$= E\left[e^{t(X_1+X_2)}\right] \quad = E(e^{tX_1}) \cdot E(e^{tX_2})$

$= (q+Pe^t)^{n_1} \cdot (q+Pe^t)^{n_2}$

$= (q+Pe^t)^{n_1+n_2}$

This mgf also have binomial distribution with parameter (n_1+n_2, P)

Q:- Let X_1, X_2 and X_3 are the independent poisson random variables having mean 2, 1 and 4

a) Find mgf of $Y = X_1+X_2+X_3$

b) How Y is distributed

c) Compute $P(3 \leq Y \leq 9)$

$M_Y(t) = E(e^{tY}) = E\left[e^{t(X_1+X_2+X_3)}\right].$

$= E(e^{tX_1}) \cdot E(e^{tX_2}) \cdot E(e^{tX_3})$

$= e^{\lambda_1(e^t-1)} \cdot e^{\lambda_2(e^t-1)} \cdot e^{\lambda_3(e^t-1)}$

$= e^{2(e^t-1)} \cdot e^{1(e^t-1)} \cdot e^{4(e^t-1)}$

$= e^{7(e^t-1)}$

Distribution of Y is also poisson with parameter $\lambda = 7$.

$$P(3 \le Y \le 9) = \frac{e^{-7}7^3}{3!} + \frac{e^{-7}7^4}{4!} + \frac{e^{-7}7^5}{5!} + \frac{e^{-7}7^6}{6!} + \frac{e^{-7}7^7}{7!} + \frac{e^{-7}7^8}{8!} + \frac{e^{-7}7^9}{9!}$$

$$= 0.0521 + 0.0912 + 0.1277 + 0.1490 + 0.1490 + 0.1303 + 0.1014$$

$$= 0.8007.$$

Q:- Let XYZ are jointly continuous random variables with joint pdf

$f(xyz) = \frac{1}{3}(x+2y+3z) \quad 0 \le x, y, z \le 1$

Find joint pdf of x and y.

Joint pdf of x and y = ?

$$f(x,y) = \int_0^1 \frac{1}{3}(x+2y+3z)\,dz$$

$$= \int_0^1 \frac{x}{3}\,dz + \frac{2}{3}\int_0^1 y\,dz + \frac{3}{3}\int_0^1 z\,dz$$

$$= \frac{x}{3}\int_0^1 dz + \frac{2}{3}y\int_0^1 dz + \frac{3}{3}\int_0^1 z\,dz$$

$$= \frac{x}{3}z\Big|_0^1 + \frac{2}{3}y\,z\Big|_0^1 + \frac{3z^2}{3\cdot 2}\Big|_0^1$$

$$= \frac{x}{3} + \frac{2}{3}y + \frac{1}{2}$$

$= \frac{1}{3}(x+2y+\frac{3}{2})$

Q:- Let xyz are 3 independent random variables with $x \sim N(\mu, \sigma^2)$ and y and $z \sim U(0,2)$. We also have expected value of $(x^2y + xyz) = 13$ and expected value of $(xy^2 + x^2z) = 14$. Find μ and σ.

Mean and variance of uniform distribution

$$E(y) = E(z) = \frac{a+b}{2} = 2/2 = 1$$

$$V(y) = V(z) = \frac{(b-a)^2}{12} = \frac{4}{12} = \frac{1}{3}$$

Now

$$E(x^2y + xyz) = 13$$

$$E(x^2y) + E(xyz) = 13$$

$$[E(x^2) \cdot E(y)] + [E(x) \cdot E(Y) \cdot E(z)] = 13$$

$$[(\sigma^2 + \mu^2) 1] + \mu \cdot 1 = 13$$

$$\sigma^2 + \mu^2 + \mu = 13 \quad \text{— (A)}$$

$\because E(x) = \mu$

$E(x^2) = [E(x)]^2 + V(x) = \sigma^2 + \mu^2$

Again

$$E(xy^2 + x^2z) = 14$$

$$[E(x) \cdot E(y^2)] + [E(x^2) \cdot E(z)] = 14$$

$$\because E(y^2) = V(y) + \{E(y)\}^2$$
$$= \frac{1}{3} + 1 = 4/3$$

$$\mu \cdot 4/3 + (\mu^2 + \sigma^2) = 14$$

$$\frac{4\mu}{3} + \mu^2 + \sigma^2 = 14$$

$$4\mu + 3\mu^2 + 3\sigma^2 = 42 \quad \text{— (B)}$$

Solving (A) and (B)

