Probability

In our daily life we often make the statements such as:

- It will probably rain today
- I will probably go abroad this year
- He is almost certain that he will win this game

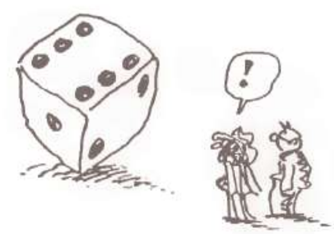
All these statements are related with uncertainty and can be measured numerically by means of "probability". Thus we may simply define probability as ***"the numeric measure of uncertainty is called probability"***.

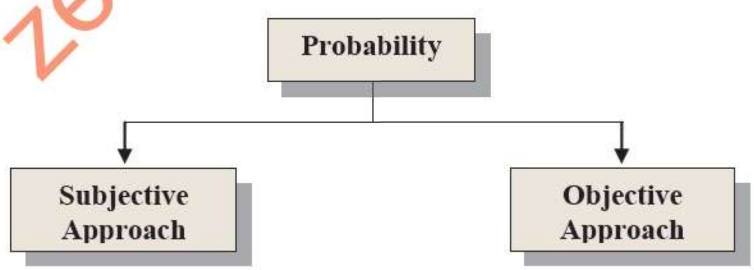
Though probability started with gambling, it has been used extensively in the fields of Physical Sciences, Commerce, Biological Sciences, Medical Sciences, Weather Forecasting, etc.

Definition of Probability

Usually probability of an event is defined by adopting any of the following two approaches:

1) Subjective approach
2) Objective approach





Subjective Approach

In subjective approach the probability of an event is defined as ***"the measure of believe in the occurrence of an event by a particular person"***. Probability in this sense is purely subjective, and is based on whatever evidence is available to the person.

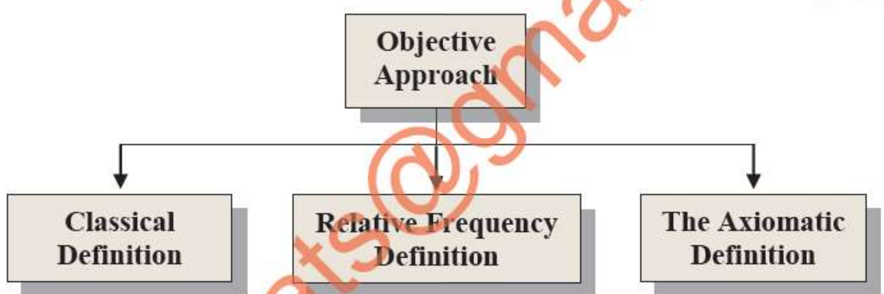
For example:

- A sports-writer may say that there is a 70% probability that Australia will win the world cup.
- A physician might say that, there is a 30% chance the patient will need an operation etc.

Objective Approach

In Objective approach, the probability of an event is defined in the following three ways:

- Classical or Priori or Theoretical Definition of Probability
- Relative Frequency or Empirical or Experimental Definition of Probability
- The Axiomatic Definition of Probability



Classical Definition

"If a random experiment can produce "n" mutually exclusive and equally likely outcomes, and if "m" of these outcomes are favorable to the occurrence of an event "A", then the probability of the event "A" is equal to the ratio m/n" If we take P(A) as "the probability of A" then:

$$P(A) = \frac{m}{n} = \frac{No.\, of\ \ favourable\, outcomes}{No.\, of\ \ possible\, outcomes}$$

- For example, when a fair Coin is tossed, then we know in **advance** that the possible outcomes are Head and Tail. Since the Head and Tail are **equally likely**, therefore, the probability of each is 1/2 or 0.5.

Historical Note

The classical definition was formulated by the French mathematician P.S. Laplace

Relative Frequency Definition

"If "m" is the number of occurrences of an event "A" in large number of trials "n", then the probability of "A" is the relative frequency of "m" and "n" as the number of trials grows infinitely large" If we take P(A) as "the probability of A" then:

$$P(A) = \lim_{n\to\infty}\left(\frac{m}{n}\right)$$

- For example, if a coin has been **loaded** (unfair), then the probability of Head and Tail will not be equal to 0.5 i.e. the Head and Tail are **not equally likely**. Thus for experiments not having equally likely outcomes if we flip the coin 10 times, say, and observe 4 heads, then, based on this information, we say that the chance of observing a head will be 4/10 or 0.4, which is not the same as 0.5. If, however, we flip the coin a large number of times, we would expect about 50 percent of the flips result in a head.

