ECONOMIC APPLICATION OF HOMOGENOUS LINEAR DIFFERENTIAL EQUATIONS OF FIRST ORDER — THE DOMAR MODEL OF ECONOMIC GROWTH

Prof. E.D. Domar presented his model of economic growth in 1946. It got much more popularity in the name of Harrod-Domar Model when Prof. R. Harrod also presented the ideas similar to Domar. However, we shall discuss here the Domar growth model. The Domar model wishes to find the rate of growth of investment which could maintain equilibrium at full employment. In other words, the Domar model wishes to stipulate the type of time path required to prevail if a certain equilibrium condition of the economy is to be satisfied. In connection with the determination of required growth path we shall discuss the followings. (1) The frame work of Domar model (2) The time path of investment (3) the concept of Razor's edge in the model.

1. The Frame Work of Domar Model

1. Any change in the rate of investment per year I(t) will have a dual effect. It means that change in investment will affect aggregate demand (or NI) as well as the productive capacity of the economy.

2. The effect of change in investment on aggregate demand operates through the multiplier effect. It means that the increase in I(t) will increase the level of income per year y (t) by a multiple of increment in I(t). In other words, the increase in income (ΔY) will be equal to increase in investment (ΔI) multiplied by the value of multiplier $\left(=\frac{1}{1-MPC}=\frac{1}{MPS}=\frac{1}{s}\right)$.

It is as: $\Delta Y = k\,(\Delta I)$

This equation represents discrete time. If we introduce the continuous time the above equation becomes as : (or dividing both sides by dt we get the continuous effect on Y as result of change in investment) a $\frac{dY}{dt} = k\frac{dI}{dt}$.

Putting the value of k $\quad \frac{dY}{dt} = \frac{dI}{dt} \cdot \frac{1}{s}$ ... (1)

3. The capacity effect of investment is to be measured by the change in the rate of potential output the economy is capable of producing. Assuming a constant capacity-capital ratio, we write $\frac{\kappa}{K} = \rho$ where κ (the Greek letter kappa) stands for capacity or potential output per year, and ρ (the Greek letter rho) denotes the given capacity-capital ratio. This implies that with a capital stock K (t) the economy is potentially capable of producing an annual product, or income, amounting to $\kappa = \rho K$ dollars.

From $k = \rho K$ (the production function), it follows that $dk = \rho dK$ Dividing both the sides by dt, we get

$$\frac{dk}{dt} = \rho \frac{dK}{dt} . \text{As } \frac{dK}{dt} = q, \text{ then putting it: } \frac{dk}{dt} = \rho q \quad \dots (2)$$

4. In Domar model, equilibrium is defined to be a situation in which productive capacity is fully utilized. In other words, the equilibrium requires that aggregate demand to be exactly equal to the potential output producible in a year, i.e., $Y = k$. If our initial start is from an equilibrium situation where $Y = k$, the balancing requires that they should grow at the same rate, i.e., the change in aggregate demand (dY) with respect to time (dt) and change in capacity (dk) with respect to time must be equal i.e.,

$$\frac{dY}{dt} = \frac{dk}{dt} \quad \dots (3)$$

(2) Time Path of Investment :-

After having seen the balancing requirement, now it is to be discussed that what kind of time path of investment $I(t)$ can satisfy the equilibrium conditions at all times. To answer it we substitute eqns. (1) & (2) in 3.

$$\frac{1}{s}\frac{dI}{dt} = \rho I \quad \text{——— (4)}$$

$$\frac{dI}{dt} - \rho s I.$$

$$\frac{1}{I}\frac{dI}{dt} - \rho s. \quad \text{——— (5)}$$

This is a first order linear diff equation. this equation specifies a definite pattern of change for I and this will help us to find the equilibrium or required investment path.

taking integral on both sides of (5)

$$\int \frac{1}{I}\frac{dI}{dt}\cdot dt = \int \rho s\, dt$$

$$\int \frac{1}{I} dI = \int \rho s\, dt$$

$$\ln I + C_1 = \rho s t + C_2$$

$$\ln I = \rho s t + C_2 - C_1$$

$$\ln I = \rho s t + C$$

taking expo. on both side

$$e^{\ln I} = e^{\rho s t + C}$$

$$A = e^{\rho s t} \cdot e^{C}$$

$$I = A e^{\rho s t} \qquad (\because e^{C} = A)$$

$$I(t) = A e^{\rho s t} \longleftarrow (6)\ (G.S)$$

Put $t=0$ $\quad I(0) = A e^{\rho s(0)}$

$$I(0) = A e^{0}$$

$A = I(0)$ Put in (6)

$I(t) = I(0) e^{\rho s t}$ this is the required investment path, where $I(0)$ represents initial rate of investment.

The above equation suggests that in order to maintain a balance between capacity and demand overtime the rate of investment flow must grow precisely at the exponential rate of ρ s — along a path as illustrated in Fig. 1. It is told that the required rate of growth of investment will be larger, if the values of capacity–capital ratio and MPS are higher. Again, if the values of ρ and s are known to us, we will be able to know the required growth path.