Normal Distribution

Let "X" be a continuous random variable with interval $(-\infty, +\infty)$ is said to be normal distribution having its probability density function (p.d.f) is given as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \qquad -\infty \leq X \leq +\infty$$

It has two parameters (μ, σ^2).

Where $\mu = Mean$ $\quad \sigma^2 = Variance$ $\quad \sigma = Standard\ deviation$

$\pi = Constant\ Approximately\ equal\ to = \frac{22}{7} = 3.14159$

$e = Constant\ Approximately\ equal\ to = 2.71828$

$X = Abscissa\ i.e.\ value\ marked\ on\ X-axis$

$Y = Ordinate\ hieght\ i.e.\ value\ marked\ on\ Y-axis$

Properties of normal distribution

i) It is continuous distribution and its range $(-\infty, +\infty)$

ii) Total area under the curve is unity

iii) It is bell shape distribution

iv) It is symmetrical distribution and its mean, median and mode are identical

v) It is unimodal distribution and maximum ordinate of the curve at $X = \mu$ is $\frac{1}{\sigma\sqrt{2\pi}}$

vi) It has two points of inflection which are equidistant from mean" μ "are $\left(\mu-\sigma, \frac{1}{\sigma\sqrt{2\pi e}}\right)$ and $\left(\mu+\sigma, \frac{1}{\sigma\sqrt{2\pi e}}\right)$

vii) The mean deviation of normal distribution is approximately $\frac{4}{5}$ of its Standard deviation i.e. $M.D = \frac{4}{5}\sigma = 0.7979\sigma$

viii) The Quartile deviation of normal distribution is approximately $\frac{2}{3}$ of its Standard deviation i.e. $Q.D = \frac{2}{3}\sigma = 0.6745\sigma$

ix) All odd order moments about mean equal to zero i.e. $\mu_1 = \mu_3 = \mu_5 = ... = 0$

x) The expression of even order moments about mean are $\mu_{2n} = \left(\frac{\sigma^2}{2}\right)^n \frac{(2n)!}{n!}$

xi) If X is $N(\mu, \sigma^2)$ and if $Y = a + bX$ then Y is $N(a + \mu b, b^2\sigma^2)$

xii) The sum of independent normal variables is a normal variable

xiii) The property of normal distribution

i) $P(\mu - 0.6745\sigma < X < \mu + 0.6745\sigma) = 0.50\ or\ 50\%$

ii) $P(\mu - \sigma < X < \mu + \sigma) = 0.6827\ or\ 68.27\%$

iii) $P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9544\ or\ 95.44\%$

iv) $P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973\ or\ 99.73\%$

xiv) The two quartiles are equidistant from the mean

$Q_1 = \mu - 0.6745\sigma$ Lower quartile

$Q_3 = \mu + 0.6745\sigma$ Upper quartile

xv) The normal curve approaches but never touches the base line. So the curve is asymptotic to the horizontal line.

xvi) It has two parameters (μ, σ^2)

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Standard normal variate

Ans: Any variable having zero mean and unit variance is called standard normal variate or variable. i.e. $Z = \frac{X-\mu}{\sigma}$

Standard normal distribution

The normal probability distribution of "Z" which has zero mean and unit variance is called the standardized normal distribution. It is denoted by $Z \to N(0,1)$ and its p.d.f given as $f(Z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}Z^2}$ $\quad -\infty \le Z \le +\infty$

Properties of standard normal distribution

i) It is continuous distribution and its range $(-\infty,+\infty)$

ii) Total area under the curve is unity

iii) It is bell shape distribution

iv) It is symmetrical distribution and its mean, median and mode ere equal to zero

v) The mean deviation of standard normal distribution is approximately $\frac{4}{5}$ i.e.

$M.D = \frac{4}{5} = 0.7979$

vi) The Quartile deviation of standard normal distribution is approximately $\frac{2}{3}$

i.e. $Q.D = \frac{2}{3} = 0.6745$

vii) All odd order moments about mean equal to zero i.e. $\mu_1 = \mu_3 = \mu_5 = ... = 0$

viii) The two quartiles of standard normal distribution

$Q_1 = -0.6745$ Lower quartile

$Q_3 = 0.6745$ Upper quartile

ix) It has two parameters $Z \to N(0,1)$

Theorem.No.1: Show that total area under the curve is unity

Proof: Let by definition

$$Total\ \ Area = \int_{-\infty}^{\infty} f(x)d(x)$$

$X \to N(\mu,\sigma^2)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \qquad -\infty \le X \le \infty$$

$$Total\ \ Area = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} d(x)$$

$$Total\ \ Area = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} d(x) \qquad (i)$$

Put $Z = \frac{X-\mu}{\sigma}$

$Z\sigma = x - \mu$

$X = Z\sigma + \mu$

$d(x) = Zd(z)$ And Limits remains same $-\infty \le Z \le \infty$

$$Total\ \ Area = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}Z^2} \sigma\ d(z)$$

$$Total\ \ Area = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}Z^2} d(z) \qquad (ii)$$

As $f(z) = e^{-\frac{1}{2}Z^2}$

Put $Z = -Z$

$f(-Z) = e^{-\frac{1}{2}(-Z)^2} = e^{-\frac{1}{2}Z^2} = f(Z)$ Its even function then eq.(ii) becomes

$$Total\ \ Area = \frac{2}{\sqrt{2\pi}}\int_0^\infty e^{-\frac{1}{2}Z^2}\ \ d(z) \qquad \text{(iii)}$$

Put $t = \frac{1}{2}Z^2$

$\sqrt{2t} = Z$

$\frac{1}{2}(2t)^{-\frac{1}{2}}2d(t) = d(z)$

$(2t)^{-\frac{1}{2}}d(t) = d(z)$ And Limits remains same $0 \le t \le \infty$

$$Total\ \ Area = \frac{2}{\sqrt{2\pi}}\int_0^\infty (2t)^{-\frac{1}{2}}e^{-t}\ \ d(t)$$

$$Total\ \ Area = \frac{2}{\sqrt{2\pi}}\int_0^\infty \frac{t^{\frac{1}{2}-1}}{2^{\frac{1}{2}}}e^{-t}\ \ d(t)$$

$$Total\ \ Area = \frac{2}{\sqrt{2}\sqrt{\pi}}\int_0^\infty \frac{t^{\frac{1}{2}-1}}{\sqrt{2}}e^{-t}\ \ d(t)$$

$$Total\ \ Area = \frac{1}{\sqrt{\pi}}\int_0^\infty t^{\frac{1}{2}-1}e^{-t}\ \ d(t) \qquad \text{(iv)}$$

As we know that Gamma function is

$$\Gamma\alpha\,\beta^\alpha = \int_0^\infty t^{\alpha-1}e^{-t/\beta}\ \ d(t) \qquad \text{v)}$$

Comparing (iv) and (v) then we get

$\alpha = \frac{1}{2}$ *And* $\beta = 1$ Then eq.(iv) becomes

$Total\ \ Area = \frac{1}{\sqrt{\pi}}\Gamma\frac{1}{2}1^{\frac{1}{2}} = \frac{1}{\sqrt{\pi}}\Gamma\frac{1}{2} = \frac{1}{\sqrt{\pi}}\sqrt{\pi} = 1$ Therefore $\Gamma\frac{1}{2} = \sqrt{\pi}$