$$\begin{aligned} \sigma^2 + \mu^2 + \mu &= 13 \\ 3\sigma^2 + 3\mu^2 + 4\mu &= 42 \end{aligned} \Rightarrow \begin{aligned} 3\sigma^2 + 3\mu^2 + 3\mu &= 39 \\ \underline{\overset{-}{3\sigma^2} + \overset{-}{3\mu^2} + \overset{-}{4\mu} = \overset{-}{42}} \end{aligned}$$

$$-\mu = -3$$
$$\mu = 3$$

Putting value of $\mu = 3$ in eq (4)

$$\sigma^2 + 9 + 3 = 13$$

$$\sigma^2 + 12 = 13$$

$$\sigma^2 = 1, \quad \sigma = 1.$$

Law of large number:-

Law of large number is used to established the Consistency of results

Detail:- In probability theory. Law of large number is a theorem that describe the results of performing same experiment a large number of times. According to this law the average of results obtained from a large number of trails should be closed to expected value of results and will tends to become closer as more trails are performed

∵ Law of large no. also called law of average

There are two types of law of large number

1) weak law of large number (WLLN):-

2) strong law of large number (SLLN):-

1) Weak law of large number:-

Let $X_1, X_2, \dots, X_n$ are independent trails with expected value of X_i $[E(X_i)] = \mu$ and finite variance σ^2. Let S_n = Sum of indep. trails $= X_1 + X_2 + \dots + X_n$. Then for any arbitrary value $\epsilon > 0$ the WLLN states that the sample average $\left(\frac{S_n}{n}\right)$ converges in probability towards the expected value.

Mathematically:-

$$P\left[\left(\frac{S_n}{n} - \mu\right) > \epsilon\right] = 0 \quad \text{as } n \to \infty$$

$\because \frac{S_n}{n} = \bar{x}$

$$P\left[\left(\frac{S_n}{n} - \mu\right) \leq \epsilon\right] = 1 \quad \text{as } n \to \infty.$$

$\because \mu = E(\bar{x})$

$$\bar{x} \xrightarrow{P} \mu$$

Here P means "converges in probability".

2) Strong law of large number:-

Let $X_1, X_2, \dots, X_n$ are the independent trails with $E(X_i) = \mu$ and finite variance σ^2. $S_n = X_1 + X_2 + \dots + X_n$

than for any no. $\epsilon > 0$ the strong law of large number states that the sample average converges almost surely to the expected value

Mathematically :-

$$\frac{S_n}{n} \xrightarrow{a.s} \mu$$

$\because$ a.s = almost surety

(یقینی)

Q:- State and prove law of large number.

Statement :-

Let S_n be a sequence of independent random variables with finite μ and finite variance σ^2. Then for any number $\epsilon > 0$

$$P\left\{\left[\frac{S_n}{n} - E\left(\frac{S_n}{n}\right)\right] > \epsilon\right\} \xrightarrow{P} 0 \qquad n \to \infty$$

$$\lim_{n \to \infty} P\left\{\left[\frac{S_n}{n} - E\left(\frac{S_n}{n}\right)\right] > \epsilon\right\} \xrightarrow{P} 0$$

