simply the **convection resistance** of the surface (Fig. 3–4). Note that when the convection heat transfer coefficient is very large ($h \rightarrow \infty$), the convection resistance becomes *zero* and $T_s \approx T_\infty$. That is, the surface offers *no resistance to convection*, and thus it does not slow down the heat transfer process. This situation is approached in practice at surfaces where boiling and condensation occur. Also note that the surface does not have to be a plane surface. Equation 3–8 for convection resistance is valid for surfaces of any shape, provided that the assumption of $h = \text{constant}$ and uniform is reasonable.

When the wall is surrounded by a gas, the *radiation effects*, which we have ignored so far, can be significant and may need to be considered. The rate of radiation heat transfer between a surface of emissivity ε and area A_s at temperature T_s and the surrounding surfaces at some average temperature T_{surr} can be expressed as

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4) = h_{\text{rad}}A_s(T_s - T_{\text{surr}}) = \frac{T_s - T_{\text{surr}}}{R_{\text{rad}}} \quad (\text{W}) \quad (3-9)$$

where

$$R_{\text{rad}} = \frac{1}{h_{\text{rad}}A_s} \quad (\text{K}/\text{W}) \quad (3-10)$$

is the *thermal resistance* of a surface against radiation, or the *radiation resistance*, and

$$h_{\text{rad}} = \frac{\dot{Q}_{\text{rad}}}{A_s(T_s - T_{\text{surr}})} = \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}}) \quad (\text{W}/\text{m}^2 \cdot \text{K}) \quad (3-11)$$

is the **radiation heat transfer coefficient**. Note that both T_s and T_{surr} *must* be in K in the evaluation of h_{rad} . The definition of the radiation heat transfer coefficient enables us to express radiation conveniently in an analogous manner to convection in terms of a temperature difference. But h_{rad} depends strongly on temperature while h_{conv} usually does not.

A surface exposed to the surrounding air involves convection and radiation simultaneously, and the total heat transfer at the surface is determined by adding (or subtracting, if in the opposite direction) the radiation and convection components. The convection and radiation resistances are parallel to each other, as shown in Fig. 3–5, and may cause some complication in the thermal resistance network. When $T_{\text{surr}} \approx T_\infty$, the radiation effect can properly be accounted for by replacing h in the convection resistance relation by

$$h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}} \quad (\text{W}/\text{m}^2 \cdot \text{K}) \quad (3-12)$$

where h_{combined} is the **combined heat transfer coefficient**. This way all the complications associated with radiation are avoided.

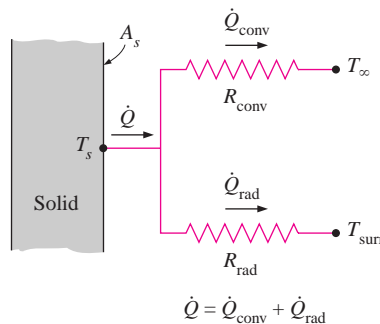


FIGURE 3-5
Schematic for convection and
radiation resistances at a surface.

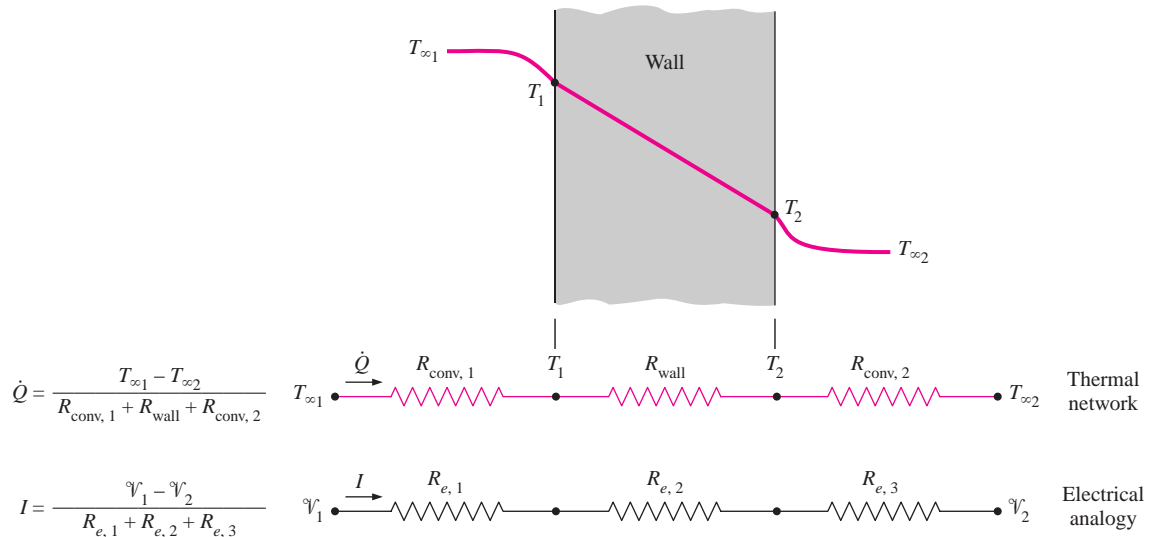


FIGURE 3-6

The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

Thermal Resistance Network

Now consider steady one-dimensional heat flow through a plane wall of thickness L , area A , and thermal conductivity k that is exposed to convection on both sides to fluids at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ with heat transfer coefficients h_1 and h_2 , respectively, as shown in Fig. 3-6. Assuming $T_{\infty 2} < T_{\infty 1}$, the variation of temperature will be as shown in the figure. Note that the temperature varies linearly in the wall, and asymptotically approaches $T_{\infty 1}$ and $T_{\infty 2}$ in the fluids as we move away from the wall.

Under steady conditions we have

$$\left(\begin{array}{c} \text{Rate of} \\ \text{heat convection} \\ \text{into the wall} \end{array} \right) = \left(\begin{array}{c} \text{Rate of} \\ \text{heat conduction} \\ \text{through the wall} \end{array} \right) = \left(\begin{array}{c} \text{Rate of} \\ \text{heat convection} \\ \text{from the wall} \end{array} \right)$$

or

$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA \frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2}) \quad (3-13)$$

which can be rearranged as

$$\begin{aligned} \dot{Q} &= \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A} \\ &= \frac{T_{\infty 1} - T_1}{R_{\text{conv}, 1}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv}, 2}} \end{aligned} \quad (3-14)$$

Adding the numerators and denominators yields (Fig. 3-7)

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \quad (\text{W}) \quad (3-15)$$

If $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \dots = \frac{a_n}{b_n} = c$

then $\frac{a_1 + a_2 + \dots + a_n}{b_1 + b_2 + \dots + b_n} = c$

For example,

$$\frac{1}{4} = \frac{2}{8} = \frac{5}{20} = 0.25$$

and

$$\frac{1 + 2 + 5}{4 + 8 + 20} = 0.25$$

FIGURE 3-7

A useful mathematical identity.