The Axiomatic Definition

Let S be a sample space with the sample points A_1, A_2 ... A_i ...A_n. To each sample point, we assign a real number, denoted by $P(A_i)$, and called the probability of A_i, that must satisfy the following basic axioms:

- **Axiom 1:** *For any event A_i* $0 \le P(A_i) \le 1$
- **Axiom 2:** $P(S) = 1$
- **Axiom 3:** *If A_i and A_j are mutually exclusive events, Then* $P(A_i \cup A_j) = P(A_i) + (A_j)$

In this case $P(A_i)$ is defined by the formula:

$$P(A_i) = \frac{n(A_i)}{n(S)} = \frac{No.\ of\ sample\ points\ in\ the\ event\ A_i}{No.\ of\ sample\ points\ in\ the\ sample\ space}$$

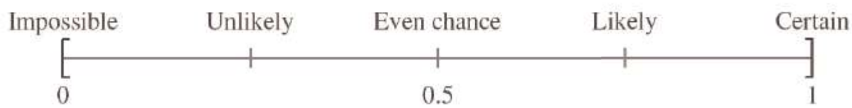
Historical Note

The Axiomatic definition was introduced in 1933 by the Russian mathematician. A.N. Kolmogorov

Subjective probability is purely subjective i.e. that two or more persons faced with the same evidence may arrive at different probabilities. On the other hand, objective probability relates to those situations where everyone will arrive at the same conclusion.

Range of Probability

If the probability of an event is **1**, the event is **certain** to occur. If the probability of an event is **0**, the event is **impossible**. A probability of **0.5** indicates that an event has an **even** chance of occurring. The following graph shows the possible range of probabilities and their meanings.



REMEMBER

The closer the probability is to 1, the more likely is an event will occur. Similarly, The closer the probability is to 0, the less likely is an event will occur.

EXAMPLE 6.24

A fair coin is tossed only once what is the probability that a Head will appear?

Solution Since a coin is tossed

Therefore $S=\{H,T\} \Rightarrow n(S)=2$

Let "A" denotes the event of getting "a Head"

Then $A=\{H\} \Rightarrow n(A)=1$

Hence $P(A)=\dfrac{n(A)}{n(S)}=\dfrac{1}{2}=0.50$

EXAMPLE 6.25

Two fair coins are tossed simultaneously, what is the probability that at least one head will appear?

Solution Since two coins are tossed

Therefore $S=\{HH,HT,TH,TT\} \Rightarrow n(S)=4$

Let "A" denotes the event of getting "at least one Head"

Then $A=\{HH,HT,TH\} \Rightarrow n(A)=3$

Hence $P(A)=\dfrac{n(A)}{n(S)}=\dfrac{3}{4}$

TIP

Probabilities should be expressed as reduced fractions or rounded to two or three decimal places. When the probability of an event is an extremely small decimal, it is permissible to round the decimal to the first nonzero digit after the point. For example, 0.0000587 would be 0.00006

EXAMPLE 6.26

A die is rolled find the probability of getting a six?

Solution Since a die is rolled

Therefore $S = \{1,2,3,4,5,6\} \Rightarrow n(S) = 6$

Let "A" denotes the event of getting "a six"

Then $A = \{6\} \Rightarrow n(A) = 1$

Hence $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{1}{6}$

REMEMBER

Probabilities can be expressed as fractions, decimals, or percentages. If you ask, "What is the probability of getting a head when a coin is tossed?" typical responses can be any of the following three. "1/2" "0.5" "50%" These answers are all equivalent.

