Example 6.13: The average grade for an exam is 74, and the standard deviation is 7. If 12% of the class is given As, and the grades are curved to follow a normal distribution, what is the lowest possible A and the highest possible B?

Solution: In this example, we begin with a known area of probability, find the z value, and then determine x from the formula $x = \sigma z + \mu$. An area of 0.12, corresponding to the fraction of students receiving As, is shaded in Figure 6.20. We require a z value that leaves 0.12 of the area to the right and, hence, an area of 0.88 to the left. From Table A.3, $P(Z < 1.18)$ has the closest value to 0.88, so the desired z value is 1.18. Hence,

$$x = (7)(1.18) + 74 = 82.26.$$

Therefore, the lowest A is 83 and the highest B is 82.

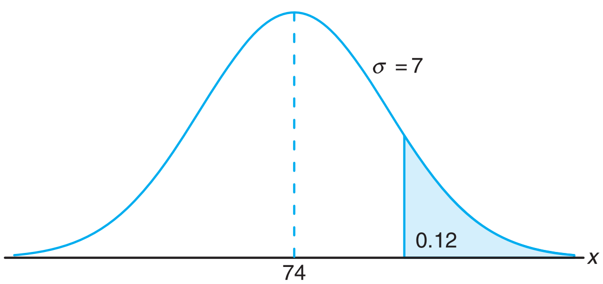

Figure 6.20: Area for Example 6.13.

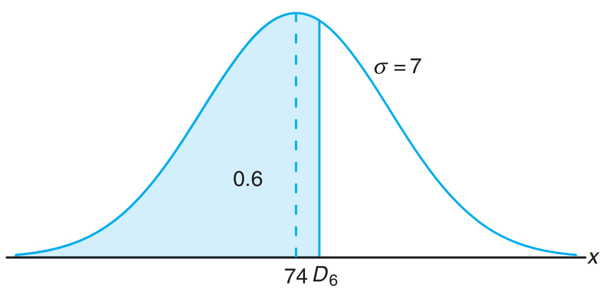

Figure 6.21: Area for Example 6.14.

Example 6.14: Refer to Example 6.13 and find the sixth decile.

Solution: The sixth decile, written D_6, is the x value that leaves 60% of the area to the left, as shown in Figure 6.21. From Table A.3 we find $P(Z < 0.25) \approx 0.6$, so the desired z value is 0.25. Now $x = (7)(0.25) + 74 = 75.75$. Hence, $D_6 = 75.75$. That is, 60% of the grades are 75 or less.

Exercises

6.1 Given a continuous uniform distribution, show that

(a) $\mu = \frac{A+B}{2}$ and

(b) $\sigma^2 = \frac{(B-A)^2}{12}$.

6.2 Suppose X follows a continuous uniform distribution from 1 to 5. Determine the conditional probability $P(X > 2.5 \mid X \le 4)$.

6.3 The daily amount of coffee, in liters, dispensed by a machine located in an airport lobby is a random variable X having a continuous uniform distribution with $A = 7$ and $B = 10$. Find the probability that on a given day the amount of coffee dispensed by this machine will be

(a) at most 8.8 liters;

(b) more than 7.4 liters but less than 9.5 liters;

(c) at least 8.5 liters.

6.4 A bus arrives every 10 minutes at a bus stop. It is assumed that the waiting time for a particular individual is a random variable with a continuous uniform distribution.