CHAPTER 06

Set Theory and Basic Probability

Chapter Contents

You should read this chapter if you need to learn about:

Set

"A well-defined collection of distinct objects is called set"

The objects in a set may be the numbers, people, letters, books, rivers etc. Sets are usually denoted by capital letters such as A, B, C etc.

- Set of vowels in English alphabets
- Set of books in a library
- Set of students in a college etc.

Finite and Infinite Sets

"A set consisting of finite number of elements is called finite set"

- Set of vowels
- Set of months of a year
- Set of days in a week etc.

On the other hand "***a set consisting of infinite number of elements is called infinite sets***"

- Set of points on a line
- Set of stars on the sky
- Set of odd numbers
- Set of even numbers etc.

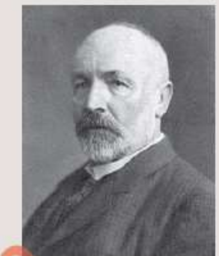
Null Set or Empty Set

"A set that contains no elements is called an empty set or null set"

A null set is denoted by the symbol ϕ (phi) or by $\{\ \}$.

- Number of male students in a girl's college
- Set of first year statistics students older than 200 years etc.

Historical Note



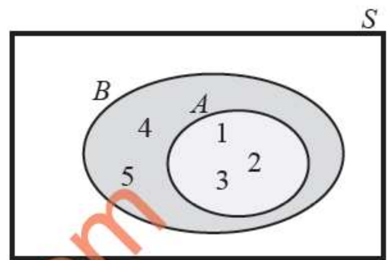
Georg Cantor

Richard Dedekind

The modern study of set theory was initiated by two German Mathematicians Georg Cantor and Richard Dedekind in the 1870s.

Sub-Set

"If each element of a set A is also the elements of set B then A is said to be the subset of B written as: $A \subset B$"



If $A = \{1,2,3\}$
And $B = \{1,2,3,4,5\}$
Then $A \subset B$

Proper Sub-Set

We call "A", a proper sub-set of "B" if:

- "A" is a sub-set of "B"
- $A \neq B$

Written as $A \subseteq B$

Every set is a subset of itself and the null set is a subset of every set.

If $A = \{1,2,3\}$
And $B = \{1,2,3,4,5\}$
Then $A \subseteq B$

Improper Sub-Set

We call "A", an improper sub-set of "B" if:

- "A" is a sub-set of "B"
- $A = B$

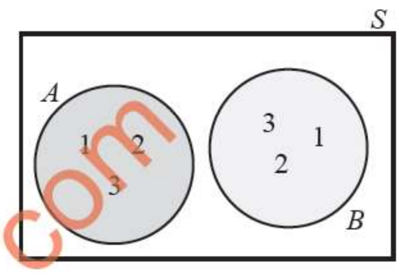
If $A = \{1,2,3\}$
And $B = \{1,2,3\}$
Then A is an improper sub set of B.

Equal Sets

"Two sets "A" and "B" are said to be equal, if they contain exactly the same elements"

In other words

If $A \subset B$ and $B \subset A$ then A = B



If $A = \{1, 2, 3\}$
And $B = \{3, 1, 2\}$
Then $A = B$

Power Set

"The set of all possible sub-sets of a set is called power set and is denoted by P (A)"

The number of subsets in power set may be counted by 2^n.

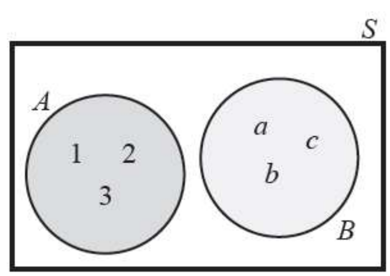
If $A = \{1, 2, 3\}$ then power set contains $2^3 = 8$ subsets i.e.

$$P(A) = \{\{\ \}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

Disjoint Sets

"If there is no element common in between the two "A" and "B", then they are called disjoint sets"

Disjoint sets are also called **mutually exclusive** sets.



If $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$

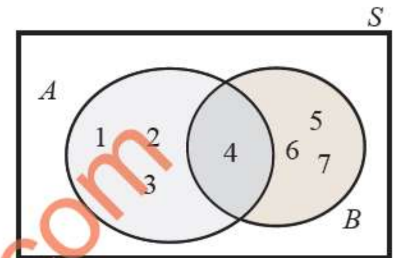
Then A and B are disjoint sets.

