4. Negative binomial distribution

Let Bernoulli trials are independent and repeated each with probability of success (p) are performed until a certain number say "k" of success occur. The random variable of interest is the no. of trials needed to observe the "kth" success i.e. "X" is the trial number at which the "kth" success is observed them "X" is called negative binomial random variable. It is also called "Pascal random variable".

A random variable "X" is said to have negative distribution. If its probability function can be given as

$P(X)=\overset{X-1}{\underset{K-1}{C}}P^k q^{X-k}$ Where X=K,K+1,K+2,……\

Where P=Probability of success
q=Probability of failure\

OR

$P(X)=\overset{X+K-1}{\underset{X}{C}}P^k q^X$ Where X=0,1,2,……

Derivation of negative binomial distribution

In order to find the expression of the probability that "x-trials" are made achieve "k-successes" with the condition that the last trial must be success in exactly "x" independent trials with the condition that it end with success such that

$$P(x)=P[S.S....S(k-1)\,times]P[f.f.f....f(x-k)\,times]P(S)$$
$$P(x)=[P(S)P(S)....P(S)(k-1)times]P[P(f).P(f)P(f)....P(f)(x-k)\,times]P(S)$$
$$P(x)=[PPP....P(k-1)times]P[qqq....q(x-k)\,times]P$$
$$P(x)=P^{k-1}q^{x-k}P$$
$$P(x)=P^k q^{x-k}$$

Since therefore the last trial must be a success therefore the total number of mutually exclusive ways in which (k-1) success and (x-k) failure preceding the last success can occurs in any order is given by $\overset{X-1}{\underset{K-1}{C}}$ then the formula for the probability of the kth success occurs of the xth trial given as

$$P(X)=\overset{X-1}{\underset{K-1}{C}}P^k q^{X-k}$$

Show that total probability of negative binomial distribution is one

Proof: Let by definition

$$Total\ \Pr obability=\sum_{X=k}^{\infty}P(X)$$

As

$$P(X)=\overset{X-1}{\underset{K-1}{C}}P^k q^{X-k}$$
$$Total\ \Pr obability=\sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}}P^k q^{X-k}$$
$$Total\ \Pr obability=\overset{K-1}{\underset{K-1}{C}}P^K q^{K-k}+\overset{K+1-1}{\underset{K-1}{C}}P^K q^{K+1-k}+\overset{K+2-1}{\underset{K-1}{C}}P^K q^{K+2-k}+...$$
$$Total\ \Pr obability=P^K+\overset{K}{\underset{K-1}{C}}P^K q^1+\overset{K+1}{\underset{K-1}{C}}P^K q^2+...$$
$$Total\ \Pr obability=P^K+\frac{k!}{(k-1)!}P^K q^1+\frac{(k+1)!}{(k-1)!2!}P^K q^2+...$$
$$Total\ \Pr obability=P^K+kP^K q^1+\frac{k(k+1)}{2!}P^K q^2+...$$
$$Total\ \Pr obability=P^K\left(1+kq^1+\frac{k(k+1)}{2!}q^2+...\right)$$
$$Total\ \Pr obability=P^K\left(1+(-k)(-q^1)+\frac{-k(-k-1)}{2!}q^2+...\right)$$
$$Total\ \Pr obability=P^K(1-q)^{-k}\quad \text{Therefore } 1+(-k)(-q^1)+\frac{-k(-k-1)}{2!}q^2+...=(1-q)^{-k}$$
$$Total\ \Pr obability=P^K P^{-k}=1$$ Hence proved

Derive m.g.f of negative binomial distribution and use it to find mean and variance

Solution: Let by definition

$$M_0(t)=E(e^{tx})=\sum_{X=k}^{\infty}e^{tx}P(X)$$

As

$$P(X)=\overset{X-1}{\underset{K-1}{C}}P^k q^{X-k}$$

$$M_0(t)=E(e^{tx})=\sum_{X=k}^{\infty}e^{tx}\overset{X-1}{\underset{K-1}{C}}P^k q^{X-k}$$

$$M_0(t)=E(e^{tx})=\sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}}(e^t)^x P^k q^{X-k}$$

$$M_0(t)=E(e^{tx})=P^k\sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}}(e^t)^{x-k+k} q^{X-k}$$

$$M_0(t)=E(e^{tx})=P^k(e^t)^k\sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}}(e^t)^{x-k} q^{X-k}$$

$$M_0(t)=E(e^{tx})=(Pe^t)^k\sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}}(e^t q)^{x-k}$$

$$M_0(t)=E(e^{tx})=(Pe^t)^k\left(1+k(e^t q)^1+\frac{k(k+1)}{2!}(e^t q)^2+...\right)$$

$$M_0(t)=E(e^{tx})=(Pe^t)^k\left(1+(-k)(-q^1e^t)+\frac{-k(-k-1)}{2!}(qe^t)^2+...\right)$$