$Total\ \ Area = 1$ Hence proved

Theorem.No.2: Show that mean of normal distribution is $"\mu"$

Proof: Let by definition

$$E(X) = \int_{-\infty}^\infty Xf(x)d(x)$$

$X \to N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \qquad -\infty \le X \le \infty$$

$$E(X) = \int_{-\infty}^\infty X\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\ d(x)$$

$$E(X) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^\infty Xe^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\ d(x) \qquad \text{(i)}$$

Put $Z = \frac{X-\mu}{\sigma}$

$Z\sigma = x - \mu$

$X = Z\sigma + \mu$

$d(x) = \sigma\ d(z)$ And Limits remains same $-\infty \le Z \le \infty$

$$E(X) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^\infty (\sigma Z + \mu)e^{-\frac{1}{2}Z^2}\ \ \sigma\ d(z)$$

$$E(X) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty \sigma Ze^{-\frac{1}{2}Z^2}\ \ d(z) + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty \mu e^{-\frac{1}{2}Z^2}\ \ d(z)$$

$$E(X)=\frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty} Ze^{-\frac{1}{2}Z^2}\ d(z)+\frac{\mu}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{1}{2}Z^2}\ d(z) \qquad \text{(ii)}$$

As $f(z)=e^{-\frac{1}{2}Z^2}$ $\qquad f(z)=Ze^{-\frac{1}{2}Z^2}$

Put $Z=-Z$

$$f(-Z)=(-Z)e^{-\frac{1}{2}(-Z)^2}=-Ze^{-\frac{1}{2}Z^2}=-f(Z)$$

$$f(-Z)=e^{-\frac{1}{2}(-Z)^2}=e^{-\frac{1}{2}Z^2}=f(Z)$$

1st function is odd and 2nd is even function then eq. (ii) becomes

$$E(X)=\frac{\sigma}{\sqrt{2\pi}}(0)+\frac{2\mu}{\sqrt{2\pi}}\int_{0}^{\infty} e^{-\frac{1}{2}Z^2}\ d(z)$$

$$E(X)=\frac{2\mu}{\sqrt{2\pi}}\int_{0}^{\infty} e^{-\frac{1}{2}Z^2}\ d(z) \qquad \text{(iii)}$$

Put $t=\frac{1}{2}Z^2$

$$\sqrt{2t}=Z$$

$$\frac{1}{2}(2t)^{-\frac{1}{2}}2d(t)=d(z)$$

$(2t)^{-\frac{1}{2}}d(t)=d(z)$ $\qquad$ And $\quad$ Limits remains same $\quad 0\le t\le\infty$

$$E(X)=\frac{2\mu}{\sqrt{2\pi}}\int_{0}^{\infty}(2t)^{-\frac{1}{2}}e^{-t}\ d(t)$$

$$E(X)=\frac{2\mu}{\sqrt{2\pi}}\int_{0}^{\infty}\frac{t^{\frac{1}{2}-1}}{2^{\frac{1}{2}}}e^{-t}\ d(t)$$

$$E(X)=\frac{2\mu}{\sqrt{2}\sqrt{\pi}}\int_{0}^{\infty}\frac{t^{\frac{1}{2}-1}}{\sqrt{2}}e^{-t}\ d(t)$$

$$E(X)=\frac{\mu}{\sqrt{\pi}}\int_{0}^{\infty}t^{\frac{1}{2}-1}e^{-t}\ d(t) \qquad \text{(iv)}$$

As we know that Gamma function is

$$\Gamma(\alpha)\beta^{\alpha}=\int_{0}^{\infty}t^{\alpha-1}e^{-t/\beta}\ d(t) \qquad \text{v)}$$

Comparing (iv) and (v) then we get

$\alpha=\frac{1}{2}$ $\quad$ *And* $\quad \beta=1$ $\qquad$ Then eq. (iv) becomes

$E(X)=\frac{\mu}{\sqrt{\pi}}\Gamma\left(\frac{1}{2}\right)1^{\frac{1}{2}}=\frac{\mu}{\sqrt{\pi}}\Gamma\left(\frac{1}{2}\right)=\frac{\mu}{\sqrt{\pi}}\sqrt{\pi}=\mu$ $\qquad$ Therefore $\qquad \Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$

$E(X)=\mu$ $\qquad$ Hence proved

Theorem.No.3: Show that Variance of normal distribution is $"\sigma^2"$

Proof: Let by definition

$$Var(X)=E[X-\mu]^2=\int_{-\infty}^{\infty}[X-\mu]^2 f(x)d(x)$$

$$X\to N(\mu,\sigma^2)$$

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \qquad -\infty\le X\le\infty$$

$$Var(X)=\int_{-\infty}^{\infty}[X-\mu]^2\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}d(x)$$

$$Var(X)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}[X-\mu]^2e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}d(x) \qquad \text{(i)}$$

Put $Z = \frac{X-\mu}{\sigma}$

$Z\sigma = x - \mu$

$X = Z\sigma + \mu$

$d(x) = \sigma\ d(z)$ And Limits remains same $-\infty \le Z \le \infty$

$$Var(X) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}(\sigma Z)^2 e^{-\frac{1}{2}Z^2}\ \sigma\ d(z)$$

$$Var(X) = \frac{\sigma^2}{\sqrt{2\pi}}\int_{-\infty}^{\infty} Z^2 e^{-\frac{1}{2}Z^2}\ d(z) \qquad \text{(ii)}$$

As $f(z) = Z^2 e^{-\frac{1}{2}Z^2}$

Put $Z = -Z$

$$f(-Z) = (-Z)^2 e^{-\frac{1}{2}(-Z)^2} = Z^2 e^{-\frac{1}{2}Z^2} = f(Z)$$

$$f(-Z) = e^{-\frac{1}{2}(-Z)^2} = e^{-\frac{1}{2}Z^2} = f(Z)$$

It is even function then eq. (ii) becomes

$$Var(X) = \frac{2\sigma^2}{\sqrt{2\pi}}\int_{0}^{\infty} Z^2 e^{-\frac{1}{2}Z^2}\ d(z) \qquad \text{(iii)}$$

Put $t = \frac{1}{2}Z^2$

$2t = Z^2$

$\sqrt{2t} = Z$

$\frac{1}{2}(2t)^{-\frac{1}{2}}2d(t) = d(z)$

$(2t)^{-\frac{1}{2}}d(t) = d(z)$ And Limits remains same $0 \le t \le \infty$

$$Var(X) = \frac{2\sigma^2}{\sqrt{2\pi}}\int_{0}^{\infty} 2t(2t)^{-\frac{1}{2}} e^{-t}\ d(t)$$

$$Var(X) = \frac{4\sigma^2}{\sqrt{2\pi}}\int_{0}^{\infty} \frac{t^{\frac{1}{2}}}{2^{\frac{1}{2}}} e^{-t}\ d(t)$$

$$Var(X) = \frac{4\sigma^2}{\sqrt{2}\sqrt{\pi}}\int_{0}^{\infty} \frac{t^{\frac{1}{2}-1}}{\sqrt{2}} e^{-t}\ d(t)$$

$$Var(X) = \frac{4\sigma^2}{2\sqrt{\pi}}\int_{0}^{\infty} t^{\frac{1}{2}-1} e^{-t}\ d(t)$$

$$Var(X) = \frac{2\sigma^2}{\sqrt{\pi}}\int_{0}^{\infty} t^{\frac{3}{2}-1} e^{-t}\ d(t) \qquad \text{(iv)}$$