Proof:-

$$S_n = x_1 + x_2 + \cdots + x_n$$

then

$$\frac{S_n}{n} = \bar{x}_n$$

$$E(\bar{x}_n) = E\left(\frac{S_n}{n}\right)$$

$$= E\left(\frac{X_1 + X_2 + \cdots + X_n}{n}\right)$$

$$= \frac{1}{n}\left(E(X_1) + E(X_2) + \cdots + E(X_n)\right)$$

$$= \frac{1}{n}\left(\mu + \mu + \cdots + \mu\right)$$

$$= \frac{n\mu}{n} = \mu$$

$$V(\bar{x}_n) = V\left(\frac{S_n}{n}\right) = \frac{1}{n^2} V(S_n)$$

$$= \frac{1}{n^2}\left\{V(X_1) + V(X_2) + \cdots + V(X_n)\right\}$$

$$= \frac{1}{n^2}\left\{\sigma^2 + \sigma^2 + \cdots + \sigma^2\right\} = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

According to chebyshev's inequality

$$P\left\{\left[\bar{x}_n - E(\bar{x}_n)\right] > \epsilon\right\} \leq \frac{V(X_n)}{\epsilon^2}$$

$$P\left[(\bar{x}_n - \mu) > \epsilon\right] \leq \frac{\sigma^2}{n\epsilon^2}$$

Applying limit on both sides

$$\lim_{n \to \infty} P\left[(\bar{x}_n - \mu) > \epsilon\right] \leq \lim_{n \to \infty} \frac{\sigma^2}{n\epsilon^2}$$

$$\lim_{n \to \infty} P\left[(\bar{x}_n - \mu) > \epsilon\right] \leq 0$$

Classical Central Limit theorem:-

As sample size increases the sampling distribution of sample mean approches to the normal distribution with mean as the population and standard deviation is the population standard deviation by square root of n (sample size).

$$\bar{x} \sim N(\mu, S/\sqrt{n})$$

$$\bar{x} - \mu \sim N(0, S/\sqrt{n})$$

$$\frac{\bar{x} - \mu}{S/\sqrt{n}} \sim N(0, 1)$$

Central limit theorem and properties:-

Let $x_1, x_2, ---, x_n$ be a sequence of n indepent and identically distributed random variables having finite population mean μ and variance σ^2. Than the CLT states that as sample size increase the distribution of sample statistic of the random variables approches to normal distribution with mean $\mu_{\bar{x}} = \mu$ and variance $\sigma_{\bar{x}}^2 = \sigma^2/n$. If the sample average or sample mean we define $\bar{x}_n$ be

$\bar{x}_n \sim N(\mu, \sigma^2/n)$. In a similar way

$$Z_n = \frac{\bar{x}_n - \mu}{\sigma/\sqrt{n}}$$

<u>Proof:-</u>

We know that moment generating function (mgf) of standard normal distribution

$$M_{Z_n}(t) = e^{t^2/2}$$

Let $M_{Z_n}(t)$ denote the mgf of Z_n and we have to show when $n \to \infty$ $M_{Z_n}(t) \to M_x(t) = e^{t^2/2}$

$\because Z_n = \dfrac{\bar{x}_n - \mu}{\sigma/\sqrt{n}}$

By defination:-

$$M_{Z_n}(t) = E[e^{t(Z_n)}]$$

$$= E\left[e^{t\cdot\left(\frac{(\bar{x}-\mu)\sqrt{n}}{\sigma}\right)}\right]$$

$$= E\left[e^{t\cdot\left(\frac{\Sigma x}{n} - \mu\right)\sqrt{n}/\sigma}\right]$$

$$= E\left[e^{t\cdot\frac{1}{n}\Sigma(x-\mu)\sqrt{n}/\sigma}\right]$$

$$= E\left[\prod_{i=1}^{n} e^{t\cdot\frac{1}{\sqrt{n}}\left(\frac{x-\mu}{\sigma}\right)}\right] \quad \text{--- (1)}$$

$\because$ Since x_i's are independent

Let $Y_i = \frac{x_i - \mu}{\sigma}$ than

$M_{Y_i}(t)$ is the mgf of X_i and

Y_i has the same distribution

as X_i. Now ① becomes as

$$M_{Z_n}(t) = \prod_{i=1}^{n} E\left(e^{t \cdot Y_i/\sqrt{n}}\right)$$

$$= \prod_{i=1}^{n} M_{Y_i}(t/\sqrt{n})$$

$$= \left[M_{Y_i}(t/\sqrt{n})\right]^n \quad —— ②$$

∵ By using property of mgf when variables are iid than product changes into power