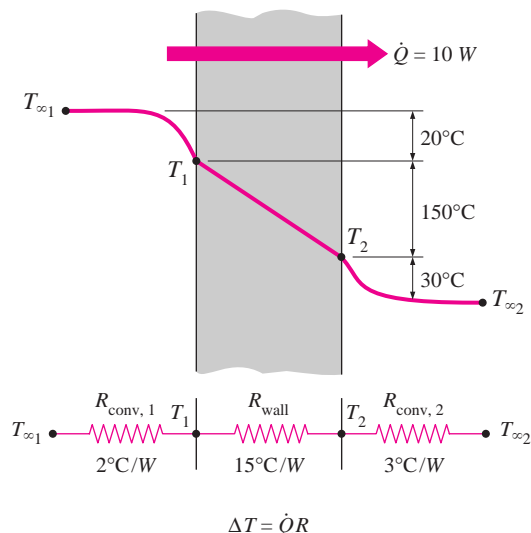


FIGURE 3–8

The temperature drop across a layer is proportional to its thermal resistance.

where

$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{wall}} + R_{\text{conv},2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \quad (\text{°C/W}) \quad (3-16)$$

Note that the heat transfer area A is constant for a plane wall, and the rate of heat transfer through a wall separating two mediums is equal to the temperature difference divided by the total thermal resistance between the mediums. Also note that the thermal resistances are in *series*, and the equivalent thermal resistance is determined by simply *adding* the individual resistances, just like the electrical resistances connected in series. Thus, the electrical analogy still applies. We summarize this as *the rate of steady heat transfer between two surfaces is equal to the temperature difference divided by the total thermal resistance between those two surfaces*.

Another observation that can be made from Eq. 3–15 is that the ratio of the temperature drop to the thermal resistance across any layer is constant, and thus the temperature drop across any layer is proportional to the thermal resistance of the layer. The larger the resistance, the larger the temperature drop. In fact, the equation $\dot{Q} = \Delta T/R$ can be rearranged as

$$\Delta T = \dot{Q}R \quad (\text{°C}) \quad (3-17)$$

which indicates that the *temperature drop* across any layer is equal to the *rate of heat transfer* times the *thermal resistance* across that layer (Fig. 3–8). You may recall that this is also true for voltage drop across an electrical resistance when the electric current is constant.

It is sometimes convenient to express heat transfer through a medium in an analogous manner to Newton's law of cooling as

$$\dot{Q} = UA \Delta T \quad (\text{W}) \quad (3-18)$$

where U is the **overall heat transfer coefficient**. A comparison of Eqs. 3–15 and 3–18 reveals that

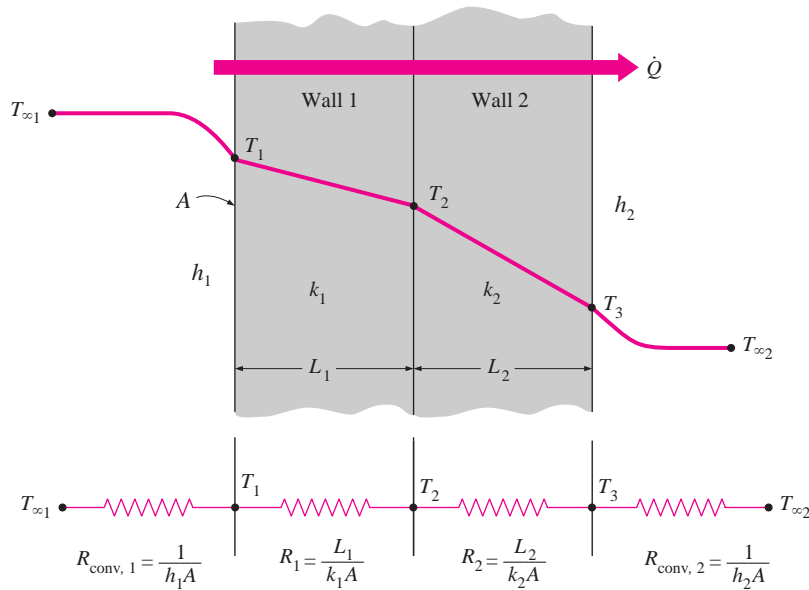


FIGURE 3-9
The thermal resistance network for heat transfer through a two-layer plane wall subjected to convection on both sides.

$$UA = \frac{1}{R_{\text{total}}} \quad (3-19)$$

Therefore, for a unit area, the overall heat transfer coefficient is equal to the inverse of the total thermal resistance.

Note that we do not need to know the surface temperatures of the wall in order to evaluate the rate of steady heat transfer through it. All we need to know is the convection heat transfer coefficients and the fluid temperatures on both sides of the wall. The *surface temperature* of the wall can be determined as described above using the thermal resistance concept, but by taking the surface at which the temperature is to be determined as one of the terminal surfaces. For example, once \dot{Q} is evaluated, the surface temperature T_1 can be determined from

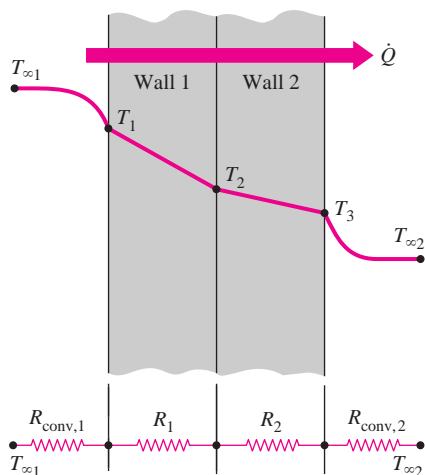
$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv}, 1}} = \frac{T_{\infty 1} - T_1}{1/h_1 A} \quad (3-20)$$

Multilayer Plane Walls

In practice we often encounter plane walls that consist of several layers of different materials. The thermal resistance concept can still be used to determine the rate of steady heat transfer through such *composite* walls. As you may have already guessed, this is done by simply noting that the conduction resistance of each wall is L/kA connected in series, and using the electrical analogy. That is, by dividing the *temperature difference* between two surfaces at known temperatures by the *total thermal resistance* between them.