Overlapping sets

"If at least one element is common in between two sets such that they are not subsets of each other then they are called overlapping sets"

If $A=\{1,2,3,4\}$ and $B=\{4,5,6,7\}$

Then A and B are overlapping sets.



Universal Set

"The set which is consisted of all the elements specified for some discussion is called universal set". It is denoted by U or S.

Product Set OR Cartesian product of Sets

The Cartesian product of sets "A" and "B" denoted by $A \times B$ (read as "A" cross "B") is the set of elements that contains all the ordered pairs (x, y) where $x \in A$ and $y \in B$

If $A=\{H,T\}$ and $B=\{1,2\}$

$\Rightarrow A\times B=\{(H,1),(H,2),(T,1),(T,2)\}$

Historical Note

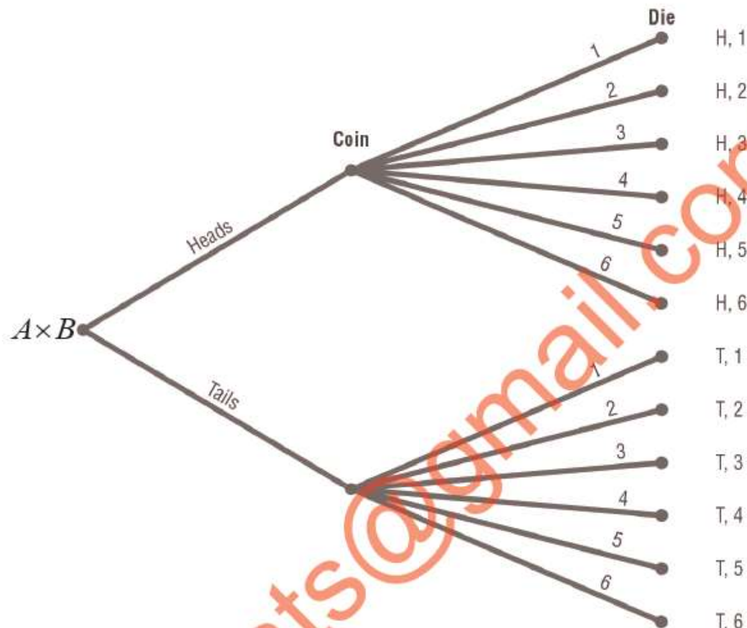
The Cartesian product is named after Rene Descartes, a French Mathematician

Tree diagram

"A systematic method of finding Cartesian product through a diagram is called tree diagram"

If for a coin, $A=\{H,T\}$ and for a die, $B=\{1,2,3,4,5,6\}$

$$\Rightarrow A\times B=\begin{Bmatrix}(H,1),(H,2),(H,3),(H,4),(H,5),(H,6)\\(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)\end{Bmatrix}$$

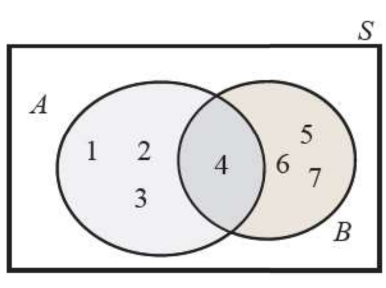


Venn diagram

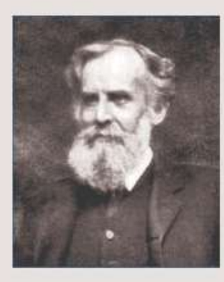
"The simple and effective way of representing the relationships between sets diagrammatically is called Venn diagram".

In Venn diagram the universal set U (or S) is represented by a rectangle and the sub sets are represented by circles inside the rectangles e.g.

If $A=\{1,2,3,4\}$ and $B=\{4,5,6,7\}$ then, they can be represented by the Venn diagram as:



Historical Note

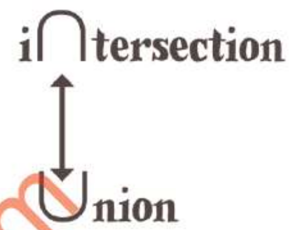


In 1880, British Philosopher John Venn introduced the Venn Diagrams.

Operations on Sets

Like algebraic operation such as addition, subtraction, multiplication and division in mathematics, we have basic operations on sets i.e.:

- Union of two sets
- Intersection of two set
- Difference of two sets
- Complement of a set

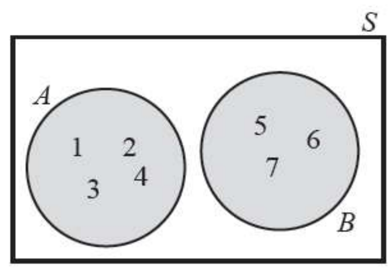


Union of sets

The union of two sets "A" and "B" is the set of all elements that belongs to "A" or to "B" or to both "A" and "B". The union of two sets "A" and "B" is denoted by $A \cup B$.

