Example No 2 :- Give the demand and Supply functions. (2A/2005 B.Z.U)

$$Qd = 40 - 2p - 2p' - p''$$

$$Qs = -5 + 3p \qquad p(0) = 12 \ \& \ p'(0) = 1$$

Find $p(t)$ on the assumption that the market is always cleared.

Solution :- Market clearance

$$Qd = Qs$$

$$40 - 2p - 2p' - p'' = -5 + 3p$$

$$40 - 2p - 2p' - p'' + 5 - 3p = 0$$

$$45 - 5p - 2p' - p'' = 0$$

$$-p'' - 2p' - 5p = -45$$

$$p'' + 2p' + 5p = 45$$

Here $a_1 = 2$, $a_2 = 5$, $b = 45$

$$P_P = \frac{b}{a_2} = \frac{45}{5} = 9 \Rightarrow \boxed{P_P = 9}$$

Check :-

$$(a_1)^2 < 4a_2$$

$$(2)^2 < 4(5)$$

$$4 < 20$$

So it is Complex real roots.

$$P_c = e^{ht}[A_5 \cos vt + A_6 \sin vt]$$

$$h = \frac{-a_1}{2} = \frac{-2}{2} = -1 \Rightarrow \boxed{h = -1}$$

$$v = \frac{\sqrt{4a_2 - (a_1)^2}}{2} \Rightarrow \frac{\sqrt{4(5)-(2)^2}}{2} = \frac{\sqrt{20-4}}{2} = \frac{\sqrt{16}}{2}$$

$$= 4/2 = 2 \Rightarrow \boxed{v = 2}$$

$$P_c = e^{-t}\left[A_5 \cos 2t + A_6 \sin 2t\right]$$

The general solution is

$$P(t) = P_c + P_p$$

$$P(t) = e^{-t}\left(A_5 \cos 2t + A_6 \sin 2t\right) + 9$$

Putting $t = 0$

$$P(0) = e^{-0}\left(A_5 \cos 2(0) + A_6 \sin 2(0)\right) + 9$$

$$P(0) = \left(A_5 \cos 0 + A_6 \sin 0\right) + 9$$

$$P(0) = A_5(1) + 0 + 9$$

$$12 = A_5 + 9 \qquad \left(\because \begin{array}{l} \cos 0 = 1 \\ \sin 0 = 0 \end{array}\right)$$

$$A_5 = 12 - 9 \Rightarrow \boxed{A_5 = 3}$$

Now differentiating the G.S w.r.to "t" by product rule.

$$(P(t))' = (e^{-t})'\left[A_5 \cos 2t + A_6 \sin 2t\right] + e^{-t}\left[A_5 \cos 2t + A_6 \sin 2t\right]' + (9)'$$

$$P'(t) = e^{-t}(-1)\left[A_5 \cos 2t + A_6 \sin 2t\right] + e^{-t}\left[A_5(-\sin 2t \cdot 2) + A_6 \cos 2t \cdot 2\right]$$

$$P'(t) = -e^{-t}\left(A_5 \cos 2t + A_6 \sin 2t\right) + e^{-t}\left(-2A_5 \sin 2t + 2A_6 \cos 2t\right)$$

Putting $t=0$

$$p'(0) = -e^{-0}(A_5\cos 2(0) + A_6\sin 2(0)) + e^{-0}(-2A_5\sin 2(0) + 2A_6\cos 2(0))$$

$$p'(0) = -e^{0}(A_5\cos 0 + A_6\sin 0) + e^{0}(-2A_5\sin 0 + 2A_6\cos 0)$$

$$p'(0) = -A_5(1) + 0 + 0 + 2A_6(1)$$

$$p'(0) = -A_5 + 2A_6 \qquad \left(\because \begin{array}{l}\cos 0 = 1\\ \sin 0 = 0\\ e^{0} = 1\end{array}\right)$$

$$1 = -3 + 2A_6 \Rightarrow 1+3 = 2A_6$$

$$2A_6 = 4 \qquad \boxed{A_6 = 2}$$

Hence the definite solution is

$$\boxed{p(t) = e^{-t}(3\cos 2t + 2\sin 2t) + 9}$$

Verification:-

$$p(t) = e^{-t}(3\cos 2t + 2\sin 2t) + 9$$

putting $t=0$

$$p(0) = e^{-0}(3\cos 2(0) + 2\sin 2(0)) + 9$$

$$p(0) = 1(3\cos 0 + 2\sin 0) + 9 \qquad (\because e^{0} = 1)$$

$$p(0) = 3(1) + 0 + 9 \qquad (\because P(0) = 12)$$

$$12 = 3 + 9$$

$$12 = 12 \quad \underline{\text{Ans.}}$$