Consider a plane wall that consists of two layers (such as a brick wall with a layer of insulation). The rate of steady heat transfer through this two-layer composite wall can be expressed as (Fig. 3-9)

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \quad (3-21)$$



$$\text{To find } T_1: \dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$$

$$\text{To find } T_2: \dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1}$$

$$\text{To find } T_3: \dot{Q} = \frac{T_3 - T_{\infty 2}}{R_{\text{conv},2}}$$

FIGURE 3-10

The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and \dot{Q} is calculated.

where R_{total} is the *total thermal resistance*, expressed as

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv},1} + R_{\text{wall},1} + R_{\text{wall},2} + R_{\text{conv},2} \\ &= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A} \end{aligned} \quad (3-22)$$

The subscripts 1 and 2 in the R_{wall} relations above indicate the first and the second layers, respectively. We could also obtain this result by following the approach used above for the single-layer case by noting that the rate of steady heat transfer \dot{Q} through a multilayer medium is constant, and thus it must be the same through each layer. Note from the thermal resistance network that the resistances are *in series*, and thus the *total thermal resistance* is simply the *arithmetic sum* of the individual thermal resistances in the path of heat flow.

This result for the *two-layer* case is analogous to the *single-layer* case, except that an *additional resistance* is added for the *additional layer*. This result can be extended to plane walls that consist of *three or more layers* by adding an *additional resistance* for each *additional layer*.

Once \dot{Q} is *known*, an unknown surface temperature T_j at any surface or interface j can be determined from

$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total},i-j}} \quad (3-23)$$

where T_i is a *known* temperature at location i and $R_{\text{total},i-j}$ is the total thermal resistance between locations i and j . For example, when the fluid temperatures $T_{\infty 1}$ and $T_{\infty 2}$ for the two-layer case shown in Fig. 3-9 are available and \dot{Q} is calculated from Eq. 3-21, the interface temperature T_2 between the two walls can be determined from (Fig. 3-10)

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_{\text{wall},1}} = \frac{T_{\infty 1} - T_2}{\frac{1}{h_1 A} + \frac{L_1}{k_1 A}} \quad (3-24)$$

The temperature drop across a layer is easily determined from Eq. 3-17 by multiplying \dot{Q} by the thermal resistance of that layer.

The thermal resistance concept is widely used in practice because it is intuitively easy to understand and it has proven to be a powerful tool in the solution of a wide range of heat transfer problems. But its use is limited to systems through which the rate of heat transfer \dot{Q} remains *constant*; that is, to systems involving *steady* heat transfer with *no heat generation* (such as resistance heating or chemical reactions) within the medium.

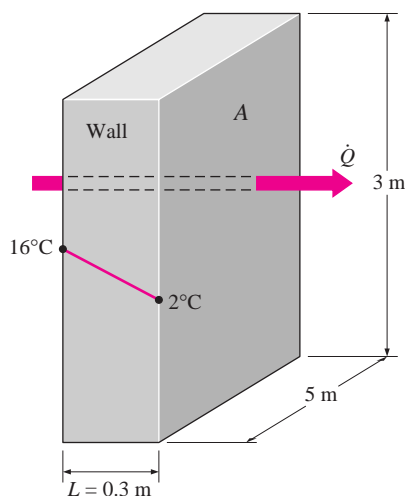


FIGURE 3-11

Schematic for Example 3-1.

EXAMPLE 3-1 Heat Loss through a Wall

Consider a 3-m-high, 5-m-wide, and 0.3-m-thick wall whose thermal conductivity is $k = 0.9 \text{ W/m} \cdot ^\circ\text{C}$ (Fig. 3-11). On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 16°C and 2°C , respectively. Determine the rate of heat loss through the wall on that day.

SOLUTION The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.

Assumptions **1** Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.9 \text{ W/m} \cdot ^\circ\text{C}$.

Analysis Noting that the heat transfer through the wall is by conduction and the area of the wall is $A = 3 \text{ m} \times 5 \text{ m} = 15 \text{ m}^2$, the steady rate of heat transfer through the wall can be determined from Eq. 3–3 to be

$$\dot{Q} = kA \frac{T_1 - T_2}{L} = (0.9 \text{ W/m} \cdot ^\circ\text{C})(15 \text{ m}^2) \frac{(16 - 2)^\circ\text{C}}{0.3 \text{ m}} = \mathbf{630 \text{ W}}$$

We could also determine the steady rate of heat transfer through the wall by making use of the thermal resistance concept from

$$\dot{Q} = \frac{\Delta T_{\text{wall}}}{R_{\text{wall}}}$$

where

$$R_{\text{wall}} = \frac{L}{kA} = \frac{0.3 \text{ m}}{(0.9 \text{ W/m} \cdot ^\circ\text{C})(15 \text{ m}^2)} = 0.02222^\circ\text{C/W}$$

Substituting, we get

$$\dot{Q} = \frac{(16 - 2)^\circ\text{C}}{0.02222^\circ\text{C/W}} = 630 \text{ W}$$

Discussion This is the same result obtained earlier. Note that heat conduction through a plane wall with specified surface temperatures can be determined directly and easily without utilizing the thermal resistance concept. However, the thermal resistance concept serves as a valuable tool in more complex heat transfer problems, as you will see in the following examples.

EXAMPLE 3–2 Heat Loss through a Single-Pane Window

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of $k = 0.78 \text{ W/m} \cdot ^\circ\text{C}$. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is -10°C . Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$ and $h_2 = 40 \text{ W/m}^2 \cdot ^\circ\text{C}$, which includes the effects of radiation.

SOLUTION Heat loss through a window glass is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

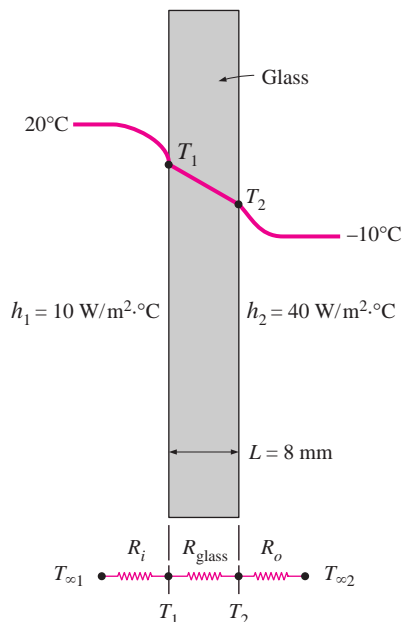


FIGURE 3-12

Schematic for Example 3-2.

Assumptions 1 Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. 2 Heat transfer through the wall is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. 3 Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.78 \text{ W/m} \cdot ^\circ\text{C}$.