As we know that Gamma function is

$$\Gamma\alpha\,\beta^{\alpha} = \int_{0}^{\infty} t^{\alpha-1}e^{-t/\beta}\ d(t) \qquad \text{v)}$$

Comparing (iv) and (v) then we get

$\alpha = \frac{3}{2}$ And $\beta = 1$ Then eq. (iv) becomes

$$Var(X) = \frac{\sigma^2}{\sqrt{\pi}}\Gamma\frac{3}{2}\,1^{\frac{3}{2}} = \frac{2\sigma^2}{\sqrt{\pi}}\Gamma\left(\frac{1}{2}+1\right)$$

$$Var(X) = \frac{2\sigma^2}{\sqrt{\pi}}\frac{1}{2}\Gamma\frac{1}{2} = \frac{\sigma^2}{\sqrt{\pi}}\sqrt{\pi} = \sigma^2$$ Therefore $\Gamma\frac{1}{2} = \sqrt{\pi}$

$Var(X) = \sigma^2$ Hence proved

Theorem.No.4: Show that Mean deviation of normal distribution is $"M.D = \frac{4}{5}\sigma"$

Proof: Let by definition

$$M.D = E|X-\mu| = \int_{-\infty}^{\infty}|X-\mu| f(x)d(x)$$

$$X \to N(\mu,\sigma^2)$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \qquad -\infty \le X \le \infty$$

$$M.D = \int_{-\infty}^{\infty}|X-\mu|\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} d(x)$$

$$M.D = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}|X-\mu|e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} d(x) \qquad \text{(i)}$$

Put $Z = \frac{X-\mu}{\sigma}$

$$Z\sigma = x-\mu$$

$$X = Z\sigma+\mu$$

$d(x) = \sigma\ d(z)$ And Limits remains same $-\infty \le Z \le \infty$

$$M.D = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}|\sigma Z|e^{-\frac{1}{2}Z^2}\ \sigma\ d(z)$$

$$Var(X) = \frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty}|Z|e^{-\frac{1}{2}Z^2}\ d(z)$$

$$M.D = \frac{\sigma}{\sqrt{2\pi}}\left[\int_{-\infty}^{0}-Ze^{-\frac{1}{2}Z^2}\ d(z)+\int_{0}^{\infty}Ze^{-\frac{1}{2}Z^2}\ d(z)\right]$$

$$M.D = \frac{\sigma}{\sqrt{2\pi}}\left[\int_{0}^{\infty}Ze^{-\frac{1}{2}Z^2}\ d(z)+\int_{0}^{\infty}Ze^{-\frac{1}{2}Z^2}\ d(z)\right]$$

$$M.D = \frac{\sigma}{\sqrt{2\pi}}\left[2\int_{0}^{\infty}Ze^{-\frac{1}{2}Z^2}\ d(z)\right]$$

$$M.D = \frac{2\sigma}{\sqrt{2\pi}}\int_{0}^{\infty}Ze^{-\frac{1}{2}Z^2}\ d(z) \qquad \text{(ii)}$$

Put $t = \frac{1}{2}Z^2$

$$2t = Z^2$$

$$\sqrt{2t} = Z$$

$$\frac{1}{2}(2t)^{-\frac{1}{2}}2d(t) = d(z)$$

$(2t)^{-\frac{1}{2}}d(t) = d(z)$ And Limits remains same $0 \le t \le \infty$

$$M.D = \frac{2\sigma}{\sqrt{2\pi}}\int_{0}^{\infty}\sqrt{2t}(2t)^{-\frac{1}{2}}e^{-t}\ d(t)$$

$$M.D = \frac{2\sigma}{\sqrt{2\pi}}\int_{0}^{\infty}\frac{\sqrt{2}\ \ t^{\frac{1}{2}}t^{-\frac{1}{2}}}{2^{\frac{1}{2}}}e^{-t}\ d(t)$$

$$M.D = \frac{2\sigma}{\sqrt{2\pi}}\int_{0}^{\infty}\frac{\sqrt{2}\ \ t^{-\frac{1}{2}+\frac{1}{2}}}{\sqrt{2}}e^{-t}\ d(t)$$

$$M.D = \frac{2\sigma}{\sqrt{2\pi}}\int_{0}^{\infty}t^{0}e^{-t}\ d(t)$$

$$Var(X)=\frac{2\sigma}{\sqrt{2\pi}}\int_0^\infty t^{1-1}e^{-t}\ d(t) \qquad \text{(iii)}$$

As we know that Gamma function is

$$\overline{)\alpha}\beta^\alpha=\int_0^\infty t^{\alpha-1}e^{-t/\beta}\ d(t) \qquad \text{(iv)}$$

Comparing (iii) and (iv) then we get

$\alpha=1$ *And* $\beta=1$ Then eq. (iv) becomes

$$M.D=\frac{2\sigma}{\sqrt{2\pi}}\overline{)1}=\frac{\sqrt{2}\sqrt{2}\sigma}{\sqrt{2}\sqrt{\pi}}=\frac{\sqrt{2}\sigma}{\sqrt{\pi}}=\sqrt{\frac{2}{\pi}}\ \sigma=0.7979\sigma=\frac{4}{5}\sigma$$ Hence proved

Theorem.No.5: Show that mean, median and mode of ND is equal to μ.

Proof: Let by definition

$$E(X)=\int_{-\infty}^{\infty}Xf(x)d(x)$$

$$X\to N(\mu,\sigma^2)$$

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \qquad -\infty\le X\le\infty$$

$$E(X)=\int_{-\infty}^{\infty}X\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}d(x)$$

$$E(X)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}Xe^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}d(x) \qquad \text{(i)}$$

Put $Z=\frac{X-\mu}{\sigma}$

$$Z\sigma=x-\mu$$

$$X=Z\sigma+\mu$$

$d(x)=\sigma\ d(z)$ And Limits remains same $-\infty\le Z\le\infty$

$$E(X)=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty}(\sigma Z+\mu)e^{-\frac{1}{2}Z^2}\ \sigma\ d(z)$$

$$E(X)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\sigma Ze^{-\frac{1}{2}Z^2}\ d(z)+\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mu e^{-\frac{1}{2}Z^2}\ d(z)$$

$$E(X)=\frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty}Ze^{-\frac{1}{2}Z^2}\ d(z)+\frac{\mu}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-\frac{1}{2}Z^2}\ d(z) \qquad \text{(ii)}$$

As $f(z)=e^{-\frac{1}{2}Z^2}$ $f(z)=Ze^{-\frac{1}{2}Z^2}$

Put $Z=-Z$

$$f(-Z)=(-Z)e^{-\frac{1}{2}(-Z)^2}=-Ze^{-\frac{1}{2}Z^2}=-f(Z)$$

$$f(-Z)=e^{-\frac{1}{2}(-Z)^2}=e^{-\frac{1}{2}Z^2}=f(Z)$$

1st function is odd and 2nd is even function then eq. (ii) becomes

$$E(X)=\frac{\sigma}{\sqrt{2\pi}}(0)+\frac{2\mu}{\sqrt{2\pi}}\int_0^\infty e^{-\frac{1}{2}Z^2}\ d(z)$$

$$E(X)=\frac{2\mu}{\sqrt{2\pi}}\int_0^\infty e^{-\frac{1}{2}Z^2}\ d(z) \qquad \text{(iii)}$$