If $A = \{1,2,3,4\}$ and $B = \{5,6,7\}$

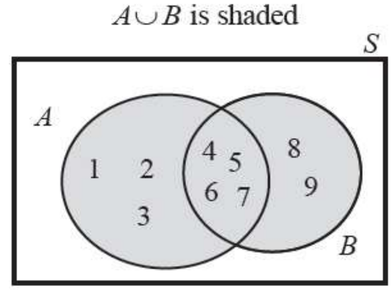
Then $A \cup B = \{1,2,3,4,5,6,7\}$



$A \cup B$ is shaded

If $A = \{1,2,3,4,5,6,7\}$ and $B = \{4,5,6,7,8,9\}$

Then $A \cup B = \{1,2,3,4,5,6,7,8,9\}$



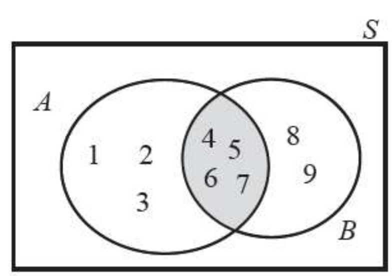
$A \cup B$ is shaded

Intersection of sets

The intersection of two sets "A" and "B" is the set of elements that belongs to both "A" and "B". The intersection of two sets "A" and "B" is denoted by $A \cap B$

If $A = \{1,2,3,4,5,6,7\}$
And $B = \{4,5,6,7,8,9\}$

Then $A \cap B = \{4,5,6,7\}$



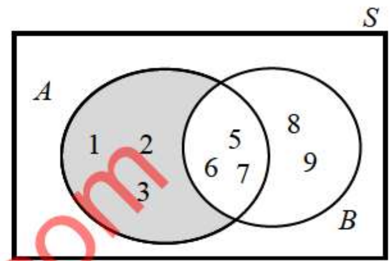
$A \cap B$ is shaded

Difference of sets

The difference of sets "A" and "B" is the set of elements that belongs to "A" but do not belongs to "B". The difference of two sets "A" and "B" is denoted by $A-B$ or $A-(A\cap B)$ or $A\cap\overline{B}$

If $A=\{1,2,3,4,5,6,7\}$
And $B=\{4,5,6,7,8,9\}$

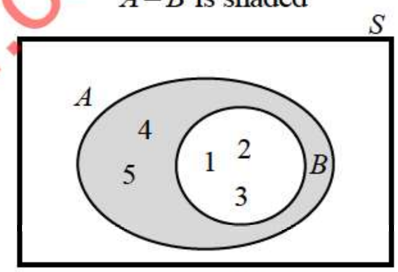
Then $A-B=\{1,2,3\}$



$A-B$ is shaded

If $A=\{1,2,3,4,5\}$
And $B=\{1,2,3\}$

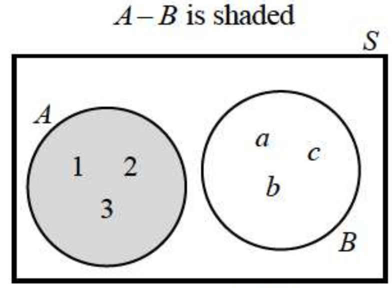
Then $A-B=\{4,5\}$



$A-B$ is shaded

If $A=\{1,2,3\}$
And $B=\{a,b,c\}$

Then $A-B=\{1,2,3\}$



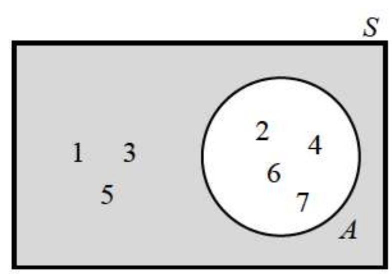
$A-B$ is shaded

Complement of a Set

The complement of a set "A" is the set of elements that belongs to "S" but do not belongs to "A" The complement of set "A" is denoted by $\overline{A}$ or A^c

If $S=\{1,2,3,4,5,6\}$
And $A=\{2,4,6,7\}$

Then $\overline{A}=S-A=\{1,3,5\}$



$\overline{A}$ is shaded