Therefore $\left(1-qe^t\right)^{-k}=\left(1+(-k)(-q^1e^t)+\frac{-k(-k-1)}{2!}(qe^t)^2+...\right)$

$$M_0(t)=(Pe^t)^k\left(1-qe^t\right)^{-k}$$

$$M_0(t)=(P)^k(e^t)^k(e^t)^{-k}\left(e^{-t}-q\right)^{-k}$$

$$M_0(t)=P^k\left(e^{-t}-q\right)^{-k}$$

Hence the required result

For mean and variance

$$E(X)=Mean=\mu_1'=\left[\frac{d}{dt}M_0(t)\right]_{t=0}$$

$$E(X)=Mean=\mu_1'=\left[\frac{d}{dt}P^k\left(e^{-t}-q\right)^{-k}\right]_{t=0}$$

$$E(X)=Mean=\mu_1'=P^k\left[\frac{d}{dt}\left(e^{-t}-q\right)^{-k}\right]_{t=0}$$

$$E(X)=Mean=\mu_1'=P^k\left[(-k)\left(e^{-t}-q\right)^{-k-1}\frac{d}{dt}\left(e^{-t}-q\right)\right]_{t=0}$$

$$E(X)=Mean=\mu_1'=P^k\left[(-k)\left(e^{-t}-q\right)^{-k-1}e^{-t}(-1)\right]_{t=0}$$

$$E(X)=Mean=\mu_1'=P^k\left[k\left(e^{-t}-q\right)^{-k-1}e^{-t}\right]_{t=0}$$

$$E(X)=Mean=\mu_1'=P^k\left[k\left(e^{0}-q\right)^{-k-1}e^{0}\right]$$

$$E(X)=Mean=\mu_1'=P^k\left[k\left(1-q\right)^{-k-1}\right]$$

$$E(X)=Mean=\mu_1'=P^k\left[kp^{-k-1}\right]$$

$$E(X)=Mean=\mu_1'=\left[kp^{-1}\right]$$

$$E(X)=\left[k+kq\left(p\right)^{-1}\right]=\frac{k}{p}$$

Now for variance

$$Var(X)=\mu_2=E(X^2)-[E(X)]^2=\mu_2'-[\mu_1']^2$$

$$E(X^2)=\mu_2'=\left[\frac{d^2}{dt^2}M_0(t)\right]_{t=0}=\frac{d}{dt}\left[\frac{d}{dt}M_0(t)\right]_{t=0}$$
$$E(X^2)=\mu_2'=\frac{d}{dt}\left[p^k k\left(e^{-t}-q\right)^{-k-1}e^{-t}\right]_{t=0}$$
$$E(X^2)=\mu_2'=kp^k\frac{d}{dt}\left[\left(e^{-t}-q\right)^{-k-1}e^{-t}\right]_{t=0}$$
$$E(X^2)=\mu_2'=kp^k\left[\left(e^{-t}-q\right)^{-k-1}\frac{d}{dt}e^{-t}+e^{-t}\frac{d}{dt}\left(e^{-t}-q\right)^{-k-1}\right]_{t=0}$$
$$E(X^2)=\mu_2'=kp^k\left[\left(e^{-t}-q\right)^{-k-1}e^{-t}(-1)+e^{-t}(-k-1)\left(e^{-t}-q\right)^{-k-1-1}\frac{d}{dt}\left(e^{-t}-q\right)\right]_{t=0}$$
$$E(X^2)=\mu_2'=kp^k\left[\left(e^{-t}-q\right)^{-k-1}e^{-t}(-1)+e^{-t}(-k-1)\left(e^{-t}-q\right)^{-k-2}e^{-t}(-1)\right]_{t=0}$$
$$E(X^2)=\mu_2'=kp^k\left[\left(e^{-t}-q\right)^{-k-1}e^{-t}(-1)+e^{-t}(k+1)\left(e^{-t}-q\right)^{-k-2}e^{-t}\right]_{t=0}$$
$$E(X^2)=\mu_2'=kp^k\left[\left(e^{0}-q\right)^{-k-1}e^{0}(-1)+e^{0}(k+1)\left(e^{0}-q\right)^{-k-2}e^{0}\right]$$
$$E(X^2)=\mu_2'=kp^k\left[\left(1-q\right)^{-k-1}(-1)+(k+1)\left(1-q\right)^{-k-2}\right]$$
$$E(X^2)=\mu_2'=kp^k\left[-p^{-k-1}+(k+1)p^{-k-2}\right]$$
$$E(X^2)=\mu_2'=kp^k\left[-p^{-k}p^{-1}+(k+1)p^{-2}p^{-k}\right]$$
$$E(X^2)=\mu_2'=kp^k p^{-k}\left[-p^{-1}+(k+1)p^{-2}\right]$$
$$E(X^2)=\mu_2'=k\left[-p^{-1}+(k+1)p^{-2}\right]$$
$$E(X^2)=\mu_2'=-kp^{-1}+k^2p^{-2}+kp^{-2}$$