Put $t=\frac{1}{2}Z^2$

$$\sqrt{2t}=Z$$

$$\frac{1}{2}(2t)^{-\frac{1}{2}}2d(t)=d(z)$$

$(2t)^{-\frac{1}{2}}d(t)=d(z)$ And Limits remains same $0\le t\le\infty$

$$E(X)=\frac{2\mu}{\sqrt{2\pi}}\int_0^\infty (2t)^{-\frac{1}{2}}e^{-t}\ d(t)$$

$$E(X)=\frac{2\mu}{\sqrt{2\pi}}\int_0^\infty \frac{t^{\frac{1}{2}-1}}{2^{\frac{1}{2}}}e^{-t}\ d(t)$$

$$E(X)=\frac{2\mu}{\sqrt{2}\sqrt{\pi}}\int_0^\infty \frac{t^{\frac{1}{2}-1}}{\sqrt{2}}e^{-t}\ d(t)$$

$$E(X)=\frac{\mu}{\sqrt{\pi}}\int_0^\infty t^{\frac{1}{2}-1}e^{-t}\ d(t) \qquad \text{(iv)}$$

As we know that Gamma function is

$$\Gamma\alpha\,\beta^{\alpha}=\int_0^\infty t^{\alpha-1}e^{-t/\beta}\ d(t) \qquad \text{(v)}$$

Comparing (iv) and (v) then we get

$\alpha=\frac{1}{2}$ And $\beta=1$ Then eq. (iv) becomes

$$E(X)=\frac{\mu}{\sqrt{\pi}}\Gamma\frac{1}{2}1^{\frac{1}{2}}=\frac{\mu}{\sqrt{\pi}}\Gamma\frac{1}{2}=\frac{\mu}{\sqrt{\pi}}\sqrt{\pi}=\mu \qquad \text{Therefore} \qquad \Gamma\frac{1}{2}=\sqrt{\pi}$$

$$E(X)=\mu$$

Now for median

Let by definition median of normal distribution (median=M)

$$\int_{-\infty}^{M} f(x)d(x)=\frac{1}{2}$$

$$X\to N(\mu,\sigma^2)$$

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \qquad -\infty\le X\le\infty$$

$$\int_{-\infty}^{M}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}d(x)=\frac{1}{2}$$

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{M}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}d(x)=\frac{1}{2}$$

Put $Z=\frac{X-\mu}{\sigma}$

$$Z\sigma=x-\mu$$

$$X=Z\sigma+\mu$$

$$d(x)=\sigma\ d(z)$$

Limits When $X\to-\infty$ *Then* $Z\to-\infty$ *And* $X\to M$ *Then* $Z\to\frac{M-\mu}{\sigma}$

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\frac{M-\mu}{\sigma}}e^{-\frac{1}{2}(Z)^2}\sigma\ d(Z)=\frac{1}{2}$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{M-\mu}{\sigma}}e^{-\frac{1}{2}(Z)^2}d(Z)=\frac{1}{2} \qquad \text{(i)}$$

As we know that normal distribution is symmetrical then we get

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0}e^{-\frac{1}{2}(Z)^2}d(Z)=\frac{1}{2}=\frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-\frac{1}{2}(Z)^2}d(Z) \qquad \text{(ii)}$$

Comparing (i) and (ii) then we get

$$\frac{M-\mu}{\sigma}=0$$

$$M=\mu$$

$Median = \mu$ Hence it is also equal to " μ "

Now for mode

The following two conditions are satisfied then mode is exist

$f'(x) = 0$

$f''(x) < 0$ Mean second derivative must be negative

As we know that probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$$

Differentiate with respect to "X"

$$f'(x) = \frac{d}{d(x)} f(x)$$

$$\frac{d}{d(x)} f(x) = \frac{d}{d(x)}\left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$\frac{d}{d(x)} f(x) = \frac{1}{\sigma\sqrt{2\pi}} \frac{d}{d(x)}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$\frac{d}{d(x)} f(x) = \frac{1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \frac{d}{d(x)} \frac{-1}{2}\left(\frac{X-\mu}{\sigma}\right)^2\right)$$

$$\frac{d}{d(x)} f(x) = \frac{1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \frac{-2}{2}\left(\frac{X-\mu}{\sigma}\right)\frac{d}{d(x)}\frac{X-\mu}{\sigma}\right)$$

$$\frac{d}{d(x)} f(x) = \frac{-1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)\frac{1}{\sigma}\frac{d}{d(x)}(X-\mu)\right)$$

$$\frac{d}{d(x)} f(x) = \frac{-1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)\frac{1}{\sigma}(1-0)\right)$$

$$\frac{d}{d(x)} f(x) = \frac{-1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)\frac{1}{\sigma}\right)$$

$$\frac{d}{d(x)} f(x) = \frac{-1}{\sigma^3\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}(X-\mu)\right) \qquad \text{(A)}$$

Now we equating the first derivative zero

$$0 = \frac{-1}{\sigma^3\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}(X-\mu)\right)$$

$$0 = e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}(X-\mu)$$

$$0 = (X-\mu)$$

$$X = \mu$$

Now we again differentiate eq. (A) with respect to "X"

$$f''(x) = \frac{d}{d(x)}\left(\frac{-1}{\sigma^3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}(X-\mu)\right)$$

$$f''(x) = \frac{-1}{\sigma^3\sqrt{2\pi}} \frac{d}{d(x)}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}(X-\mu)\right)$$

$$f''(x) = \frac{-1}{\sigma^3\sqrt{2\pi}}\left((X-\mu)\frac{d}{d(x)} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} + e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\frac{d}{d(x)}(X-\mu)\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left((X-\mu)e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\frac{d}{d(x)}\frac{-1}{2}\left(\frac{X-\mu}{\sigma}\right)^2+e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}(1-0)\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left((X-\mu)e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\frac{-2}{2}\left(\frac{X-\mu}{\sigma}\right)\frac{d}{d(x)}\left(\frac{X-\mu}{\sigma}\right)+e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left(-(X-\mu)e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)\frac{1}{\sigma}\frac{d}{d(x)}(X-\mu)+e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left(-(X-\mu)e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)\frac{1}{\sigma}(1-0)+e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left(-(X-\mu)e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma^2}\right)+e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left(-e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)^2+e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$f''(x)=\frac{1}{\sigma^3\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\left(\frac{X-\mu}{\sigma}\right)^2-1\right)$$

Put $X=\mu$ then we get $f''(x)<0$ thus the $Mode=\mu$.
Hence in normal distribution
$Mean = Median = Mode = \mu$ Proved

Theorem.No.6: Show that maximum value of $f(X)$ occurs at "$X=\mu$"