Analysis This problem involves conduction through the glass window and convection at its surfaces, and can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in Fig. 3-12. Noting that the area of the window is $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv},1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.08333^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L}{kA} = \frac{0.008 \text{ m}}{(0.78 \text{ W/m} \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.00855^\circ\text{C/W}$$

$$R_o = R_{\text{conv},2} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2 \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.02083^\circ\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$R_{\text{total}} = R_{\text{conv},1} + R_{\text{glass}} + R_{\text{conv},2} = 0.08333 + 0.00855 + 0.02083 = 0.1127^\circ\text{C/W}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{0.1127^\circ\text{C/W}} = \mathbf{266 \text{ W}}$$

Knowing the rate of heat transfer, the inner surface temperature of the window glass can be determined from

$$\begin{aligned} \dot{Q} &= \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}} \quad \longrightarrow \quad T_1 = T_{\infty 1} - \dot{Q}R_{\text{conv},1} \\ &= 20^\circ\text{C} - (266 \text{ W})(0.08333^\circ\text{C/W}) \\ &= \mathbf{-2.2^\circ\text{C}} \end{aligned}$$

Discussion Note that the inner surface temperature of the window glass will be -2.2°C even though the temperature of the air in the room is maintained at 20°C . Such low surface temperatures are highly undesirable since they cause the formation of fog or even frosts on the inner surfaces of the glass when the humidity in the room is high.

EXAMPLE 3-3 Heat Loss through Double-Pane Windows

Consider a 0.8-m-high and 1.5-m-wide double-pane window consisting of two 4-mm-thick layers of glass ($k = 0.78 \text{ W/m} \cdot ^\circ\text{C}$) separated by a 10-mm-wide stagnant air space ($k = 0.026 \text{ W/m} \cdot ^\circ\text{C}$). Determine the steady rate of heat

transfer through this double-pane window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is -10°C . Take the convection heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$ and $h_2 = 40 \text{ W/m}^2 \cdot ^\circ\text{C}$, which includes the effects of radiation.

SOLUTION A double-pane window is considered. The rate of heat transfer through the window and the inner surface temperature are to be determined.

Analysis This example problem is identical to the previous one except that the single 8-mm-thick window glass is replaced by two 4-mm-thick glasses that enclose a 10-mm-wide stagnant air space. Therefore, the thermal resistance network of this problem will involve two additional conduction resistances corresponding to the two additional layers, as shown in Fig. 3–13. Noting that the area of the window is again $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv},1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.08333^\circ\text{C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.004 \text{ m}}{(0.78 \text{ W/m} \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.00427^\circ\text{C/W}$$

$$R_2 = R_{\text{air}} = \frac{L_2}{k_2 A} = \frac{0.01 \text{ m}}{(0.026 \text{ W/m} \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.3205^\circ\text{C/W}$$

$$R_o = R_{\text{conv},2} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2 \cdot ^\circ\text{C})(1.2 \text{ m}^2)} = 0.02083^\circ\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv},1} + R_{\text{glass},1} + R_{\text{air}} + R_{\text{glass},2} + R_{\text{conv},2} \\ &= 0.08333 + 0.00427 + 0.3205 + 0.00427 + 0.02083 \\ &= 0.4332^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{0.4332^\circ\text{C/W}} = \mathbf{69.2 \text{ W}}$$

which is about one-fourth of the result obtained in the previous example. This explains the popularity of the double- and even triple-pane windows in cold climates. The drastic reduction in the heat transfer rate in this case is due to the large thermal resistance of the air layer between the glasses.

The inner surface temperature of the window in this case will be

$$T_1 = T_{\infty 1} - \dot{Q} R_{\text{conv},1} = 20^\circ\text{C} - (69.2 \text{ W})(0.08333^\circ\text{C/W}) = \mathbf{14.2^\circ\text{C}}$$

which is considerably higher than the -2.2°C obtained in the previous example. Therefore, a double-pane window will rarely get fogged. A double-pane window will also reduce the heat gain in summer, and thus reduce the air-conditioning costs.

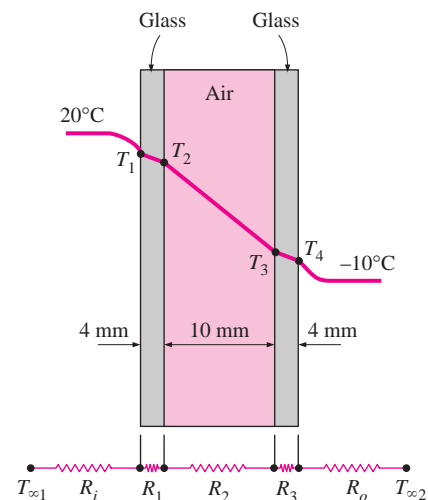


FIGURE 3–13

Schematic for Example 3–3.

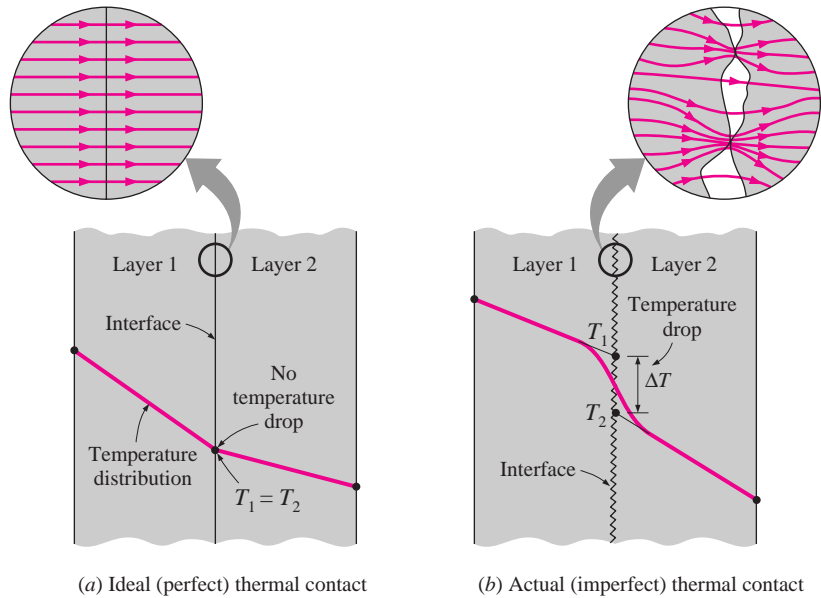


FIGURE 3-14
Temperature distribution and heat flow lines along two solid plates pressed against each other for the case of perfect and imperfect contact.

3-2 ■ THERMAL CONTACT RESISTANCE

In the analysis of heat conduction through multilayer solids, we assumed “perfect contact” at the interface of two layers, and thus no temperature drop at the interface. This would be the case when the surfaces are perfectly smooth and they produce a perfect contact at each point. In reality, however, even flat surfaces that appear smooth to the eye turn out to be rather rough when examined under a microscope, as shown in Fig. 3-14, with numerous peaks and valleys. That is, a surface is *microscopically rough* no matter how smooth it appears to be.

