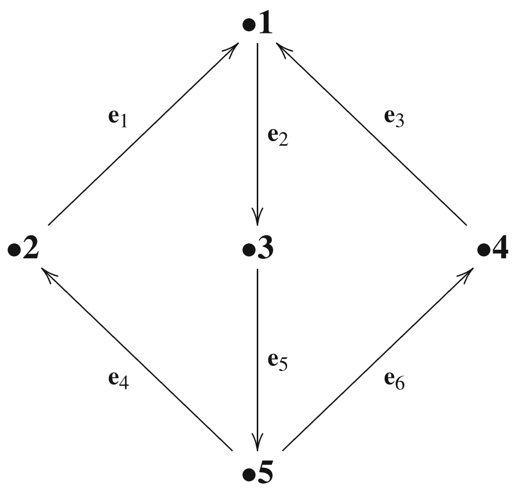
Chapter 2
Incidence Matrix

Let G be a graph with $V(G) = \{1, \dots, n\}$ and $E(G) = \{e_1, \dots, e_m\}$. Suppose each edge of G is assigned an orientation, which is arbitrary but fixed. The (*vertex-edge*) *incidence matrix* of G, denoted by $Q(G)$, is the $n \times m$ matrix defined as follows. The rows and the columns of $Q(G)$ are indexed by $V(G)$ and $E(G)$, respectively. The (i, j)-entry of $Q(G)$ is 0 if vertex i and edge e_j are not incident, and otherwise it is 1 or -1 according as e_j originates or terminates at i, respectively. We often denote $Q(G)$ simply by Q. Whenever we mention $Q(G)$ it is assumed that the edges of G are oriented.

Example 2.1 Consider the graph shown. Its incidence matrix is given by Q.



$$Q = \begin{bmatrix} -1 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 1 \end{bmatrix}$$

R.B. Bapat, *Graphs and Matrices*, Universitext,
DOI 10.1007/978-1-4471-6569-9_2