Proof: As we know that

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$$

Differentiate with respect to "X"

$$f'(x)=\frac{d}{d(x)}f(x)$$

$$\frac{d}{d(x)}f(x)=\frac{d}{d(x)}\left(\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$\frac{d}{d(x)}f(x)=\frac{1}{\sigma\sqrt{2\pi}}\frac{d}{d(x)}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$\frac{d}{d(x)}f(x)=\frac{1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\frac{d}{d(x)}\frac{-1}{2}\left(\frac{X-\mu}{\sigma}\right)^2\right)$$

$$\frac{d}{d(x)}f(x)=\frac{1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\frac{-2}{2}\left(\frac{X-\mu}{\sigma}\right)\frac{d}{d(x)}\frac{X-\mu}{\sigma}\right)$$

$$\frac{d}{d(x)}f(x)=\frac{-1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)\frac{1}{\sigma}\frac{d}{d(x)}(X-\mu)\right)$$

$$\frac{d}{d(x)}f(x)=\frac{-1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)\frac{1}{\sigma}(1-0)\right)$$

$$\frac{d}{d(x)}f(x)=\frac{-1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)\frac{1}{\sigma}\right)$$

$$\frac{d}{d(x)}f(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}(X-\mu)\right) \qquad \text{(A)}$$

Now we equating the first derivative zero

$$0=\frac{-1}{\sigma^3\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}(X-\mu)\right)$$

$$0=e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}(X-\mu)$$

$$0=(X-\mu)$$

$$X=\mu$$

Now we see that

$$For \quad X<\mu \quad Then \quad f'(x)>0$$

$$For \quad X>\mu \quad Then \quad f'(x)<0$$

$$For \quad X=\mu \quad Then \quad f'(x)=0$$

Thus the maximum of the function $f(x)$ at $X=\mu$ *which is* $\frac{1}{\sigma\sqrt{2\pi}}$

$$\left[f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}=\frac{1}{\sigma\sqrt{2\pi}} \quad at \quad X=\mu\right]$$

Theorem.No.7: Show that the points of inflexation in ND are equidistant from mean

Proof:

As we know that probability density function

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$$

Differentiate with respect to "X"

$$f'(x)=\frac{d}{d(x)}f(x)$$

$$\frac{d}{d(x)}f(x)=\frac{d}{d(x)}\left(\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$\frac{d}{d(x)}f(x)=\frac{1}{\sigma\sqrt{2\pi}}\frac{d}{d(x)}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$\frac{d}{d(x)}f(x)=\frac{1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\frac{d}{d(x)}\frac{-1}{2}\left(\frac{X-\mu}{\sigma}\right)^2\right)$$

$$\frac{d}{d(x)}f(x)=\frac{1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\frac{-2}{2}\left(\frac{X-\mu}{\sigma}\right)\frac{d}{d(x)}\frac{X-\mu}{\sigma}\right)$$

$$\frac{d}{d(x)}f(x)=\frac{-1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)\frac{1}{\sigma}\frac{d}{d(x)}(X-\mu)\right)$$

$$\frac{d}{d(x)}f(x)=\frac{-1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)\frac{1}{\sigma}(1-0)\right)$$

$$\frac{d}{d(x)}f(x)=\frac{-1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)\frac{1}{\sigma}\right)$$

$$\frac{d}{d(x)}f(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}(X-\mu)\right) \qquad \text{(A)}$$

Now we again differentiate eq. (A) with respect to "X"

$$f''(x)=\frac{d}{d(x)}\left(\frac{-1}{\sigma^3\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}(X-\mu)\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\frac{d}{d(x)}\left(e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}(X-\mu)\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left((X-\mu)\frac{d}{d(x)}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}+e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\frac{d}{d(x)}(X-\mu)\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left((X-\mu)e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\frac{d}{d(x)}\frac{-1}{2}\left(\frac{X-\mu}{\sigma}\right)^2+e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}(1-0)\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left((X-\mu)e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\frac{-2}{2}\left(\frac{X-\mu}{\sigma}\right)\frac{d}{d(x)}\left(\frac{X-\mu}{\sigma}\right)+e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left(-(X-\mu)e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)\frac{1}{\sigma}\frac{d}{d(x)}(X-\mu)+e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left(-(X-\mu)e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)\frac{1}{\sigma}(1-0)+e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left(-(X-\mu)e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma^2}\right)+e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left(-e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\frac{X-\mu}{\sigma}\right)^2+e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\right)$$

$$f''(x)=\frac{1}{\sigma^3\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\left(\frac{X-\mu}{\sigma}\right)^2-1\right)$$

Now we equating the 2nd derivative zero

$$0=\frac{1}{\sigma^3\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}\left(\left(\frac{X-\mu}{\sigma}\right)^2-1\right)$$

$$0=\left(\left(\frac{X-\mu}{\sigma}\right)^2-1\right)$$

$$1=\left(\frac{X-\mu}{\sigma}\right)^2$$

$$\sigma^2=(X-\mu)^2$$

$$\sqrt{\sigma^2}=\sqrt{(X-\mu)^2}$$

$$\pm\sigma=X-\mu$$

$$X=\mu\pm\sigma$$

At $X=\mu-\sigma$ Then we get $\left[f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}}=\frac{1}{\sigma\sqrt{2\pi\, e}}\right]$

At $X=\mu+\sigma$ Then we get $\left[f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}}=\frac{1}{\sigma\sqrt{2\pi\, e}}\right]$

Hence the two points of inflection of normal curve are

$\left[(\mu-\sigma)\ ,\ \frac{1}{\sigma\sqrt{2\pi\, e}}\right]$ And $\left[(\mu+\sigma)\ ,\ \frac{1}{\sigma\sqrt{2\pi\, e}}\right]$

Theorem.No.8: Derive moment generating function of normal distribution

Proof: Let by definition of m.g.f

$M_0(t) = E(e^{tx})$

$M_0(t) = \int_{-\infty}^{\infty} e^{tx} f(x) d(x)$

$X \to N(\mu, \sigma^2)$

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}$ $\quad -\infty \le X \le \infty$

$M_0(t) == \int_{-\infty}^{\infty} e^{tx} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} d(x)$

$M_0(t) == \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} d(x)$

Put $Z = \frac{X-\mu}{\sigma}$

$Z\sigma = x - \mu$

$X = Z\sigma + \mu$

$d(x) = Zd(z)$ And Limits remains same $-\infty \le Z \le \infty$

$M_0(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t(\sigma Z+\mu)} e^{-\frac{1}{2}(Z)^2} \sigma\, d(z)$

$M_0(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tZ\sigma} e^{t\mu} e^{-\frac{1}{2}(Z)^2} d(z)$

$M_0(t) = \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(Z)^2 + tZ\sigma} d(z)$

$M_0(t) = \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(Z^2 - 2tZ\sigma\right)} d(z)$

$M_0(t) = \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(Z^2 - tZ\sigma + (\sigma t)^2 - (\sigma t)^2\right)} d(z)$

$M_0(t) = \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(Z^2 - 2tZ\sigma + (\sigma t)^2\right)} e^{-\frac{1}{2}\left(-(\sigma t)^2\right)} d(z)$

$M_0(t) = \frac{e^{t\mu} e^{\frac{1}{2}(\sigma t)^2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(Z-\sigma t)^2} d(z)$

$M_0(t) = \frac{e^{\mu t + \frac{1}{2}(\sigma t)^2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(Z-\sigma t)^2} d(z)$