EXAMPLE 6.27

Two dice are rolled, find the probability that the sum is:

(1)	Exactly "5"	(2)	At least "9"	(3)	At most "4"
(4)	Even	(5)	Less than "3		

Solution Since two dice are rolled therefore:

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

1) The sum is "Exactly "5"

Let "A" be an event of getting "sum is exactly 5"

Then $A = \begin{Bmatrix} (1,4),(2,3), \\ (3,2),(4,1) \end{Bmatrix} \Rightarrow n(A) = 4$

Hence $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{4}{36}$

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

2) The sum is "At least "9"

Let "B" be an event of getting "sum is at least 9" then

$$S=\begin{Bmatrix}(1,1)&(1,2)&(1,3)&(1,4)&(1,5)&(1,6)\\(2,1)&(2,2)&(2,3)&(2,4)&(2,5)&(2,6)\\(3,1)&(3,2)&(3,3)&(3,4)&(3,5)&(3,6)\\(4,1)&(4,2)&(4,3)&(4,4)&(4,5)&(4,6)\\(5,1)&(5,2)&(5,3)&(5,4)&(5,5)&(5,6)\\(6,1)&(6,2)&(6,3)&(6,4)&(6,5)&(6,6)\end{Bmatrix}$$

$$B=\begin{Bmatrix}(3,6),(4,5),(5,4),(6,3),(4,6),\\(5,5),(6,4),(5,6),(6,5),(6,6)\end{Bmatrix}$$

$$\Rightarrow n(B)=10$$

Hence $P(B)=\dfrac{n(B)}{n(S)}=\dfrac{10}{36}$

3) The sum is "At most "4"

Let "C" be an event of getting "sum is at most 4" then

$$S=\begin{Bmatrix}(1,1)&(1,2)&(1,3)&(1,4)&(1,5)&(1,6)\\(2,1)&(2,2)&(2,3)&(2,4)&(2,5)&(2,6)\\(3,1)&(3,2)&(3,3)&(3,4)&(3,5)&(3,6)\\(4,1)&(4,2)&(4,3)&(4,4)&(4,5)&(4,6)\\(5,1)&(5,2)&(5,3)&(5,4)&(5,5)&(5,6)\\(6,1)&(6,2)&(6,3)&(6,4)&(6,5)&(6,6)\end{Bmatrix}$$

$$C=\begin{Bmatrix}(1,1),(1,2),(1,3),\\(2,1),(2,2),(3,1)\end{Bmatrix}\Rightarrow n(C)=6$$

Hence $P(C)=\dfrac{n(C)}{n(S)}=\dfrac{6}{36}$

4) The sum is "Even"

Let "D" be an event of getting "sum is even" then

$$D=\begin{Bmatrix}(1,1),(1,3),(2,2),\\(3,1),(1,5),(2,4),\\(3,3),(4,2),(5,1),\\(2,6),(3,5),(4,4),\\(5,3),(6,2),(4,6),\\(5,5),(6,4),(6,6)\end{Bmatrix}\Rightarrow n(D)=18$$

$$S=\begin{Bmatrix}(1,1)&(1,2)&(1,3)&(1,4)&(1,5)&(1,6)\\(2,1)&(2,2)&(2,3)&(2,4)&(2,5)&(2,6)\\(3,1)&(3,2)&(3,3)&(3,4)&(3,5)&(3,6)\\(4,1)&(4,2)&(4,3)&(4,4)&(4,5)&(4,6)\\(5,1)&(5,2)&(5,3)&(5,4)&(5,5)&(5,6)\\(6,1)&(6,2)&(6,3)&(6,4)&(6,5)&(6,6)\end{Bmatrix}$$

Hence $P(D)=\dfrac{n(D)}{n(S)}=\dfrac{18}{36}$

5) The sum is "Less than "3"