We know that

$$M_X(t) = M_0(t) + \frac{t}{1!} M'(t) + \frac{t^2}{2!} M''(t)$$

$$+ \frac{t^3}{3!} M'''(t) + \cdots$$

By using this relation

$$M_{Y_i}(t/\sqrt{n}) = 1 + \frac{t}{\sqrt{n}}\left(\frac{\mu_1}{\sigma}\right) + \frac{\mu_2}{2\sigma^2}\left(\frac{t}{\sqrt{n}}\right)^2$$

$$+ \frac{\mu_3}{3!\sigma^3}\left(\frac{t}{\sqrt{n}}\right)^3 + \cdots$$

Since all odd order moments about mean are zero so

$$= 1 + 0 + \frac{\sigma^2}{2!\,\sigma^2} \cdot \left(\frac{t}{\sqrt{n}}\right)^2 + \cdots$$

$$= 1 + t^2/2n + \cdots$$

$$= 1 + P/n \qquad \because t^2/2 = P$$

$$= (1 + P/n)^n$$

applying limit on both ~~sides we have~~

$$\lim_{n\to\infty} M_{z_n}(t) = \lim_{n\to\infty} (1 + P/n)^n$$

$$= e^{P} \qquad P = t^2/2$$

$$= e^{t^2/2} = M_x(t)$$

~~This indicates that for~~ z_n has the mgf as the standard normal distribution

$$M_{z_n}(t) \to M_x(t) \text{ when } n \to \infty$$

Q:- Let S_n be the number of successes in n bernoulli trails with probability $P = \frac{1}{2}$ for success on each trail then. Show that with the help of chebyshev inequality for any $\epsilon > 0$

a) $P\left[\left|\frac{S_n}{n} - P\right| > \epsilon\right] \leq \frac{P(1-P)}{n\epsilon^2}$

b) Find max possible value for $P(1-P)$. If P lies between 0 and 1 and show that $P\left[\left|\frac{S_n}{n} - \frac{1}{2}\right| > \epsilon\right] \leq \frac{1}{4n\epsilon^2}$

we Know that

$$P\left[|\bar{x} - E(\bar{x})| > \epsilon\right] \le \frac{V(\bar{x})}{\epsilon^2}$$

In our case

$$\bar{x} = \frac{S_n}{n}$$

$$P\left[\left|\frac{S_n}{n} - E\left(\frac{S_n}{n}\right)\right| > \epsilon\right] \le \frac{V(S_n/n)}{\epsilon^2} \quad \text{— ①}$$

$$E\left(\frac{S_n}{n}\right) = \frac{1}{n} E(S_n) = \frac{1}{n} E\{X_1 + X_2 + \cdots + X_n\}$$

$$= \frac{n\mu}{n} = \mu = P$$

$$V\left(\frac{S_n}{n}\right) = \frac{1}{n^2} V(X_1 + X_2 + \cdots + X_n)$$

$$= \frac{1}{n^2}(\sigma^2 + \sigma^2 + \cdots + \sigma^2)$$

$$= \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n} = \frac{P(1-P)}{n}$$

Now ① becomes As

$$P\left[\left|\frac{S_n}{n} - P\right| > \epsilon\right] \le \frac{P(1-P)}{n\epsilon^2} \quad \text{—— ②}$$

b) Since P lies between 0 and 1 and we also Know that $P(1-P)$ has its maximum value at $P = \frac{1}{2}$ Now using eq ② we have

$$P\left[\left|\frac{S_n}{n} - \frac{1}{2}\right| > \epsilon\right] \le \frac{1}{4n\epsilon^2}$$