Put $w = z - t\sigma$

$dw = dz$

Limits remain same

$M_0(t) = \frac{e^{\mu t + \frac{1}{2}(\sigma t)^2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}w^2} d(w)$

As $f(w) = e^{-\frac{1}{2}w^2}$

Put $w = -w$

$f(-w) = e^{-\frac{1}{2}(-w)^2} = e^{-\frac{1}{2}w^2} = f(w)$ Its even function then we gets

$M_0(t) = \frac{2e^{\mu t + \frac{1}{2}(\sigma t)^2}}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-\frac{1}{2}w^2} d(w)$

Put $y = \frac{1}{2}w^2$

$\sqrt{2y} = w$

$$\frac{1}{2}(2y)^{-\frac{1}{2}}2d(y)=d(w)$$

$(2y)^{-\frac{1}{2}}d(y)=d(w)$ And Limits remains same $0\le y\le\infty$

$$M_0(t)=\frac{2e^{\mu t+\frac{1}{2}(\sigma t)^2}}{\sqrt{2\pi}}\int_0^\infty e^{-y}(2y)^{-\frac{1}{2}}d(y)$$

$$M_0(t)=\frac{2e^{\mu t+\frac{1}{2}(\sigma t)^2}}{\sqrt{2\pi}}\int_0^\infty \frac{y^{\frac{1}{2}-1}}{2^{\frac{1}{2}}}e^{-y}d(y)$$

$$M_0(t)=\frac{2e^{\mu t+\frac{1}{2}(\sigma t)^2}}{\sqrt{2\pi}}\int_0^\infty \frac{y^{\frac{1}{2}-1}}{\sqrt{2}}e^{-y}d(y)$$

$$M_0(t)=\frac{2e^{\mu t+\frac{1}{2}(\sigma t)^2}}{2\sqrt{\pi}}\int_0^\infty y^{\frac{1}{2}-1}e^{-y}d(y)$$

$$M_0(t)=\frac{e^{\mu t+\frac{1}{2}(\sigma t)^2}}{\sqrt{\pi}}\int_0^\infty y^{\frac{1}{2}-1}e^{-y}d(y) \qquad \text{(i)}$$

As we know that Gamma function is

$$\Gamma\alpha\,\beta^\alpha=\int_0^\infty y^{\alpha-1}e^{-y/\beta}\ d(y) \qquad \text{(ii)}$$

Comparing (i) and (v) then we get

$\alpha=\frac{1}{2}$ *And* $\beta=1$ Then eq. (ii) becomes

$$M_0(t)=\frac{e^{\mu t+\frac{1}{2}(\sigma t)^2}}{\sqrt{\pi}}\Gamma\frac{1}{2}1^{\frac{1}{2}}$$

$M_0(t)=\frac{e^{\mu t+\frac{1}{2}(\sigma t)^2}}{\sqrt{\pi}}\sqrt{\pi}$ Therefore $\Gamma\frac{1}{2}=\sqrt{\pi}$

$M_0(t)=e^{\mu t+\frac{1}{2}(\sigma t)^2}$ Required result

Theorem.No.9: Derive moment generating function of standardized ND

Proof: Let by definition of m.g.f

$$M_0(t)=E(e^{tZ})$$

$$M_0(t)=\int_{-\infty}^\infty e^{tz}f(z)d(z)$$

$$z\to N(0,1)$$

$f(z)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(z)^2}$ $-\infty\le Z\le\infty$

$$M_0(t)==\int_{-\infty}^\infty e^{tZ}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(Z)^2}d(Z)$$

$$M_0(t)==\frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty e^{tZ}e^{-\frac{1}{2}(Z)^2}d(Z)$$

$$M_0(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty e^{-\frac{1}{2}(Z)^2+tZ}d(z)$$

$$M_0(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty e^{-\frac{1}{2}\left(Z^2-2t\right)}d(z)$$

$$M_0(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty e^{-\frac{1}{2}\left(Z^2-tZ+(t)^2-(t)^2\right)}d(z)$$

$$M_0(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^\infty e^{-\frac{1}{2}\left(Z^2-2tZ+(t)^2\right)}e^{-\frac{1}{2}\left(-(t)^2\right)}d(z)$$

$$M_0(t)=\frac{e^{\frac{1}{2}(t)^2}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-\frac{1}{2}(Z-t)^2}d(z)$$

$$M_0(t)=\frac{e^{\mu t+\frac{1}{2}(\sigma t)^2}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-\frac{1}{2}(Z-t)^2}d(z)$$

Put $w=z-t$

$dw=dz$

Limits remain same

$$M_0(t)=\frac{e^{\frac{1}{2}(t)^2}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-\frac{1}{2}w^2}d(w)$$

As $f(w)=e^{-\frac{1}{2}w^2}$

Put $w=-w$

$f(-w)=e^{-\frac{1}{2}(-w)^2}=e^{-\frac{1}{2}w^2}=f(w)$ Its even function then we gets

$$M_0(t)=\frac{2e^{\frac{1}{2}(t)^2}}{\sqrt{2\pi}}\int_{0}^{\infty}e^{-\frac{1}{2}w^2}d(w)$$

Put $y=\frac{1}{2}w^2$

$\sqrt{2y}=w$

$\frac{1}{2}(2y)^{-\frac{1}{2}}2d(y)=d(w)$

$(2y)^{-\frac{1}{2}}d(y)=d(w)$ And Limits remains same $0\le y\le\infty$

$$M_0(t)=\frac{2e^{\frac{1}{2}(t)^2}}{\sqrt{2\pi}}\int_{0}^{\infty}e^{-y}(2y)^{-\frac{1}{2}}d(y)$$

$$M_0(t)=\frac{2e^{\frac{1}{2}t^2}}{\sqrt{2\pi}}\int_{0}^{\infty}\frac{y^{\frac{1}{2}-1}}{2^{\frac{1}{2}}}e^{-y}d(y)$$

$$M_0(t)=\frac{2e^{\frac{1}{2}t^2}}{\sqrt{2\pi}}\int_{0}^{\infty}\frac{y^{\frac{1}{2}-1}}{\sqrt{2}}e^{-y}d(y)$$

$$M_0(t)=\frac{e^{\frac{1}{2}t^2}}{\sqrt{\pi}}\int_{0}^{\infty}y^{\frac{1}{2}-1}e^{-y}d(y)$$

$$M_0(t)=\frac{e^{\frac{1}{2}t^2}}{\sqrt{\pi}}\int_{0}^{\infty}y^{\frac{1}{2}-1}e^{-y}d(y) \qquad \text{(i)}$$

As we know that Gamma function is

$$\Gamma\alpha\,\beta^{\alpha}=\int_{0}^{\infty}y^{\alpha-1}e^{-y/\beta}\ d(y) \qquad \text{(ii)}$$

Comparing (i) and (v) then we get

$\alpha=\frac{1}{2}$ *And* $\beta=1$ Then eq. (ii) becomes

$$M_0(t)=\frac{e^{\frac{1}{2}t^2}}{\sqrt{\pi}}\Gamma\frac{1}{2}\,1^{\frac{1}{2}}$$

$M_0(t)=\frac{e^{\frac{1}{2}t^2}}{\sqrt{\pi}}\sqrt{\pi}$ Therefore $\Gamma\frac{1}{2}=\sqrt{\pi}$

$M_0(t)=e^{\frac{1}{2}t^2}$ Required result

Theorem.No.10: Show those odd order moments of a normal distribution are equal to zero and derive the expression of even odder moments about mean.