When two such surfaces are pressed against each other, the peaks will form good material contact but the valleys will form voids filled with air. As a result, an interface will contain numerous *air gaps* of varying sizes that act as *insulation* because of the low thermal conductivity of air. Thus, an interface offers some resistance to heat transfer, and this resistance per unit interface area is called the **thermal contact resistance**, R_c . The value of R_c is determined experimentally using a setup like the one shown in Fig. 3-15, and as expected, there is considerable scatter of data because of the difficulty in characterizing the surfaces.

Consider heat transfer through two metal rods of cross-sectional area A that are pressed against each other. Heat transfer through the interface of these two rods is the sum of the heat transfers through the *solid contact spots* and the *gaps* in the noncontact areas and can be expressed as

$$\dot{Q} = \dot{Q}_{\text{contact}} + \dot{Q}_{\text{gap}} \quad (3-25)$$

It can also be expressed in an analogous manner to Newton’s law of cooling as

$$\dot{Q} = h_c A \Delta T_{\text{interface}} \quad (3-26)$$

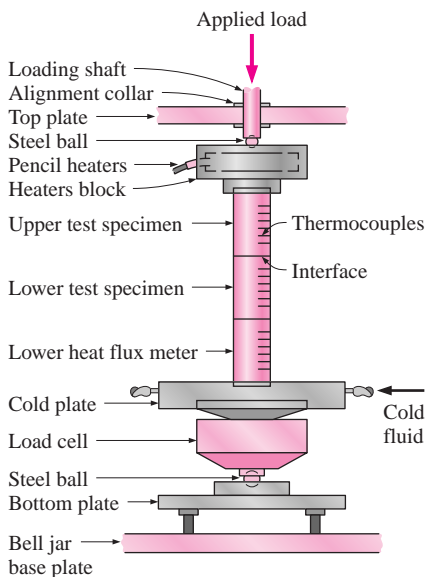


FIGURE 3-15
A typical experimental setup for the determination of thermal contact resistance (from Song et al., Ref. 11).

where A is the apparent interface area (which is the same as the cross-sectional area of the rods) and $\Delta T_{\text{interface}}$ is the effective temperature difference at the interface. The quantity h_c , which corresponds to the convection heat transfer coefficient, is called the **thermal contact conductance** and is expressed as

$$h_c = \frac{\dot{Q}/A}{\Delta T_{\text{interface}}} \quad (\text{W/m}^2 \cdot ^\circ\text{C}) \quad (3-27)$$

It is related to thermal contact resistance by

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{\text{interface}}}{\dot{Q}/A} \quad (\text{m}^2 \cdot ^\circ\text{C/W}) \quad (3-28)$$

That is, thermal contact resistance is the inverse of thermal contact conductance. Usually, thermal contact conductance is reported in the literature, but the concept of thermal contact resistance serves as a better vehicle for explaining the effect of interface on heat transfer. Note that R_c represents thermal contact resistance *per unit area*. The thermal resistance for the entire interface is obtained by dividing R_c by the apparent interface area A .

The thermal contact resistance can be determined from Eq. 3–28 by measuring the temperature drop at the interface and dividing it by the heat flux under steady conditions. The value of thermal contact resistance depends on the *surface roughness* and the *material properties* as well as the *temperature* and *pressure* at the interface and the *type of fluid* trapped at the interface. The situation becomes more complex when plates are fastened by bolts, screws, or rivets since the interface pressure in this case is nonuniform. The thermal contact resistance in that case also depends on the plate thickness, the bolt radius, and the size of the contact zone. Thermal contact resistance is observed to *decrease* with *decreasing surface roughness* and *increasing interface pressure*, as expected. Most experimentally determined values of the thermal contact resistance fall between 0.000005 and 0.0005 $\text{m}^2 \cdot ^\circ\text{C/W}$ (the corresponding range of thermal contact conductance is 2000 to 200,000 $\text{W/m}^2 \cdot ^\circ\text{C}$).

When we analyze heat transfer in a medium consisting of two or more layers, the first thing we need to know is whether the thermal contact resistance is *significant* or not. We can answer this question by comparing the magnitudes of the thermal resistances of the layers with typical values of thermal contact resistance. For example, the thermal resistance of a 1-cm-thick layer of an insulating material per unit surface area is

$$R_{c, \text{insulation}} = \frac{L}{k} = \frac{0.01 \text{ m}}{0.04 \text{ W/m} \cdot ^\circ\text{C}} = 0.25 \text{ m}^2 \cdot ^\circ\text{C/W}$$

whereas for a 1-cm-thick layer of copper, it is

$$R_{c, \text{copper}} = \frac{L}{k} = \frac{0.01 \text{ m}}{386 \text{ W/m} \cdot ^\circ\text{C}} = 0.000026 \text{ m}^2 \cdot ^\circ\text{C/W}$$

Comparing the values above with typical values of thermal contact resistance, we conclude that thermal contact resistance is significant and can even dominate the heat transfer for good heat conductors such as metals, but can be

TABLE 3-1

Thermal contact conductance for aluminum plates with different fluids at the interface for a surface roughness of $10\ \mu\text{m}$ and interface pressure of 1 atm (from Fried, Ref. 5)

Fluid at the Interface	Contact Conductance, h_c , $\text{W/m}^2 \cdot ^\circ\text{C}$
Air	3640
Helium	9520
Hydrogen	13,900
Silicone oil	19,000
Glycerin	37,700

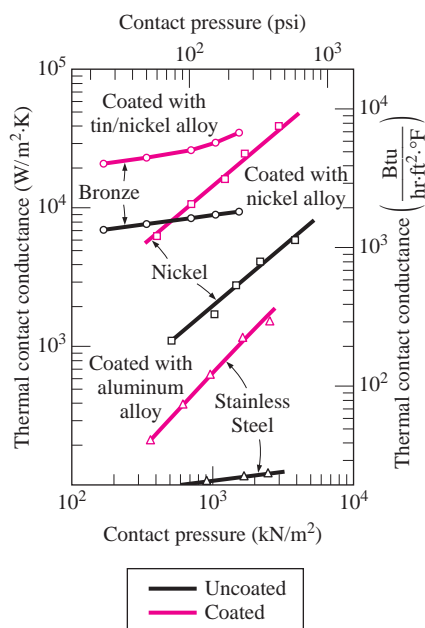


FIGURE 3-16

Effect of metallic coatings on thermal contact conductance (from Peterson, Ref. 10).

disregarded for poor heat conductors such as insulations. This is not surprising since insulating materials consist mostly of air space just like the interface itself.

The thermal contact resistance can be minimized by applying a thermally conducting liquid called a *thermal grease* such as silicon oil on the surfaces before they are pressed against each other. This is commonly done when attaching electronic components such as power transistors to heat sinks. The thermal contact resistance can also be reduced by replacing the air at the interface by a *better conducting gas* such as helium or hydrogen, as shown in Table 3-1.

