Exact Differential Equation

Given a function of two variables $F(y,t)$, its total differential is is

$$dF(y,t) = \frac{\partial F}{\partial y}(dy) + \frac{\partial F}{\partial t}(dt)$$

when this diff. eqn. is set equal to zero, then the resulting equation will become $\frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial t}dt = 0$ is known as exact diff. equation, because its left side is exactly the diff. of the function $F(y,t)$.

Gr. F: $M\,dy + N\,dt = 0$

Here $M = \frac{\partial F}{\partial y}$

& $N = \frac{\partial F}{\partial t}$.

Condition to check Ex. Diff. Equ.

$$\frac{\partial M}{\partial t} = \frac{\partial N}{\partial y}$$

Exp: $2yt\,dy + y^2\,dt = 0$

Here $M = 2yt$ and $N = y^2$

$$\frac{\partial M}{\partial t} = 2y \quad \text{and} \quad \frac{\partial N}{\partial y} = 2y$$

Hence $\frac{\partial M}{\partial t} = \frac{\partial N}{\partial y}$ (Exact differential equation)

1. Verify that each of the following differential equations is exact, and solve by the four-step procedure:

(a) $2yt^3\,dy + 3y^2t^2\,dt = 0$

(b) $3y^2t\,dy + (y^3 + 2t)\,dt = 0$

(c) $t(1 + 2y)\,dy + y(1 + y)\,dt = 0$

(d) $\dfrac{dy}{dt} + \dfrac{2y^4t + 3t^2}{4y^3t^2} = 0$ [*Hint*: First convert to the form of (15.17).]