Let "D" be an event of getting "is less than 3" then

$E=\{(1,1)\} \Rightarrow n(E)=1$

Hence $P(E)=\dfrac{n(E)}{n(S)}=\dfrac{1}{36}$

$$S=\begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

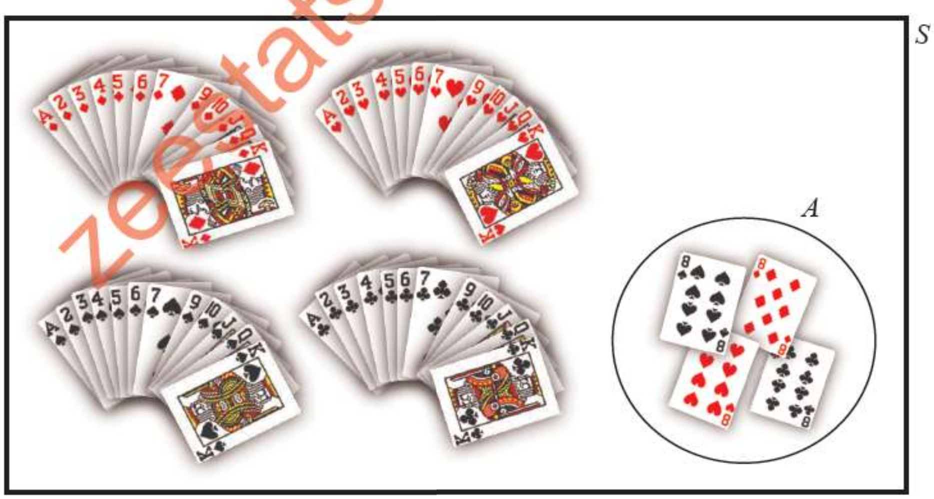
EXAMPLE 6.28

A card is drawn at random from an ordinary pack of 52 playing cards. Find the probability that the card drawn is "8"?

Solution Since a card is drawn therefore

$S=\{\textit{the pack of 52 cards}\} \Rightarrow n(S)=\binom{52}{1}=52$

Eights	Others	Total
4	48	52



Let "A" be the event that "the card is eight" Then $n(A)=\binom{4}{1}=4$

Hence $P(A)=\dfrac{n(A)}{n(S)}=\dfrac{4}{52}=\dfrac{1}{13}$

EXAMPLE 6.29

A basket contains 5 white and 4 black balls; what is the probability of selecting 3 white balls?

White	Black	Total
5	4	9

Solution Since "3" balls are selected out of "9"

Therefore $n(S)=\binom{9}{3}=84$

Let "W" be the event of "selecting 3 white balls"

Then $n(W)=\binom{5}{3}=10$

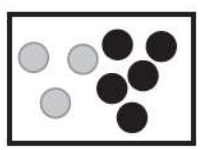
Hence $P(W)=\dfrac{n(W)}{n(S)}=\dfrac{10}{84}$

EXAMPLE 6.30

A box contains 3 gray and 5 black balls. If 4 balls are drawn together from the box then find the probability of getting:

(i) At least 2 black balls (ii) At most 2 gray balls.

Solution Since 4 balls are drawn from 8 balls:

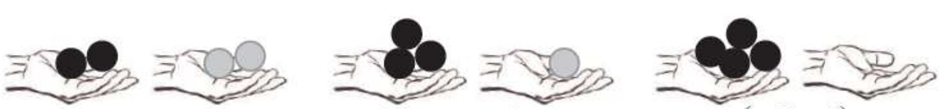


Gray	Black	Total
3	5	8

Therefore $n(S)=\binom{8}{4}=70$

Let "A" be an event of getting "at least 2 black balls i.e. two or more black balls:

Now "A" can occur in the following mutually exclusive ways:



$\binom{2\ black}{2\ gray}$ *OR* $\binom{3\ black}{1\ gray}$ *OR* $\binom{4\ black}{0\ gray}$

$$\therefore n(A) = \binom{5}{2}\binom{3}{2} \text{ or } \binom{5}{3}\binom{3}{1} \text{ or } \binom{5}{4}\binom{3}{0} \Rightarrow n(A) = \binom{5}{2}\binom{3}{2} + \binom{5}{3}\binom{3}{1} + \binom{5}{4}\binom{3}{0} = 65$$

Hence $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{65}{70} = \dfrac{13}{14}$

Let "B" be an event of getting "at most 2 black balls i.e. two or less black balls:

Now "B" can occur in the following mutually exclusive ways:

$$\begin{pmatrix} 2\,black \\ 2\,gray \end{pmatrix} \quad OR \quad \begin{pmatrix} 1\,black \\ 3\,gray \end{pmatrix}$$

$$\therefore n(B) = \binom{5}{2}\binom{3}{2} \text{ or } \binom{5}{1}\binom{3}{3} \Rightarrow n(B) = \binom{5}{2}\binom{3}{2} + \binom{5}{1}\binom{3}{3} = 60$$

Hence $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{60}{70} = \dfrac{6}{7}$