$\because P(1-P) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Q:- Let X be a random variable with $E(X)=0$ and $V(X)=1$. What integer value of K will assure us that

$$P\{|X|>K\}\leq 0.01$$

Using chebyshev inequality

$$P[|X-E(X)|>\epsilon]\leq \frac{V(X)}{\epsilon^2}$$

here $E(X)=0$ and $V(X)=1$

$$P[|X-0|>\epsilon]\leq \frac{1}{\epsilon^2}$$

let for any no K

$$P[|X|>K]\leq \frac{1}{K^2} \quad \text{--- (1)}$$

It is given that

$$P[|X|>K]\leq 0.01 \quad \text{--- (2)}$$

Comparing eq (1) and (2)

$$\frac{1}{K^2}=0.01$$

$$K^2=100$$

$$K=10.$$

Q:- Let S_{100} be the number of heads that turn up in 100 tosses of a fair coin. Use Central limit theorem to estimate probability of

a) $P(S_{100} < 45) = ?$

b) $P(45 \leq S_{100} < 55) = ?$

c) $P(S_{100} > 63) = ?$ d) $P(S_{100} < 47) = ?$

we have

$$Z_n = \frac{S_n - np}{\sqrt{npq}}$$

Here $np = 100 \times \frac{1}{2} = 50$

$$\sqrt{npq} = \sqrt{100 \times \frac{1}{2} \times \frac{1}{2}} = 5$$

Now

a) $P(S_{100} < 45) = P\left(\frac{S_{100} - 50}{5} < \frac{45-50}{5}\right)$

$$P(Z_{100} < -1) = P[Z < -1]$$

$$= 0.15866$$

b) $P(45 \leq S_{100} < 55)$

$$= P\left[\frac{45-50}{5} \leq \frac{S_{100}-50}{5} \leq \frac{55-50}{5}\right]$$

$$= P[-1 \leq Z_{100} < 1]$$

$= P[0 \le Z \le 1] - P[0 \le Z \le -1]$

$= 0.84134 - 0.15866 \quad = 0.68268$

using standard normal probabilities

c) $P(S_{100} > 63) = P\left[\frac{S_{100} - 50}{5} > \frac{63-50}{5}\right]$

$P[Z_{100} > 2.6] = P[Z > 2.6]$

$= 1 - P[0 \le Z \le 2.6]$

$= 1 - 0.99534 \quad = \quad 0.00466$

d) $P(S_{100} < 47)$

$= P\left(\frac{S_{100} - 50}{5} < \frac{47-50}{5}\right)$

$= P[Z_{100} < -0.6]$

$= P[Z < -0.6] \quad = \quad 0.27425$

Q:- There are 100 men on a plane. let X_i be the weight in pounds of i^{th} man on the plane. Suppose that X_i's are independent and identically distributed and $E(X_i) = \mu = 170$ and $\sigma_{X_i} = \sigma = 30$. Find the probability that total weight of the men on the plane exceeds 18000 pounds.

Let $w = X_1 + X_2 + \dots + X_{100}$

= weight of all persons.

$E(X_i) = \mu = 170$

$\sigma_{X_i} = 30$

~~$P(w > 18000) = ?$~~

Now $E(w) = n\mu = n \times 170 = 100 \times 170$

$= 17000$

$V(w) = n\sigma^2 = 100 \times (30)^2 = 90{,}000$

$\sigma_w = 300$

$P(w > 18000) = P\left(\frac{w - 17000}{300} > \frac{18000 - 17000}{300}\right)$

$= P(Z > 3.33)$

$= 1 - P(0 \le Z \le 3.33)$

$= 1 - 0.99957 \qquad = 0.00043$

using standard area probabilities

Q:- Let $X_1, X_2, \dots, X_{25}$ be independent identically distributed with following pmf

$P(x) = 0.6$ if $x = 1$

$P(x) = 0.4$ if $x = -1$

and let $Y = X_1 + X_2 + \dots + X_n$

using Central limit theorem with

Continuity Correction Factor estimate probability of Y lies b/w 4 and 6.

∵ In probability theory, a continuity correction is an adjustment that is made when a discrete distribution is approximated by a continuous distribution.

