Q:NO: What is equations reducible to the linear form.

ANS If the differentiable equations $dy/dt = h(y,t)$ happens or becomes in specific form which is non-linear form

$$dy/dt + Ry = Ty^m \longrightarrow (1)$$

Where R and T are two functions of "t" and m is only or any number other then 0 and 1 then the equation reffered to as a Bernoulli equation. The reduction procedure is relatively Simple.

first of all we divide the equation (1) by y^m to get

$$\frac{1}{y^m}\frac{dy}{dt} + \frac{Ry}{y^m} = \frac{Ty^m}{y^m}$$

$$y^{-m}\frac{dy}{dt} + R \cdot y \cdot y^{-m} = T$$

$$y^{-m}\frac{dy}{dt} + Ry^{1-m} = T \longrightarrow (2)$$

Let $z = y^{1-m}$

So that $dz/dt = dz/dy \cdot dy/dt$ (chain rule) — (A)

Now $dz/dy = (1-m)y^{1-m-1} \Rightarrow dz/dy = (1-m)y^{-m}$

Put in (A)

$$dz/dt = (1-m)y^{-m} \cdot dy/dt \qquad \div \text{ by } (1-m)$$

$$\frac{1}{1-m} dz/dt = dy/dt \, y^{-m}$$

thus the equation (2) can be written as.

$$\frac{1}{1-m} dz/dt + Rz = T \qquad \because z = y^{1-m}$$

Now multiplying by $(1-m)$ on both sides:

$$(1-m)\frac{1}{1-m} dz/dt + (1-m)Rz = T(1-m)$$

$$\boxed{dz/dt + (1-m)Rz = (1-m)T}$$

Example 1:- Solve the Bernoulli equation: $\frac{dy}{dt} + ty = 3ty^2$.

Here $m = 2$, $z = y^{1-n} = y^{1-2} = y^{-1}$, $R = t$ and $T = 3t$.

The linear version of Bernoulli's equation is as: $\frac{dz}{dt} + (1-m)Rz = (1-m)T$ or $\frac{dz}{dt} + (1-m)Rz = (1-m)T$

Putting their values: $$\frac{dz}{dt} + (1-2)\,tz = (1-2)\,3t$$

$$\frac{dz}{dt} + tz - 2tz = 3t - 6t \Rightarrow \frac{dz}{dt} - tz = -3t.$$

Thus $\frac{dz}{dt} - tz = -3t$ is the linear differential equation of the form: $\frac{dy}{dt} + uy = w$.

Reference above solved equation: $y = z$, $u = -t$ and $w = -3t$.

Applying the general formula: $z(t) = e^{\int u\,dt}\left(A + \int -3t\, e^{\int u\,dt}\,dt\right)$

$$= e^{\int -t\,dt}\left(A + \int -3t\, e^{\int -t\,dt}\,dt\right) = e^{\int t\,dt}\left(A + \int -3t\, e^{-(1/2)t^2}\,dt\right) = e^{1/2\,t^2}\left(A - \int 3t\, e^{-1/2\,t^2}\,dt\right)$$

Finding $\int -3t\, e^{-1/2\,t^2}\,dt$. Let $u = -\frac{1}{2}t^2$, $\frac{du}{dt} = -t \Rightarrow dt = \frac{du}{-t}$.

By following Integration by substitution: $-\int 3t\, e^{-1/2\,t^2}\,dt$

$$\int -3t\, e^{u}\,\frac{du}{-t} = -3\int e^{u}\,du = -3\,e^{u} = -3e^{-1/2\,t^2}.$$

Thus $z(t) = e^{1/2\,t^2}\left(A + 3e^{-1/2\,t^2}\right) \Rightarrow z(t) = A\,e^{1/2\,t^2} + 3.$

As $y^{-1} = z \Rightarrow y = z^{-1}$. Then putting the value $y = z^{-1} = (Ae^{1/2\,t^2} + 3)^{-1}$.

Example 2 :- We solve the following Bernoulli equation . $\frac{dy}{dt} - y = ty^2$.

Here $m = 2$, so $z = y^{1-n} = y^{1-2} = y^{-1}$, $R = -1$, $T = t$ and $(1-m) = -1$.

Substituting the value:

$$\frac{dz}{dt} + (1-n)Rz = (1-n)T \Rightarrow \frac{dz}{dt} + (1-2)(-1)(z) = (1-2)\,t \Rightarrow \frac{dz}{dt} + z = -t.$$

Thus $\frac{dz}{dt} + z = -t$ is the linear differential equation of the form $\frac{dy}{dt} + uy = w$.

Reference above solved equation: $-y = z$, $u = 1$ and $w = -t$. Applying the formula:

$$z(t) = e^{\int 1\,dt}\left(A + \int -t\, e^{\int 1\,dt}\,dt\right) = e^{-t}\left(A + \int -t\, e^{t}\,dt\right)$$

$z(t) = e^{-t}(A - te^{t} + e^{t}) = Ae^{-t} - t + 1$. Since $z(t) = y(t)^{-1}$. Thus $y(t) = (Ae^{-t} - t + 1)^{-1}$.