Test Yourself

1) A fair coin is tossed only once what is the probability that a Tail will appear?
2) Two fair coins are tossed, what is the probability that at least two head will appear?
3) A die is rolled find the probability of getting a four?
4) Two dice are rolled, find the probability that the sum is:

   (i) Exactly "4" (ii) At least "10" (iii) At most "5"
   (iv) Odd (v) Less than "2"

5) A card is drawn at random from an ordinary pack of 52 playing cards. Find the probability that the card drawn is "picture"?
6) A basket contains 6 white and 3 black balls; what is the probability of selecting 4 white balls?
7) A box contains 4 gray and 6 black balls. If 4 balls are drawn together from the box then find the probability of getting:

   (i) At least 2 black balls (ii) At most 3 gray balls.

Addition Rule of prbability for Mutually Exclusive Events

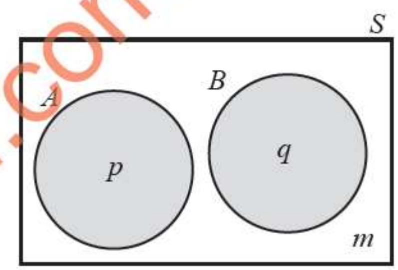
Statement: *Let "A" and "B" are two mutually exclusive events then the probability that "A" or "B" occurs is equal to the probability that "A" occurs plus the probability that "B" occurs i.e.*

$$P(A \text{ or } B) = P(A) + P(B)$$

OR

$$P(A \cup B) = P(A) + P(B)$$

Proof: To prove the theorem, consider the two Mutually Exclusive events "A" and "B" in the Venn-diagram:



$n(A \cup B) = p + q$ is shaded

It is clear from the Venn-diagram that:

$n(S) = m$
$n(A) = p$
$n(B) = q$
$n(A \cup B) = p + q$

Now

$$P(A) = \frac{n(A)}{n(S)} = \frac{p}{m}$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{q}{m}$$

Therefore

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{p+q}{m} = \frac{p}{m} + \frac{q}{m} = P(A) + P(B)$$

$\Rightarrow P(A \cup B) = P(A) + P(B)$ Hence proved

EXAMPLE 6.31

Suppose that we roll a pair of dice, what is the probability of getting a sum of 5 or a sum of 11?

Solution Since a pair of dice is rolled therefore:

$$S=\begin{Bmatrix}(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6)\\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6)\\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6)\\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6)\\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6)\\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6)\end{Bmatrix}$$

Let "A" be an event of getting "sum is exactly 5" then

$$A=\begin{Bmatrix}(1,4),(2,3),\\ (3,2),(4,1)\end{Bmatrix}\Rightarrow n(A)=4$$

$$\Rightarrow P(A)=\frac{n(A)}{n(S)}=\frac{4}{36}$$

$$S=\begin{Bmatrix}(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6)\\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6)\\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6)\\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6)\\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6)\\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6)\end{Bmatrix}$$

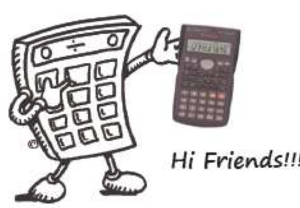
Let "B" be an event of getting "sum is 11"

Then $B=\{(5,6),(6,5)\}\Rightarrow n(B)=2$

$$\Rightarrow P(B)=\frac{n(B)}{n(S)}=\frac{2}{36}$$

Now we have to find $P(A\ or\ B)$ and since the two events "A" and "B" are mutually exclusive (because they cannot occur together)

$$\therefore P(A\ or\ B)=P(A\cup B)=P(A)+P(B)=\frac{4}{36}+\frac{2}{36}=\frac{6}{36}$$



EXAMPLE 6.32

A card is drawn from a well-shuffled deck of 52 cards; find the probability that the card is a red or black queen?