$$Var(X)=\mu_2=E(X^2)-[E(X)]^2=\mu_2'-[\mu_1']^2$$
$$Var(X)=-kp^{-1}+k^2p^{-2}+kp^{-2}-\left[kp^{-1}\right]^2$$
$$Var(X)=-kp^{-1}+k^2p^{-2}+kp^{-2}-k^2p^{-2}$$
$$Var(X)=-kp^{-1}+kp^{-2}$$
$$Var(X)=kp^{-2}-kp^{-1}$$
$$Var(X)=k\left(\frac{1}{p^2}-\frac{1}{p}\right)$$
$$Var(X)=k\left(\frac{1-p}{p^2}\right)$$
$$Var(X)=k\frac{q}{p^2}$$

Prove that in negative binomial distribution

$$\mu_{r+1}=q\left[\frac{rk}{p^2}\mu_{r-1}-\frac{d}{dp}\mu_r\right]$$

Also find first four moments about mean use it to find moments ratios.

Solution:

Let we consider

$$\mu_r=E(X-mean)^r$$
$$\mu_r=\sum_X (X-mean)^r P(X)$$

As $X\to NB(P)$

$$P(X)=\overset{X-1}{\underset{K-1}{C}}P^kq^{X-k}\qquad \text{Where}\qquad X=K,K+1,K+2,\ldots\ldots$$

$$E(X)=Mean=\frac{k}{p}$$
$$\mu_r=\sum_{X=k}^{\infty}(X-\frac{k}{p})^r\overset{X-1}{\underset{K-1}{C}}P^kq^{X-k}$$

$$\mu_r = \sum_{X=k}^{\infty}(X-\frac{k}{p})^r \overset{X-1}{\underset{K-1}{C}} P^k(1-p)^{X-k}$$

Differentiate with respect to "P"

$$\frac{d}{dp}\mu_r = \frac{d}{dp}\left[\sum_{X=k}^{\infty}(X-\frac{k}{p})^r \overset{X-1}{\underset{K-1}{C}} P^k(1-p)^{X-k}\right]$$

$$\frac{d}{dp}\mu_r = \sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}} P^k(1-p)^{X-k}\frac{d}{dp}(X-\frac{k}{p})^r + \sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}}(X-\frac{k}{p})^r(\text{q})^{X-k}\frac{d}{dp}P^k + \sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}}(X-\frac{k}{p})^r P^k\frac{d}{dp}(1-p)^{X-k}$$

$$\frac{d}{dp}\mu_r = \sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}} P^k(1-p)^{X-k} r(X-\frac{k}{p})^{r-1}(\frac{k}{p^2}) + \sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}}(X-\frac{k}{p})^r(1-p)^{X-k}kP^{k-1}$$
$$+\sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}}(X-\frac{k}{p})^r P^k(x-k)(1-p)^{X-k-1}(-1)$$