Another way to minimize the contact resistance is to insert a *soft metallic foil* such as tin, silver, copper, nickel, or aluminum between the two surfaces. Experimental studies show that the thermal contact resistance can be reduced by a factor of up to 7 by a metallic foil at the interface. For maximum effectiveness, the foils must be very thin. The effect of metallic coatings on thermal contact conductance is shown in Fig. 3-16 for various metal surfaces.

There is considerable uncertainty in the contact conductance data reported in the literature, and care should be exercised when using them. In Table 3-2 some experimental results are given for the contact conductance between similar and dissimilar metal surfaces for use in preliminary design calculations. Note that the *thermal contact conductance* is *highest* (and thus the contact resistance is lowest) for *soft metals* with *smooth surfaces* at *high pressure*.

EXAMPLE 3-4 Equivalent Thickness for Contact Resistance

The thermal contact conductance at the interface of two 1-cm-thick aluminum plates is measured to be $11,000\ \text{W/m}^2 \cdot ^\circ\text{C}$. Determine the thickness of the aluminum plate whose thermal resistance is equal to the thermal resistance of the interface between the plates (Fig. 3-17).

SOLUTION The thickness of the aluminum plate whose thermal resistance is equal to the thermal contact resistance is to be determined.

Properties The thermal conductivity of aluminum at room temperature is $k = 237\ \text{W/m} \cdot ^\circ\text{C}$ (Table A-3).

Analysis Noting that thermal contact resistance is the inverse of thermal contact conductance, the thermal contact resistance is

$$R_c = \frac{1}{h_c} = \frac{1}{11,000\ \text{W/m}^2 \cdot ^\circ\text{C}} = 0.909 \times 10^{-4}\ \text{m}^2 \cdot ^\circ\text{C/W}$$

For a unit surface area, the thermal resistance of a flat plate is defined as

$$R = \frac{L}{k}$$

where L is the thickness of the plate and k is the thermal conductivity. Setting $R = R_c$, the equivalent thickness is determined from the relation above to be

$$L = kR_c = (237\ \text{W/m} \cdot ^\circ\text{C})(0.909 \times 10^{-4}\ \text{m}^2 \cdot ^\circ\text{C/W}) = 0.0215\ \text{m} = \mathbf{2.15\ \text{cm}}$$

TABLE 3-2

Thermal contact conductance of some metal surfaces in air (from various sources)

Material	Surface Condition	Roughness, μm	Temperature, $^{\circ}\text{C}$	Pressure, MPa	h_c ,* $\text{W/m}^2 \cdot ^{\circ}\text{C}$
Identical Metal Pairs					
416 Stainless steel	Ground	2.54	90–200	0.3–2.5	3800
304 Stainless steel	Ground	1.14	20	4–7	1900
Aluminum	Ground	2.54	150	1.2–2.5	11,400
Copper	Ground	1.27	20	1.2–20	143,000
Copper	Milled	3.81	20	1–5	55,500
Copper (vacuum)	Milled	0.25	30	0.7–7	11,400
Dissimilar Metal Pairs					
Stainless steel– Aluminum		20–30	20	10 20	2900 3600
Stainless steel– Aluminum		1.0–2.0	20	10 20	16,400 20,800
Steel Ct-30– Aluminum	Ground	1.4–2.0	20	10 15–35	50,000 59,000
Steel Ct-30– Aluminum	Milled	4.5–7.2	20	10 30	4800 8300
Aluminum–Copper	Ground	1.3–1.4	20	5 15	42,000 56,000
Aluminum–Copper	Milled	4.4–4.5	20	10 20–35	12,000 22,000

*Divide the given values by 5.678 to convert to $\text{Btu/h} \cdot \text{ft}^2 \cdot ^{\circ}\text{F}$.

Discussion Note that the interface between the two plates offers as much resistance to heat transfer as a 2.3-cm-thick aluminum plate. It is interesting that the thermal contact resistance in this case is greater than the sum of the thermal resistances of both plates.

EXAMPLE 3-5 Contact Resistance of Transistors

Four identical power transistors with aluminum casing are attached on one side of a 1-cm-thick 20-cm \times 20-cm square copper plate ($k = 386 \text{ W/m} \cdot ^{\circ}\text{C}$) by screws that exert an average pressure of 6 MPa (Fig. 3-18). The base area of each transistor is 8 cm^2 , and each transistor is placed at the center of a 10-cm \times 10-cm quarter section of the plate. The interface roughness is estimated to be about $1.5 \mu\text{m}$. All transistors are covered by a thick Plexiglas layer, which is a poor conductor of heat, and thus all the heat generated at the junction of the transistor must be dissipated to the ambient at 20°C through the back surface of the copper plate. The combined convection/radiation heat transfer coefficient at the back surface can be taken to be $25 \text{ W/m}^2 \cdot ^{\circ}\text{C}$. If the case temperature of

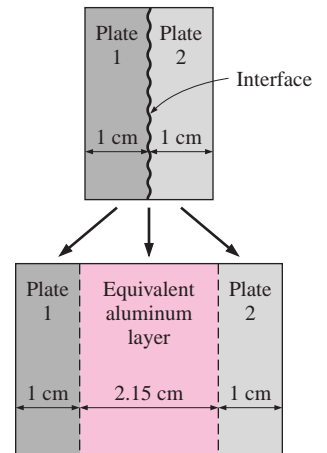


FIGURE 3-17
Schematic for Example 3-4.

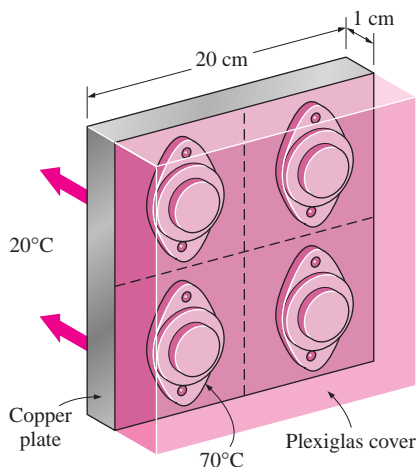


FIGURE 3–18
Schematic for Example 3–5.

the transistor is not to exceed 70°C , determine the maximum power each transistor can dissipate safely, and the temperature jump at the case-plate interface.

SOLUTION Four identical power transistors are attached on a copper plate. For a maximum case temperature of 70°C , the maximum power dissipation and the temperature jump at the interface are to be determined.

Assumptions 1 Steady operating conditions exist. 2 Heat transfer can be approximated as being one-dimensional, although it is recognized that heat conduction in some parts of the plate will be two-dimensional since the plate area is much larger than the base area of the transistor. But the large thermal conductivity of copper will minimize this effect. 3 All the heat generated at the junction is dissipated through the back surface of the plate since the transistors are covered by a thick Plexiglas layer. 4 Thermal conductivities are constant.