Solution Since a card is drawn therefore

$$S = \{the\ pack\ of\ 52\ cards\} \Rightarrow n(S) = \binom{52}{1} = 52$$

Red Queens	Black Queens	Others	Total
2	2	48	52

Let "R" be the event that "red queen"

Then $n(R) = \binom{2}{1} = 2$

$$P(R) = \frac{n(R)}{n(S)} = \frac{2}{52}$$

Let "B" be the event that "black queen"

Then $n(B) = \binom{2}{1} = 2$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{52}$$

Now we have to find $P(R\ or\ B)$ and since the two events "R" and "B" are mutually exclusive (because they cannot occur together)

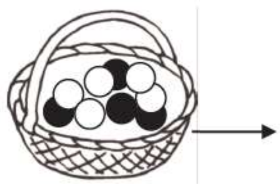
$$\therefore P(R\ or\ B) = P(R \cup B) = P(R) + P(B) = \frac{2}{52} + \frac{2}{52} = \frac{4}{52}$$

EXAMPLE 6.33

A basket contains 5 white and 4 black balls; what is the probability that a ball drawn at random is white or black balls?

Solution Since a ball is drawn out of "9"

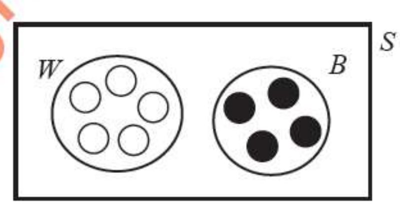
Therefore $n(S)=\binom{9}{1}=9$



White	Black	Total
5	4	9

Let "W" be the event of "drawing a white ball"

Then $n(W)=\binom{5}{1}=5$



Therefore $P(W)=\frac{n(W)}{n(S)}=\frac{5}{9}$

Let "B" be the event of "drawing a black ball"

Then $n(B)=\binom{4}{1}=4$

Therefore $P(B)=\frac{n(B)}{n(S)}=\frac{4}{9}$

Now we have to find $P(W\ or\ B)$ and since the two events "W" and "B" are mutually exclusive (because they cannot occur together)

$$\therefore P(W\ or\ B)=P(W\cup B)=P(W)+P(B)=\frac{5}{9}+\frac{4}{9}=1$$

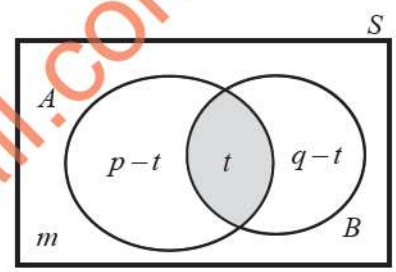
Addition Rule of prbability for Not Mutually Exclusive Events

Statement: Let "A" and "B" are two not mutually exclusive events then the probability of event "A" or "B" or "both" occuring is equal to the probability that "A" occurs plus the probabilitly that "B" occurs minus the probability that "both" events "A" and "B" occur together i.e.

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

OR $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof: To prove the theorem, consider the two Not Mutually Exclusive events "A" and "B" in the Venn-diagram:



$n(A \cap B) = t$ is shaded

$n(A \cup B) = p - t + t + q - t = p + q - t$

It is clear from the Venn-diagram that:

$n(S) = m,\ n(A) = p,\ n(B) = q$
$n(A \cup B) = p + q - t$
$n(A \cap B) = t$

Now

$$P(A) = \frac{n(A)}{n(S)} = \frac{p}{m}$$
$$P(B) = \frac{n(B)}{n(S)} = \frac{q}{m}$$
$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{t}{m}$$

Therefore

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{p+q-t}{m} = \frac{p}{m} + \frac{q}{m} - \frac{t}{m} = P(A) + P(B) - P(A \cap B)$$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$ Hence proved