$$\frac{d}{dp}\mu_r = r\sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}} P^k(1-p)^{X-k}(X-\frac{k}{p})^{r-1}(\frac{k}{p^2}) + \sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}}(X-\frac{k}{p})^r(1-p)^{X-k}P^k\frac{k}{p}$$
$$-\sum_{X=k}^{\infty}\overset{X-1}{\underset{K-1}{C}}(X-\frac{k}{p})^r P^k(x-k)(1-p)^{X-k}q^{-1}$$

$$\frac{d}{dp}\mu_r = r\sum_{X=k}^{\infty}(X-\frac{k}{p})^{r-1}\overset{X-1}{\underset{K-1}{C}} P^k(1-p)^{X-k}(\frac{k}{p^2}) + \sum_{X=k}^{\infty}(X-\frac{k}{p})^r\overset{X-1}{\underset{K-1}{C}}(1-p)^{X-k}P^k\frac{k}{p}$$
$$-\sum_{X=k}^{\infty}(X-\frac{k}{p})^r\overset{X-1}{\underset{K-1}{C}} P^k(1-p)^{X-k}(x-k)q^{-1}$$

$$\frac{d}{dp}\mu_r = \frac{rk}{p^2}\sum_{X=k}^{\infty}(X-mean)^{r-1}P(X) + \sum_{X=k}^{\infty}(X-mean)^r P(X)\frac{k}{p} - \sum_{X=k}^{\infty}(X-mean)^r P(X)(x-k)q^{-1}$$

$$\frac{d}{dp}\mu_r = \frac{rk}{p^2}\sum_{X=k}^{\infty}(X-mean)^{r-1}P(X) + \sum_{X=k}^{\infty}(X-mean)^r P(X)\left[\frac{k}{p}-\frac{(x-k)}{q}\right]$$

$$\frac{d}{dp}\mu_r = \frac{rk}{p^2}\sum_{X=k}^{\infty}(X-mean)^{r-1}P(X) + \sum_{X=k}^{\infty}(X-mean)^r P(X)\left[\frac{kq-(x-k)p}{pq}\right]$$

$$\frac{d}{dp}\mu_r = \frac{rk}{p^2}\sum_{X=k}^{\infty}(X-mean)^{r-1}P(X) + \sum_{X=k}^{\infty}(X-mean)^r P(X)\left[\frac{k(1-p)-xp+kp}{pq}\right]$$

$$\frac{d}{dp}\mu_r = \frac{rk}{p^2}\sum_{X=k}^{\infty}(X-mean)^{r-1}P(X) + \sum_{X=k}^{\infty}(X-mean)^r P(X)\left[\frac{k-kp-px+pk}{pq}\right]$$

$$\frac{d}{dp}\mu_r = \frac{rk}{p^2}\sum_{X=k}^{\infty}(X-mean)^{r-1}P(X) + \sum_{X=k}^{\infty}(X-mean)^r P(X)\left[\frac{k-px}{pq}\right]$$

$$\frac{d}{dp}\mu_r = \frac{rk}{p^2}\sum_{X=k}^{\infty}(X-mean)^{r-1}P(X) + \sum_{X=k}^{\infty}(X-mean)^r P(X)(-p)\left[\frac{x-\frac{k}{p}}{pq}\right]$$

$$\frac{d}{dp}\mu_r = \frac{rk}{p^2}\sum_{X=k}^{\infty}(X-mean)^{r-1}P(X) - \sum_{X=k}^{\infty}(X-mean)^r P(X)\left[\frac{x-mean}{q}\right]$$

$$\frac{d}{dp}\mu_r = \frac{rk}{p^2}\sum_{X=k}^{\infty}(X-mean)^{r-1}P(X) - \frac{1}{q}\sum_{X=k}^{\infty}(X-mean)^{r+1}P(X)$$

$$\frac{d}{dp}\mu_r = \frac{rk}{p^2}\mu_{r-1} - \frac{1}{q}\mu_{r+1}$$

$$\frac{1}{q}\mu_{r+1} = \frac{rk}{p^2}\mu_{r-1} - \frac{d}{dp}\mu_r$$

$$\mu_{r+1} = q\left[\frac{rk}{p^2}\mu_{r-1} - \frac{d}{dp}\mu_r\right] \qquad \text{(A)}$$