Proof: Let by definition of odd odder moments

$$\mu_{2n+1} = E[X-\mu]^{2n+1} = \int_{-\infty}^{\infty} [X-\mu]^{2n+1} f(x)d(x)$$

$$X \to N(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \qquad -\infty \le X \le \infty$$

$$\mu_{2n+1} = \int_{-\infty}^{\infty} [X-\mu]^{2n+1} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} d(x)$$

$$\mu_{2n+1} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} [X-\mu]^{2n+1} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} d(x)$$

Put $Z = \frac{X-\mu}{\sigma}$

$$Z\sigma = x - \mu$$

$$X = Z\sigma + \mu$$

$d(x) = \sigma\ d(z)$ And Limits remains same $-\infty \le Z \le \infty$

$$\mu_{2n+1} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} [\sigma Z]^{2n+1} e^{-\frac{1}{2}(Z)^2} \sigma d(Z)$$

$$\mu_{2n+1} = \frac{[\sigma]^{2n+1}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [Z]^{2n+1} e^{-\frac{1}{2}(Z)^2} d(Z) \qquad \text{(i)}$$

As $f(z) = Z^{2n+1f} e^{-\frac{1}{2}Z^2}$

Put $Z = -Z$

$f(-Z) = -f(Z)$

It is odd function then eq. (i) becomes

$$\mu_{2n+1} = \frac{[\sigma]^{2n+1}}{\sqrt{2\pi}} (0) = 0 \qquad \text{Hence proved}$$

Let by definition of even odder moments

$$\mu_{2n} = E[X-\mu]^{2n} = \int_{-\infty}^{\infty} [X-\mu]^{2n} f(x)d(x)$$

$$X \to N(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \qquad -\infty \le X \le \infty$$

$$\mu_{2n} = \int_{-\infty}^{\infty} [X-\mu]^{2n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} d(x)$$

$$\mu_{2n} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} [X-\mu]^{2n} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} d(x)$$

Put $Z = \frac{X-\mu}{\sigma}$

$Z\sigma = x - \mu$ $\qquad X = Z\sigma + \mu$

$d(x) = \sigma\ d(z)$ And Limits remains same $-\infty \le Z \le \infty$

$$\mu_{2n} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} [\sigma Z]^{2n} e^{-\frac{1}{2}(Z)^2} \sigma d(Z)$$

$$\mu_{2n} = \frac{[\sigma]^{2n}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [Z]^{2n} e^{-\frac{1}{2}(Z)^2} d(Z) \qquad \text{(i)}$$

As $f(z) = Z^{2n} e^{-\frac{1}{2}Z^2}$

Put $Z = -Z$

$f(-Z) = f(Z)$

It is even function then eq. (i) becomes

$$\mu_{2n} = \frac{2\sigma^{2n}}{\sqrt{2\pi}} \int_0^\infty [Z]^{2n} e^{-\frac{1}{2}(Z)^2} d(Z)$$

$$\mu_{2n} = \frac{2\sigma^{2n}}{\sqrt{2\pi}} \int_0^\infty [Z]^{2n} e^{-\frac{1}{2}(Z)^2} d(Z)$$

$y = \frac{1}{2} Z^2$ $\qquad\qquad$ $\sqrt{2y} = Z$

$$\frac{1}{2}(2y)^{-\frac{1}{2}} 2d(y) = d(Z)$$

$(2y)^{-\frac{1}{2}} d(y) = d(Z)$ $\qquad$ And $\quad$ Limits remains same $\quad 0 \le Z \le \infty$

$$\mu_{2n} = \frac{2\sigma^{2n}}{\sqrt{2\pi}} \int_0^\infty \left[\sqrt{2y}\right]^{2n} e^{-y} (2y)^{-\frac{1}{2}} d(y)d(y)$$

$$\mu_{2n} = \frac{2\sigma^{2n}}{\sqrt{2\pi}} \int_0^\infty [2y]^{n} e^{-y} (2y)^{-\frac{1}{2}} d(y)$$

$$\mu_{2n} = \frac{[2]^n \sigma^{2n}}{\sqrt{\pi}} \int_0^\infty y^{n-\frac{1}{2}} e^{-y} d(y)$$

$$\mu_{2n} = \frac{[2]^n \sigma^{2n}}{\sqrt{\pi}} \int_0^\infty y^{n+\frac{1}{2}-1} e^{-y} d(y)$$

Comparing by Gamma function then we get

$$\mu_{2n} = \frac{[2]^n \sigma^{2n}}{\sqrt{\pi}} \Gamma\left(n + \frac{1}{2}\right)$$

$$\mu_{2n} = \frac{[2]^n \sigma^{2n}}{\sqrt{\pi}} (n - \frac{1}{2})(n - \frac{3}{2})(n - \frac{5}{2}).....\frac{5}{2}, \frac{3}{2}, \frac{1}{2} \Gamma\frac{1}{2}$$

$$\mu_{2n} = \frac{[2]^n \sigma^{2n}}{\sqrt{\pi}} (n - \frac{1}{2})(n - \frac{3}{2})(n - \frac{5}{2}).....\frac{5}{2}, \frac{3}{2}, \frac{1}{2} \sqrt{\pi}$$

$$\mu_{2n} = [2]^n \sigma^{2n} \frac{(2n-1)(2n-3)(2n-5).....5,3,1}{2 \times 2 \times 2 \times .... \times 2}$$

$$\mu_{2n} = [2]^n \sigma^{2n} \frac{(2n-1)(2n-3)(2n-5).....5,3,1}{2^n}$$

$$\mu_{2n} = \sigma^{2n} (2n-1)(2n-3)(2n-5).....5,3,1$$

Multiplying and dividing by $2n(2n-2)(2n-4)....4,2$

$$\mu_{2n} = \sigma^{2n} \frac{2n(2n-1)(2n-2)(2n-3)(2n-4)(2n-5).....5,4,3,2,1}{2n(2n-2)(2n-4)....4,2}$$

$$\mu_{2n} = \sigma^{2n} \frac{2n(2n-1)(2n-2)(2n-3)(2n-4)(2n-5).....5,4,3,2,1}{(2 \times 2 \times ... \times 2)n(n-)(n-2)....2,1}$$

$$\mu_{2n} = \sigma^{2n} \frac{(2n)!}{2^n n!} = \left(\frac{\sigma^2}{2}\right)^n \frac{(2n)!}{n!}$$

Theorem.No.12: Show that Mean deviation of normal distribution is

$"Q.D = \frac{2}{3}\sigma = 0.67\sigma"$

Proof: As we know that $Q.D = \frac{Q_3 - Q_1}{2}$ $\qquad$ (A)

First we find $"Q_3 \quad And \quad Q_1"$

Let “Q_1” be a point that contain 25% area below it. Then we get the ordinate from μ to Q_1 is 0.5-0.25=0.25