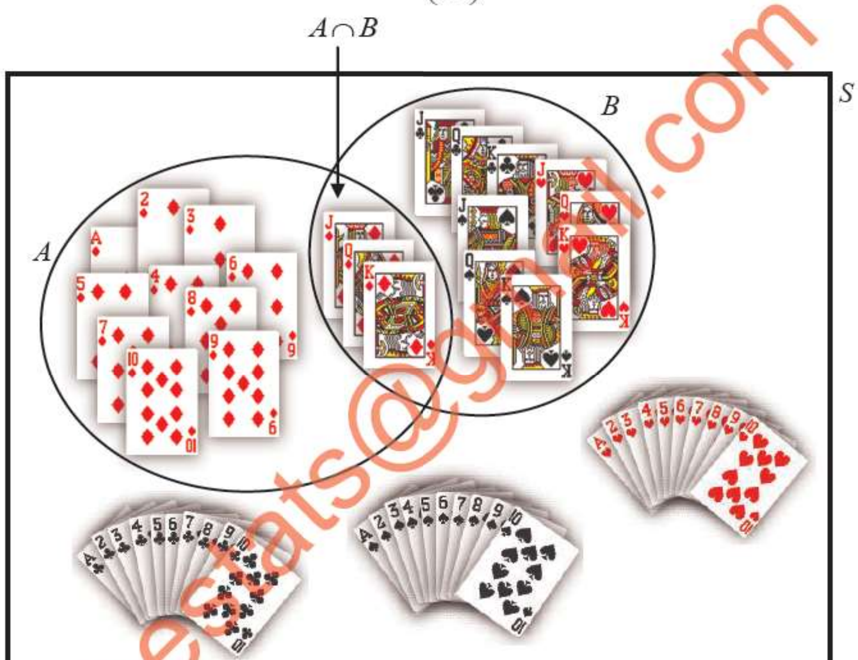
EXAMPLE 6.34

If a card is selected at random from a deck of 52 plyaing cards, what is the proability that the card is a diamond or a picture card or both?

Solution Since a card is drawn, therefore

$S=\{the\ pack\ of\ 52\ cards\}\Rightarrow n(S)=\binom{52}{1}=52$

Diamonds	Picture	Others	Total
13	12	37	52



Let "A" be the event that "a diamond card"

Then $n(A)=\binom{13}{1}=13$

$P(A)=\dfrac{n(A)}{n(S)}=\dfrac{13}{52}$

Let "B" be the event that "a picture card"

Then $n(B)=\binom{12}{1}=12$

$P(B)=\dfrac{n(B)}{n(S)}=\dfrac{12}{52}$

Since the two events "A" and "B" are not mutually exclusive (because they can occur together), therefore $n(A\cap B)=3$

Now the probability of both "A" and "B" occur together is: $P(A\cap B)=\dfrac{n(A\cap B)}{n(S)}=\dfrac{3}{52}$

Hence $\quad P(A\ or\ B\ or\ both)=P(A\cup B)=P(A)+P(B)-P(A\cup B)=\dfrac{13}{52}+\dfrac{12}{52}-\dfrac{3}{52}=\dfrac{22}{52}$

EXAMPLE 6.35

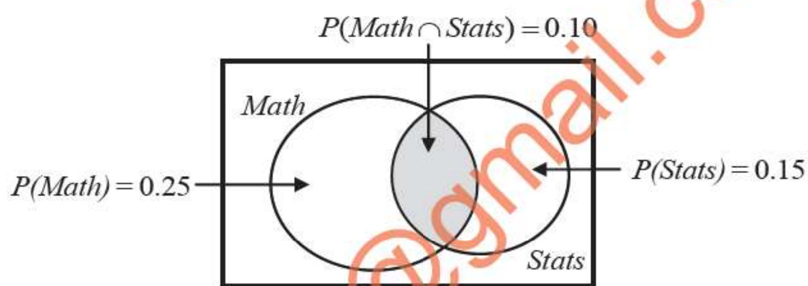
In a certain college 25% of the students failed math, 15% of the students failed stats and 10% of the students failed both mant and stats. A student is selected at random; what is the prbability the he/she failed math or stats?

Solution Given that

25% of students who failed Math $\Rightarrow P(Math) = 0.25$

15% of students who failed Stats $\Rightarrow P(Stats) = 0.15$

10% of students who failed both Math **and** Stats $\Rightarrow P(Math \cap Stats) = 0.10$



