Discrete Uniform Distribution

A discrete random variable "X" is said to have a uniform distribution if its p.m.f is defined as:

$$f(x) = \begin{cases} \dfrac{1}{N} & ; x = 1, 2, \ldots, N \\ 0 & ; otherwise \end{cases}$$

Note: "N" is the **parameter** of the discrete uniform distribution.

Mean, Variance and S.D of Discrete Uniform Distribution

Measure	Formula
Mean	$\mu = \dfrac{N+1}{2}$
Variance	$\sigma^2 = \dfrac{N^2-1}{12}$
Standard Deviation	$\sigma = \sqrt{\dfrac{N^2-1}{12}}$

Prove that the mean of the discrete uniform distribution is $\frac{N+1}{2}$

Proof: We know that: $Mean = \mu = E(X) = \sum_{x=1}^{N} x f(x)$

$$\Rightarrow Mean = \mu = E(X) = \sum_{x=1}^{N} x \cdot \frac{1}{N} \qquad \left(\because f(x) = \frac{1}{N}\right)$$

$$= \frac{1}{N}\sum_{x=1}^{N} x$$

$$= \frac{1}{N}(1 + 2 + \ldots. + N)$$

$$\left(\because 1 + 2 + \ldots. + N = \frac{N(N+1)}{2}\right) \qquad = \frac{1}{N}\left(\frac{N(N+1)}{2}\right)$$

$$= \frac{N+1}{2}$$

μ (meu)

EXAMPLE 8.11

From the following series find mean and variance:
1001, 1002, 1003… 1009

Solution Here N = 9

Now $Mean = \frac{N+1}{2} = \frac{9+1}{2} = \frac{10}{2} = 5$

And $Variance = \frac{N^2 - 1}{12} = \frac{9^2 - 1}{12} = \frac{81-1}{12} = \frac{80}{12} = 6.66$

TIP: For consecutive natural numbers we may use discrete uniform distribution to find its mean and variance.

Continuous Uniform Distribution

A continuous random variable "X" is said to have a uniform distribution over the interval (a, b) if its p.d.f is defined as:

$$f(x) = \begin{cases} \dfrac{1}{b-a} & ; a \le x \le b \\ 0 & ; otherwise \end{cases}$$

Note: "*a*" and "*b*" are the **parameters** of the continuous uniform distribution

Mean, Variance and S.D of Continuous Uniform Distribution

Measure	Formula
Mean	$\mu = \frac{a+b}{2}$
Variance	$\sigma^2 = \frac{(b-a)^2}{12}$
Standard Deviation	$\sigma = \frac{(b-a)}{\sqrt{12}}$

EXAMPLE 8.12

Let X has a continuous uniform distribution under the interval (2, 5) find its Mean and Variance?

Solution Since the interval is (2, 5) therefore $2 < x < 5$

Now $Mean = \frac{a+b}{2} = \frac{2+5}{2} = \frac{7}{2} = 3.5$ $\quad (here \quad a=2 \quad and \quad b=5)$

And $Variance = \frac{(b-a)^2}{12} = \frac{(5-2)^2}{12} = \frac{(3)^2}{12} = \frac{9}{12} = 0.75$

Important!!!

While reading probability problems, pay special attention to key phrases that translate into mathematical symbols. The following table lists various phrases and their corresponding mathematical equivalents:

Math Symbol	Phrases
$>$	"greater than" or "more than" or "exceed" or "better than" or "taller than" or "above"
$<$	"less than" or "smaller than" or "below" or "under" or "fewer than"
$\geq$	"at least" or "greater than or equal to" or "no less than"
$\leq$	"at most" or "less than or equal to" or "no more than"
$=$	"exactly" or "equal" or "is"