Properties The thermal conductivity of copper is given to be $k = 386 \text{ W/m} \cdot ^{\circ}\text{C}$. The contact conductance is obtained from Table 3–2 to be $h_c = 42,000 \text{ W/m}^2 \cdot ^{\circ}\text{C}$, which corresponds to copper-aluminum interface for the case of $1.3\text{--}1.4 \mu\text{m}$ roughness and 5 MPa pressure, which is sufficiently close to what we have.

Analysis The contact area between the case and the plate is given to be 8 cm^2 , and the plate area for each transistor is 100 cm^2 . The thermal resistance network of this problem consists of three resistances in series (interface, plate, and convection), which are determined to be

$$R_{\text{interface}} = \frac{1}{h_c A_c} = \frac{1}{(42,000 \text{ W/m}^2 \cdot ^{\circ}\text{C})(8 \times 10^{-4} \text{ m}^2)} = 0.030^{\circ}\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.01 \text{ m}}{(386 \text{ W/m} \cdot ^{\circ}\text{C})(0.01 \text{ m}^2)} = 0.0026^{\circ}\text{C/W}$$

$$R_{\text{conv}} = \frac{1}{h_o A} = \frac{1}{(25 \text{ W/m}^2 \cdot ^{\circ}\text{C})(0.01 \text{ m}^2)} = 4.0^{\circ}\text{C/W}$$

The total thermal resistance is then

$$R_{\text{total}} = R_{\text{interface}} + R_{\text{plate}} + R_{\text{ambient}} = 0.030 + 0.0026 + 4.0 = 4.0326^{\circ}\text{C/W}$$

Note that the thermal resistance of a copper plate is very small and can be ignored altogether. Then the rate of heat transfer is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{(70 - 20)^{\circ}\text{C}}{4.0326^{\circ}\text{C/W}} = 12.4 \text{ W}$$

Therefore, the power transistor should not be operated at power levels greater than 12.4 W if the case temperature is not to exceed 70°C .

The temperature jump at the interface is determined from

$$\Delta T_{\text{interface}} = \dot{Q} R_{\text{interface}} = (12.4 \text{ W})(0.030^{\circ}\text{C/W}) = 0.37^{\circ}\text{C}$$

which is not very large. Therefore, even if we eliminate the thermal contact resistance at the interface completely, we will lower the operating temperature of the transistor in this case by less than 0.4°C .

3-3 ■ GENERALIZED THERMAL RESISTANCE NETWORKS

The *thermal resistance* concept or the *electrical analogy* can also be used to solve steady heat transfer problems that involve parallel layers or combined series-parallel arrangements. Although such problems are often two- or even three-dimensional, approximate solutions can be obtained by assuming one-dimensional heat transfer and using the thermal resistance network.

Consider the composite wall shown in Fig. 3-19, which consists of two parallel layers. The thermal resistance network, which consists of two parallel resistances, can be represented as shown in the figure. Noting that the total heat transfer is the sum of the heat transfers through each layer, we have

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (3-29)$$

Utilizing electrical analogy, we get

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{total}}} \quad (3-30)$$

where

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2} \quad (3-31)$$

since the resistances are in parallel.

Now consider the combined series-parallel arrangement shown in Fig. 3-20. The total rate of heat transfer through this composite system can again be expressed as

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \quad (3-32)$$

where

$$R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}} \quad (3-33)$$

and

$$R_1 = \frac{L_1}{k_1 A_1}, \quad R_2 = \frac{L_2}{k_2 A_2}, \quad R_3 = \frac{L_3}{k_3 A_3}, \quad R_{\text{conv}} = \frac{1}{h A_3} \quad (3-34)$$

Once the individual thermal resistances are evaluated, the total resistance and the total rate of heat transfer can easily be determined from the relations above.

The result obtained will be somewhat approximate, since the surfaces of the third layer will probably not be isothermal, and heat transfer between the first two layers is likely to occur.

Two assumptions commonly used in solving complex multidimensional heat transfer problems by treating them as one-dimensional (say, in the

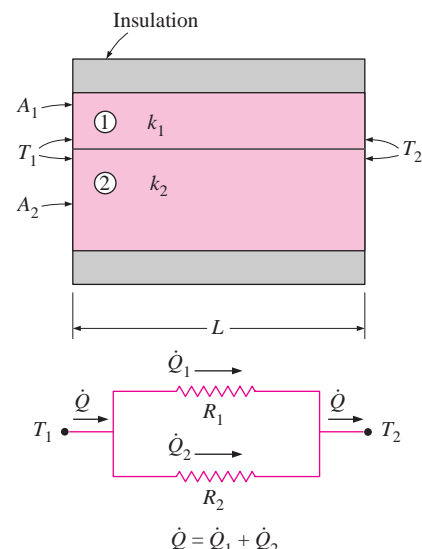


FIGURE 3-19

Thermal resistance network for two parallel layers.

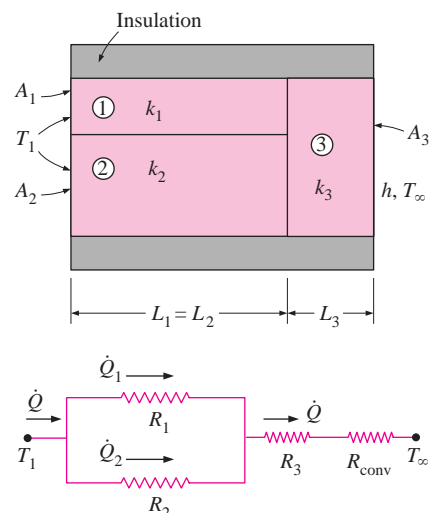


FIGURE 3-20

Thermal resistance network for combined series-parallel arrangement.

x -direction) using the thermal resistance network are (1) any plane wall normal to the x -axis is *isothermal* (i.e., to assume the temperature to vary in the x -direction only) and (2) any plane parallel to the x -axis is *adiabatic* (i.e., to assume heat transfer to occur in the x -direction only). These two assumptions result in different resistance networks, and thus different (but usually close) values for the total thermal resistance and thus heat transfer. The actual result lies between these two values. In geometries in which heat transfer occurs predominantly in one direction, either approach gives satisfactory results.

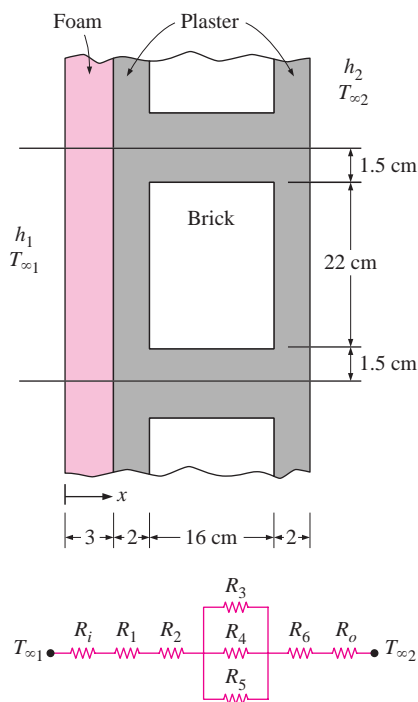


FIGURE 3-21
Schematic for Example 3-6.