Now since the two subjects are **not mutually exclusive**, therefore

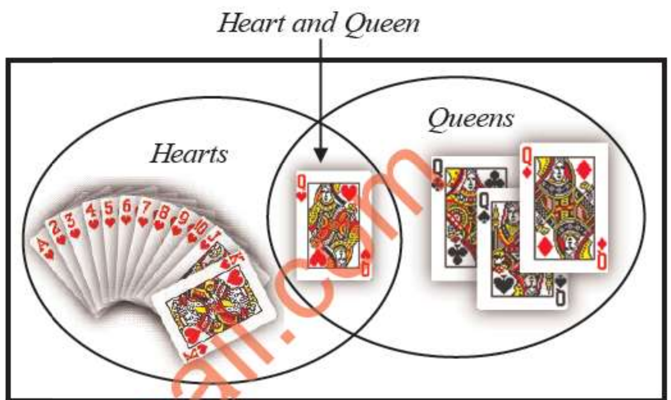
$$\begin{aligned} P(a\ student\ \ failed\ Math\ or\ Stats\ ) &= P(Math \cup Stats) \\ &= P(Math) + P(Stats) - P(Math \cap Stats) \\ &= 0.25 + 0.15 - 0.10 = 0.30 \end{aligned}$$

Test Yourself

1) Suppose that we roll a pair of dice, what is the probability of getting a sum of 5 or a sum of 11?
2) A card is drawn from a well-shuffled deck of 52 cards; find the probability that the card is a red or black King?
3) A basket contains 7 white and 3 black balls; what is the probability that a ball drawn at random is white or black balls?
4) If a card is selected at random from a deck of 52 plyaing cards, what is the proability that the card is a Heart or a picture card or both?
5) A customer enters a food store. The probability that the customer buys bread is 0.60, milk is 0.50 and both bread and milk is 0.30. What is the probability that the customer would buy either bread or milk or both?

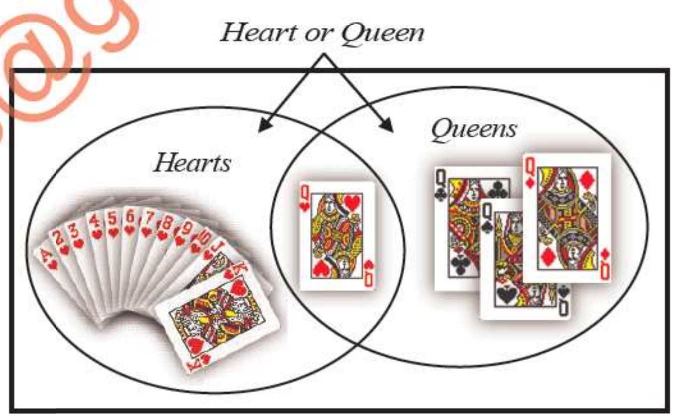
Understand the meaning of the words "AND" and "OR"!!!

The word **"AND"** has a single meaning.

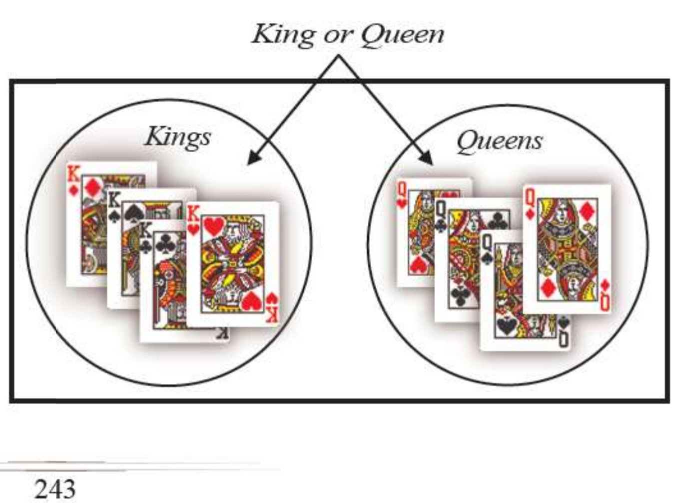


- For example, if you were asked to find the probability of getting **a queen and a heart** when you were drawing a single card from a deck, you would be looking for the queen of hearts. Here the word **"and"** means **"at the same time."**

The word **"OR"** has two meanings.



- For example, if you were asked to find the probability of selecting **a queen or a heart** when one card is selected from a deck, you would be looking for **one of the 4 queens or one of the 13 hearts**. In this case, **the queen of hearts would be included in both cases and counted twice**. In this case, both events can occur at the same time; we say that this is an example of the **inclusive or.**



- On the other hand, if you were asked to find the probability of getting **a queen or a king**, you would be looking for **one of the 4 queens or one of the 4 kings**. In this case, both events cannot occur at the same time, and we say that this is an example of the **exclusive or**.