Hence proved

Now we find first four moments about mean

Put "r=1" in eq (A)

$$\mu_{1+1} = q\left[\frac{rk}{p^2}\mu_{1-1} - \frac{d}{dp}\mu_1\right]$$

$$\mu_2 = q\left[\frac{rk}{p^2}\mu_0 - \frac{d}{dp}\mu_1\right]$$ Therefore $\mu_1 = 0 \quad and \quad \mu_0 = 1$

$$\mu_2 = \frac{qk}{p^2}$$

Put "r=2" in eq (A)

$$\mu_{2+1} = q\left[\frac{2k}{p^2}\mu_{2-1} - \frac{d}{dp}\mu_2\right]$$

$$\mu_3 = q\left[\frac{2k}{p^2}\mu_1 - \frac{d}{dp}\mu_2\right]$$ Therefore $\mu_1 = 0 \quad and \quad \mu_0 = 1$ and $\mu_2 = \frac{qk}{p^2}$

$$\mu_3 = q\left[0 - \frac{d}{dp}\frac{qk}{p^2}\right] = -q\left[\frac{d}{dp}\frac{(1-p)k}{p^2}\right] = -qK\left[\frac{p^2\frac{d}{dp}(1-p) - (1-p)\frac{d}{dp}p^2}{(p^2)^2}\right] = -qK\left[\frac{p^2(-1) - (1-p)2p}{(p^2)^2}\right]$$

$$\mu_3 = -qK\left[\frac{-p^2 - 2p + 2p^2}{(p^2)^2}\right] = -qK\left[\frac{p^2 - 2p}{p^4}\right] = -qKp\left[\frac{p-2}{p^4}\right] = -qK\left[\frac{p-2}{p^3}\right] = qK\left[\frac{2-p}{p^3}\right]$$

Put "r=3" in eq (A)

$$\mu_{3+1} = q\left[\frac{3k}{p^2}\mu_{3-1} - \frac{d}{dp}\mu_3\right]$$

$$\mu_4 = q\left[\frac{3k}{p^2}\mu_2 - \frac{d}{dp}\mu_3\right]$$ Therefore $\mu_2 = \frac{qk}{p^2}$ and $\mu_3 = qK\left[\frac{2-p}{p^3}\right]$

$$\mu_4 = q\left[\frac{3k}{p^2}\frac{qk}{p^2} - \frac{d}{dp}\frac{qk(2-p)}{p^3}\right] = q\left[\frac{3qk^2}{p^4} - k\frac{d}{dp}(1-p)(2-p)p^{-3}\right]$$

$$\mu_4 = q\left[\frac{3qk^2}{p^4}\right] - kq\left[(2-p)p^{-3}\frac{d}{dp}(1-p) + (1-p)p^{-3}\frac{d}{dp}(2-p) + (1-p)(2-p)\frac{d}{dp}p^{-3}\right]$$

$$\mu_4 = q\left[\frac{3qk^2}{p^4}\right] - kq\left[(2-p)p^{-3}(-1) + (1-p)p^{-3}(-1) + (1-p)(2-p)(-3)p^{-4}\right]$$

$$\mu_4 = q\left[\frac{3qk^2}{p^4}\right] + kq\left[(2-p)p^{-3} + (1-p)p^{-3} + 3(1-p)(2-p)p^{-4}\right]$$

$$\mu_4 = q\left[\frac{3qk^2}{p^4}\right] + kq\left[2p^{-3} - p^{-2} + p^{-3} - p^{-2} - (3-3p)(2p^{-4} - p^{-3})\right]$$

$$\mu_4 = q\left[\frac{3qk^2}{p^4}\right] + kq\left[3p^{-3} - 2p^{-2} - 6p^{-4} + 3p^{-3} + 6p^{-3} - 3p^{-2}\right]$$

$$\mu_4 = q\left[\frac{3qk^2}{p^4}\right] + kq\left[3p^{-3} - 2p^{-2} - 6p^{-4} + 3p^{-3} + 6p^{-3} - 3p^{-2}\right]$$

$$\mu_4 = \frac{qk}{p^4}(3kq - 6p + 6 + p^2)$$

If $K \to \infty$ then negative binomial distribution tends to Poisson distribution