EXAMPLE 3-6 Heat Loss through a Composite Wall

A 3-m-high and 5-m-wide wall consists of long 16-cm \times 22-cm cross section horizontal bricks ($k = 0.72 \text{ W/m} \cdot ^\circ\text{C}$) separated by 3-cm-thick plaster layers ($k = 0.22 \text{ W/m} \cdot ^\circ\text{C}$). There are also 2-cm-thick plaster layers on each side of the brick and a 3-cm-thick rigid foam ($k = 0.026 \text{ W/m} \cdot ^\circ\text{C}$) on the inner side of the wall, as shown in Fig. 3-21. The indoor and the outdoor temperatures are 20°C and -10°C , and the convection heat transfer coefficients on the inner and the outer sides are $h_1 = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$ and $h_2 = 25 \text{ W/m}^2 \cdot ^\circ\text{C}$, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

SOLUTION The composition of a composite wall is given. The rate of heat transfer through the wall is to be determined.

Assumptions 1 Heat transfer is steady since there is no indication of change with time. 2 Heat transfer can be approximated as being one-dimensional since it is predominantly in the x -direction. 3 Thermal conductivities are constant. 4 Heat transfer by radiation is negligible.

Properties The thermal conductivities are given to be $k = 0.72 \text{ W/m} \cdot ^\circ\text{C}$ for bricks, $k = 0.22 \text{ W/m} \cdot ^\circ\text{C}$ for plaster layers, and $k = 0.026 \text{ W/m} \cdot ^\circ\text{C}$ for the rigid foam.

Analysis There is a pattern in the construction of this wall that repeats itself every 25-cm distance in the vertical direction. There is no variation in the horizontal direction. Therefore, we consider a 1-m-deep and 0.25-m-high portion of the wall, since it is representative of the entire wall.

Assuming any cross section of the wall normal to the x -direction to be *isothermal*, the thermal resistance network for the representative section of the wall becomes as shown in Fig. 3-21. The individual resistances are evaluated as:

$$R_i = R_{\text{conv}, 1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot ^\circ\text{C})(0.25 \times 1 \text{ m}^2)} = 0.4^\circ\text{C/W}$$

$$R_1 = R_{\text{foam}} = \frac{L}{kA} = \frac{0.03 \text{ m}}{(0.026 \text{ W/m} \cdot ^\circ\text{C})(0.25 \times 1 \text{ m}^2)} = 4.6^\circ\text{C/W}$$

$$R_2 = R_6 = R_{\text{plaster, side}} = \frac{L}{kA} = \frac{0.02 \text{ m}}{(0.22 \text{ W/m} \cdot ^\circ\text{C})(0.25 \times 1 \text{ m}^2)} = 0.36^\circ\text{C/W}$$

$$R_3 = R_5 = R_{\text{plaster, center}} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.22 \text{ W/m} \cdot ^\circ\text{C})(0.015 \times 1 \text{ m}^2)} = 48.48^\circ\text{C/W}$$

$$R_4 = R_{\text{brick}} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.72 \text{ W/m} \cdot ^\circ\text{C})(0.22 \times 1 \text{ m}^2)} = 1.01^\circ\text{C/W}$$

$$R_o = R_{\text{conv},2} = \frac{1}{h_2 A} = \frac{1}{(25 \text{ W/m}^2 \cdot ^\circ\text{C})(0.25 \times 1 \text{ m}^2)} = 0.16^\circ\text{C/W}$$

The three resistances R_3 , R_4 , and R_5 in the middle are parallel, and their equivalent resistance is determined from

$$\frac{1}{R_{\text{mid}}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03 \text{ W/}^\circ\text{C}$$

which gives

$$R_{\text{mid}} = 0.97^\circ\text{C/W}$$

Now all the resistances are in series, and the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_i + R_1 + R_2 + R_{\text{mid}} + R_6 + R_o \\ &= 0.4 + 4.6 + 0.36 + 0.97 + 0.36 + 0.16 \\ &= 6.85^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the wall becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{6.85^\circ\text{C/W}} = 4.38 \text{ W} \quad (\text{per } 0.25 \text{ m}^2 \text{ surface area})$$

or $4.38/0.25 = 17.5 \text{ W per m}^2$ area. The total area of the wall is $A = 3 \text{ m} \times 5 \text{ m} = 15 \text{ m}^2$. Then the rate of heat transfer through the entire wall becomes

$$\dot{Q}_{\text{total}} = (17.5 \text{ W/m}^2)(15 \text{ m}^2) = \mathbf{263 \text{ W}}$$

Of course, this result is approximate, since we assumed the temperature within the wall to vary in one direction only and ignored any temperature change (and thus heat transfer) in the other two directions.

Discussion In the above solution, we assumed the temperature at any cross section of the wall normal to the x -direction to be *isothermal*. We could also solve this problem by going to the other extreme and assuming the surfaces parallel to the x -direction to be *adiabatic*. The thermal resistance network in this case will be as shown in Fig. 3–22. By following the approach outlined above, the total thermal resistance in this case is determined to be $R_{\text{total}} = 6.97^\circ\text{C/W}$, which is very close to the value 6.85°C/W obtained before. Thus either approach would give roughly the same result in this case. This example demonstrates that either approach can be used in practice to obtain satisfactory results.

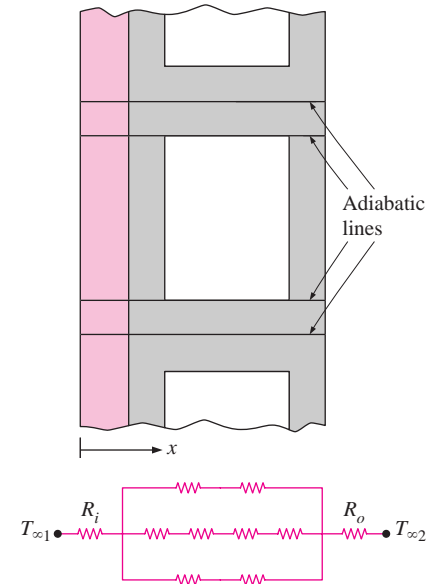


FIGURE 3–22

Alternative thermal resistance network for Example 3–6 for the case of surfaces parallel to the primary direction of heat transfer being adiabatic.