جب (Discrete distribution) سے (Continuous distribution) میں جاتے ہیں

تو (D) اور (C) کے فرق یا (Gap) کو ختم کرنے کے

لیے Continuity correction (use) کرتے ہیں۔

we have to calculate

$$P(4 \leq Y \leq 6)$$

using continuity correction.

$$P[3.5 \leq Y \leq 6.5] = ?$$

For this we need mean and variance of Y.

$$E(X_i) = E(X_1) + E(X_2)$$

$$= X_1 P(X_1) + X_2 P(X_2)$$

$$= 0.6 - 0.4 = 0.2$$

$$E(x_i^2) = \Sigma x_i^2 P(x)$$
$$= 0.6 + 0.4 = 1$$

$$V(X) = E(x_i^2) - [E(x_i)]^2$$
$$= 1 - (0.2)^2 \quad = 0.96$$

Now
$$Y = X_1 + X_2 + \cdots + X_{25}$$
$$E(Y) = 25\mu = 25(0.2) \quad = 5$$

$$V(Y) = V(X_1 + X_2 + \cdots + X_{25})$$
$$= 25\sigma^2 = 25(0.96) \quad = 24$$

$$= P[3.5 \le Y \le 6.5]$$

$$= P\left[\frac{3.5-5}{(24)^{1/2}} \le \frac{Y-5}{\sqrt{24}} \le \frac{6.5-5}{\sqrt{24}}\right]$$

$$= P\left[\frac{3.5-5}{4.89} \le \frac{Y-5}{4.89} \le \frac{6.5-5}{4.89}\right]$$

$$= P[-0.31 \le Z \le 0.31]$$

$$= 0.62172 - 0.37828$$

$$= 0.24344$$

Q :- You have invited 64 guest to a party. You need to make sandwitches for the guest, you believe that guest might need 0, 1 or 2 sandwitches with probability $\frac{1}{4}, \frac{1}{2}$ and $\frac{1}{4}$. You assume that the number of sandwitches each guest need is independent from other guest. How many sandwitches should you make so that you are 95% sure that there is no shortage.

Let X_i be the no. of sandwitches that the i^{th} person need and let $Y = X_1 + X_2 + \cdots + X_{64}$. we need Y such that

$$P[Y \le y] \geq 0.95$$

$$E(X_i) = \sum X_i P(X_i)$$

$$= X_1 P(X_1) + X_2 P(X_2) + X_3 P(X_3)$$

$$= 0 + \frac{1}{2} + 2 \cdot \frac{1}{4} = 0 + \frac{1}{2} + \frac{1}{2} = 1$$

$$E(X_i^2) = \sum X_i^2 P(X_i)$$

$$= 0 + \frac{1}{2} + 4 \cdot \frac{1}{4} = 3/2 = 1.5$$

$$V(X_i) = 1.5 - (1)^2 = 0.5$$

$$P[Y \le y] \geq 0.95$$

$$P\left[\frac{Y - E(Y)}{\sigma_y} \le \frac{y - E(Y)}{\sigma_y}\right] \geq 0.95 \quad \text{— ①}$$

$E(Y) = E(X_1 + X_2 + \cdots + X_{64})$

$= 64\,\mu = 64 \times 1 = 64$

$V(Y) = 64\sigma^2 = 64 \times 0.5 = 32.$

Now (1) becomes As.

$$P\left[Z \le \frac{y-64}{32}\right] \geq 0.95$$

$$P\left[Z \le \frac{y-64}{32}\right] = 1.645$$

$$\frac{y-64}{32} = 1.645$$

$$y = (1.645 \times 32) + 64$$

$$y = 74.$$

Q:- Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random valiable follow exponential distribution with parameter λ and $\lambda = 1$. let $\bar{x} = \frac{X_1 + X_2 + \cdots + X_n}{n}$ ~~how large should~~ n be ~~such that~~ $P(0.9 < \bar{x} < 1.1) \geq 0.95$