By S.N.V

$$Z = \frac{X - \mu}{\sigma}$$

$P(Z/P=0.250)=\frac{Q_1-\mu}{\sigma}$ Using area table inversely

$0.67=\frac{X-\mu}{\sigma}$ Sign is negative because left sided

$-0.67\sigma=Q_1-\mu$

$Q_1=\mu-0.67\sigma$

Let "Q_3" be a point that contain 75% area below it. Then we get ordinate from Q_3 to μ is 0.75-0.5=0.25

By S.N.V

$Z=\frac{X-\mu}{\sigma}$

$P(Z/P=0.250)=\frac{Q_3-\mu}{\sigma}$ Using area table inversely

$0.67=\frac{Q_3-\mu}{\sigma}$ Sign is positive because right sided

$0.67\sigma=Q_3-\mu$

$Q_3=\mu+0.67\sigma$

Substitute the values in (A) we get

$Q.D=\frac{\mu+0.67\sigma-(\mu-0.67\sigma)}{2}$

$Q.D=\frac{\mu+0.67\sigma-\mu+0.67\sigma}{2}=\frac{\mu 0.67\sigma+0.67\sigma}{2}=\frac{2(0.67\sigma)}{2}=0.67\sigma=\frac{2}{3}\sigma$ Proved

Importance of normal distribution

There are some importance of normal distribution are given below

i) Normal distribution is called probability distribution for errors of measurements.
ii) It is used in solving problems both in probability and in statistical inference.
iii) Many natural facts follow normal distribution.
iv) This distribution helps for drawing conclusions about population on the basis of sample information.
v) This distribution is used in many other subjects
vi) Many other distribution derived from normal distribution
vii) Shape of normal curve determined by its parameters (μ *and* σ^2)
a) μ Changes the position of normal curve along horizontal axis
b) σ^2 Indicates horizontal spread
viii) The graph of normal distribution is bell-shaped and symmetrical
ix) Normal distribution has two parameters μ and σ^2. It is denoted by $X\to N(\mu,\sigma^2)$
X) It is very important in applied statistics

Binomial approximation to normal

When total number of trials "n is so much large ($n\geq 30$) and $p\equiv q=\frac{1}{2}$ binomial probability tends to a continuous distribution known as normal distribution

i.e. $\lim_{n\to\infty} b(x;n,p)\to N(\mu,\sigma^2)$

Poisson approximation to normal distribution

When μ the mean of Poisson distribution is so much large (as it approaches to ∞) then Poisson distribution tends to a continuous distribution known as normal distribution

$\lim_{t\to\infty} P(x;\mu)=N(\mu,\sigma^2)$

Asymptotic curve

The normal curve approaches but never touches the base line (horizontal axis). So the curve is asymptotic to the horizontal axis as $X\to\pm\infty$

What do you mean by continuity correction?

Continuity correction: Binomial probability distribution is discrete which is the probability for a specified value "X" and the normal distribution is continuous, which is probability for an interval. Therefore, using binomial approximation to normal , a discrete value of binomial variable "X" is to be replaced by an interval from "X-0.5 " to "X+0.5" , before the "Z" values are computed. This sort of adjustment is called continuity correction.

For example:

i) $P(X \geq 240) = P(X \geq 239.5)$ ii) $P(X > 240) = P(X > 240.5)$

iii) $P(X \leq 240) = P(X \leq 240.5)$ iv) $P(X < 240) = P(X < 239.5)$

v) $P(240 \leq X \leq 260) = P(239.5 \leq X \leq 260.5)$

vi) $P(240 < X < 260) = P(240.5 < X < 259.5)$ ii) $P(X = 240) = P(239.5 \leq X \leq 240.5)$

Example: In a normal distribution the first moment about 10 is 40 and the fourth moment about 50 is 48. What is the arithmetic mean, variance and standard deviation.

Solution: The relationship between first raw moment and mean is $Mean = A + \mu_1'$

Given that $\mu_1' = E(X-10) = 40$ $\mu_4' = E(X-50)^4 = 48$

$D = (X-10)$ $A = 10$

So $Mean = A + \mu_1' = 10 + 40 = 50$

$\mu_4' = E(X-50)^4 = 48$ (i)

As we know that fourth moment about mean

$\mu_4 = E(X-\mu)^4$ Therefore $Mean = \mu = 50$,so it becomes

$\mu_4 = E(X-50)^4$ (ii)

Comparing (i) and (ii)

$\mu_4' = \mu_4$

So, we get

$\mu_4 = 48$

As we know that

$\mu_4 = 3\sigma^4$

$48 = 3\sigma^4$

$\sigma^4 = \frac{48}{3} = 16$ $\sigma^4 = (2)^4$

Then $\sigma = S\tan dard\ deviation = 2$ $\sigma^2 = Variance = 4$

Theorem.No.13: Prove that for the normal distribution, the quartile deviation, the mean deviation and standard deviation are approximately in the ratio $10:12:15$

Proof: As we know that

$Q.D = \frac{2}{3}\sigma$ $M.D = \frac{4}{5}\sigma$

$3Q.D = 2\sigma$

Dividing by 30 in each side

$\frac{3}{30}Q.D = \frac{2}{30}\sigma$

$\frac{1}{10}Q.D = \frac{1}{15}\sigma$ (A)

$5M.D = 4\sigma$

Dividing by 60 in each side

$\frac{5}{60}M.D = \frac{4}{60}\sigma$

$$\frac{1}{12}M.D = \frac{1}{15}\sigma \qquad \text{(B)}$$

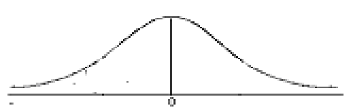
From (A) and (B)

$\frac{1}{10}Q.D = \frac{1}{12}M.D = \frac{1}{15}\sigma$ Hence the Q.D, M.D and S.D are approximately in the ratio 10:12:15 Proved

Short Question

Q.1: Give the background of normal distribution?
Ans: Normal distribution is also called "Gaussian distribution in the honor of Karl F Gauss who derived its equation. Karl Pearson named it normal distribution, in 1893. It was discovered by "Abraham de moivre (1667-1754) as a limiting form of binomial distribution, when $n \geq 30$ and $p \equiv q$

Q.2: What is the shape of normal curve?
Ans: The shape of normal curve is uni-modal, symmetrical and bell shaped as shown in the figure.



Q.3: What is the role of mean" μ " and standard deviation " σ " in the normal curve?
Ans: Mean" μ " and variance " σ^2 "are the parameters of normal distribution. Where $\mu = Mean$ is known as shape parameters and σ is known as scale parameters.

Q.4: When normal distribution changes into standard normal distribution?
Ans: When the normal random variable "X" is expressed in terms of deviations from mean in units of standard deviation the normal distribution changes into standard normal distribution.

Q.5: Define points of inflection in a normal distribution.
Ans: Those points at which concavity of curve changes are called points of inflection. There are two points of inflection which are equidistant from mean. The two points ate given as $\left(\mu - \sigma, \frac{1}{\sigma\sqrt{2\pi e}}\right)$ and $\left(\mu + \sigma, \frac{1}{\sigma\sqrt{2\pi e}}\right)$

Q.6: Explain why odd order moments about mean equal to zero for the normal distribution.
Ans: In normal curve, if nth moment is odd, the value of odd moment will always equal to zero. This is because the normal curve is symmetrical and for symmetrical distribution sum of the positive deviations from μ will always equal to sum of the negative deviations from μ and these deviations are cancel out to each other.

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