Proof: As we know the probability function of negative binomial distribution

$$P(X) = \overset{X+k-1}{\underset{x}{C}} P^k q^X$$ X=0,1,2,3,…

As we know mean of NBD is $Mean = \frac{kq}{P}$

Mean of passion distribution $Mean = \lambda$

Then we get

$$\lambda = \frac{kq}{P}$$

$$\lambda P = kq$$

$$\lambda(1-q) = kq$$

$$\lambda - \lambda q = kq$$

$$\lambda = \lambda q + kq = q(\lambda + k)$$

$$q = \frac{\lambda}{\lambda + k}$$

$$P(X) = \overset{X+k-1}{\underset{x}{C}} (1-q)^k q^X$$

$$P(X) = \frac{(X+k-1)!}{X!(X+k-1-X)!}(1-\frac{\lambda}{\lambda+k})^k (\frac{\lambda}{\lambda+k})^X$$

$$P(X) = \frac{(X+k-1)!}{X!(k-1)!}(\frac{\lambda+k-\lambda}{\lambda+k})^k (\frac{\lambda}{\lambda+k})^X$$

$$P(X) = \frac{(X+k-1)(X+k-2)(X+k-3)..(X+k-1+1-X)(X+k-1-X)!}{X!(X+k-1-X)!} \frac{k^{,k}}{(\lambda+k)^k} \frac{\lambda^X}{(\lambda+k)^X}$$

$$P(X) = \frac{k(1+\frac{X-1}{k})k(1+\frac{X-2}{k})k(1+\frac{X-3}{k})...k}{X!} \frac{k^{,k}}{k^{,k}(\frac{\lambda}{k}+1)^k} \frac{\lambda^X}{k^{,X}(\frac{\lambda}{k}+1)^X}$$

$$P(X) = \frac{k(1+\frac{X-1}{k})k(1+\frac{X-2}{k})k(1+\frac{X-3}{k})...k}{X!} \frac{1}{(\frac{\lambda}{k}+1)^k} \frac{\lambda^X}{k^{,X}(\frac{\lambda}{k}+1)^X}$$

$$P(X) = \frac{k^{,X}\left[(1+\frac{X-1}{k})(1+\frac{X-2}{k})(1+\frac{X-3}{k})...1\right]}{X!} \frac{1}{(\frac{\lambda}{k}+1)^k} \frac{\lambda^X}{k^{,X}(\frac{\lambda}{k}+1)^X}$$

$$P(X) = \frac{\left[(1+\frac{X-1}{k})(1+\frac{X-2}{k})(1+\frac{X-3}{k})...1\right]}{X!} \frac{1}{(\frac{\lambda}{k}+1)^k} \frac{\lambda^X}{(\frac{\lambda}{k}+1)^X}$$

Now applying limit $k \to \infty$

$$P(X) = \lim i_{k\to\infty} \frac{\left[(1+\frac{X-1}{k})(1+\frac{X-2}{k})(1+\frac{X-3}{k})...1\right]}{X!} \frac{1}{(\frac{\lambda}{k}+1)^k} \frac{\lambda^X}{(\frac{\lambda}{k}+1)^X}$$

$$P(X) = \frac{\lambda^X}{X!} \lim i_{k\to\infty} \frac{\left[(1+\frac{X-1}{k})(1+\frac{X-2}{k})(1+\frac{X-3}{k})...1\right]}{(\frac{\lambda}{k}+1)^k (\frac{\lambda}{k}+1)^X}$$

$$P(X) = \frac{\lambda^X}{X!} \lim i_{k\to\infty} \frac{[(1+0)(1+0)(1+0)...1]}{(\frac{\lambda}{k}+1)^k (0+1)^X}$$

$$P(X) = \frac{\lambda^X}{X!} \lim i_{k\to\infty} \frac{1}{(\frac{\lambda}{k}+1)^{k\frac{\lambda}{\lambda}}}$$

$$P(X) = \frac{\lambda^X}{X!} \frac{1}{\left[\lim i_{k\to\infty} (\frac{\lambda}{k}+1)^{\frac{K}{\lambda}}\right]^\lambda} = \frac{\lambda^X}{X!} \frac{1}{[e]^\lambda} = \frac{\lambda^X e^{-\lambda}}{X!}$$

$$P(X) = \frac{\lambda^X e^{-\lambda}}{X!}$$

Hence Proved

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