Since $X \sim \exp(1)$

$$E(X_i) = \frac{1}{\lambda} = 1$$

$$Var(X_i) = \frac{1}{\lambda^2} = 1$$

$$P[0.9 < \bar{x} < 1.1] \geq 0.95$$

let $Y = X_1 + X_2 + \cdots + X_n$

$$P\left[0.9 < \frac{Y}{n} < 1.1\right] \geq 0.95 \quad \text{— } (1)$$

Now $E(Y) = E(\Sigma x_i) = \Sigma\{E(x_i)\}$

$$= \Sigma(1) = n$$

$$\text{var}(Y) = \text{var}(\Sigma x_i) = \Sigma\{\text{var}(x_i)\}$$

$$= \Sigma(1) = n$$

Now (1) $\Rightarrow$ $P[0.9n < Y < 1.1n] \geq 0.95$

$$P\left[\frac{0.9n - n}{\sqrt{n}} < Z_Y < \frac{1.1n - n}{\sqrt{n}}\right] \geq 0.95$$

$$P\left[\frac{n(0.9-1)}{\sqrt{n}} < Z_Y < \frac{n(1.1-1)}{\sqrt{n}}\right] \geq 0.95$$

$$P[-0.1\sqrt{n} < Z_Y < 0.1\sqrt{n}] \geq 0.95$$

According to probability Rule

$$\phi(0.1\sqrt{n}) - \phi(-0.1\sqrt{n}) \geq 0.95$$

$$\because \phi(z) = 1 - \phi(-z)$$

$$\phi(0.1\sqrt{n}) - 1 + \phi(0.1\sqrt{n}) \geq 0.95$$

$$2\phi(0.1\sqrt{n}) - 1 \geq 0.95$$

$$2\phi(0.1\sqrt{n}) \geq 1.95$$

$$\Phi(0.1\sqrt{n}) \geq 1.95/2$$

$$\Phi(0.1\sqrt{n}) \geq 0.975$$

$$0.1\sqrt{n} \geq \Phi^{-1}(0.975)$$

$$0.1\sqrt{n} \geq 1.96$$

$$\sqrt{n} \geq \left(\frac{1.96}{0.1}\right) \Rightarrow n \geq \left(\frac{1.96}{0.1}\right)^2$$

$$n \geq 384.16.$$

Markov chain :- A Markov chain is a mathematical system that experiences transition from one state to another according to some probabilistic rules. The defining characteristic of a Markov chain is that no matter how the process arrived at its present state, the possible future states are fixed. In other words, the probability of transitioning to any particular state is dependent solely on the current state and time elapsed.

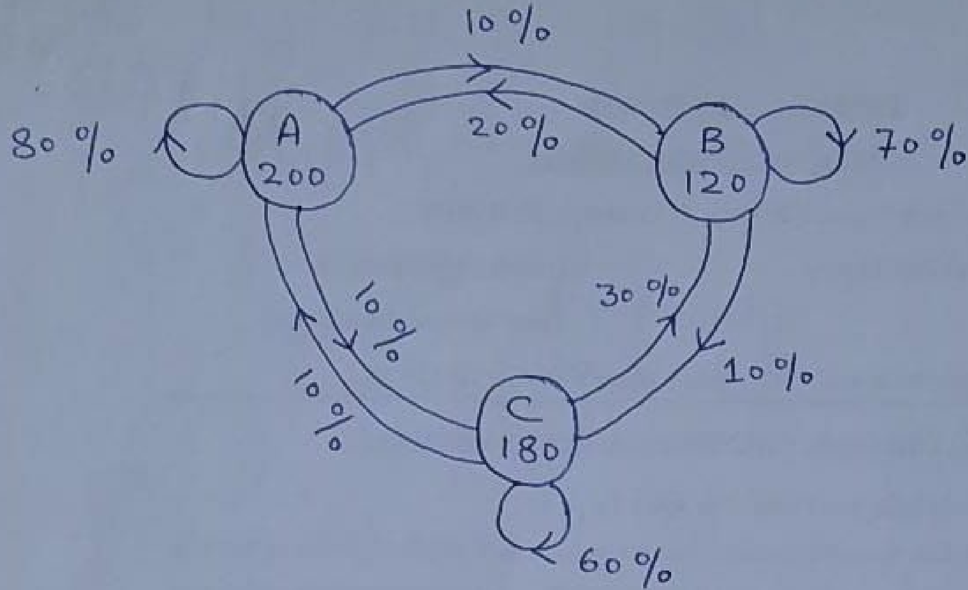
Transition Matrices :- A transition matrix P_t for Markov chain at time t is a matrix containing information on the probability of transitioning between states. In particular, given an ordering of a matrix's rows and columns by state spaces. This means each row of a matrix is a probability vector, and the sum of its entries is 1.

Most of our study of probability has dealt with independent trail process. When we study a sequence of chance experiments from an independent trails process, the possible outcomes of each experiment are the same and occur with the same probability. Further, knowledge of the outcomes of the previous experiments does not influence our predictions for the outcomes of the next experiment.

Modern probability theory studies chance process for which the knowledge of previous outcomes influences predictions for future experiments. In principle, when we observe a Sequence of chance experiments, all of the past outcomes could influence our predictions for the next experiments. For example, this should be the case in predicting a student's grades on a sequence of exams in a Course. But to allow this much generally would make it very difficult to prove general results.

In 1907, Markov began the study of an important new type of chance process. In this process, the outcome of a given experiment can effect the outcome of the next experiment. This type of process is called a Markov chain.

Question :- There are three shoping malls in a certain colony. A sample of 500 people are collected from these malls on the basis of their choices about these shoping malls. From the data given below in the form of chain (or transition diagram) calculate the Next state to illustrate the condition of these shoping malls.



[NEXT STATE] = $\begin{bmatrix} \text{Matrix of} \\ \text{Transition} \\ \text{Probabilities} \end{bmatrix}$ [Current state]

$$\begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix} = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.8 & 0.2 & 0.1 \\ 0.1 & 0.7 & 0.3 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \end{array} \begin{bmatrix} 200 \\ 120 \\ 180 \end{bmatrix} \underset{\text{OR}}{} \begin{bmatrix} 200/500 \\ 120/500 \\ 180/500 \end{bmatrix}$$

$$= \begin{bmatrix} (0.8)(0.4) + (0.2)(0.4) + (0.1)(0.36) \\ (0.1)(0.4) + (0.7)(0.24) + (0.3)(0.36) \\ (0.1)(0.4) + (0.1)(0.24) + (0.6)(0.36) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.404 \\ 0.316 \\ 0.280 \end{bmatrix}$$ → Sum should be one

$0.404 + 0.316 + 0.280 = 1$

Now the Next state is as follow

$$\begin{bmatrix} 0.404 \times 500 = 202 \\ 0.316 \times 500 = 158 \\ 0.280 \times 500 = 140 \end{bmatrix}$$

Total Sampled people

$202 + 158 + 140 = 500$

why they call it chains

$X_1 = PX_0$ → Initial State

Next OR Future State

Probability Matrix

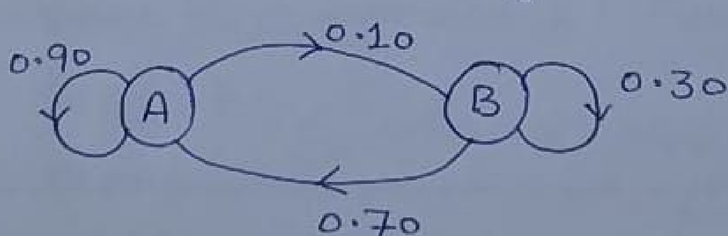
$X_2 = PX_1$ $\quad X_3 = PX_2$ $\quad$ and so on.

Question :- Suppose that an orange juice company controls 20 % of the orange juice market. ~~Suppose~~ They hire a market research company to predict the effects on an aggressive ad campaign. Suppose they conclude

→ Someone using Brand A will stay with Brand A with 90% probability, after 1 week.

→ Someone Not using Brand A will switch to Brand A with 70 % probability, after 1 week.

Transition Diagram



$$\begin{bmatrix} \text{Next} \\ \text{state} \end{bmatrix} = \begin{bmatrix} \text{Transition} \\ \text{Probability} \\ \text{Matrix} \end{bmatrix} \begin{bmatrix} \text{Current state} \end{bmatrix}$$