CONTINUOUS TIME : FIRST ORDER DIFFERENTIAL EQUATIONS

Q.NO: What is continuous time: and first order differential equations?

ANS: **CONTINUOUS TIME: FIRST ORDER DIFFERENTIAL EQUATIONS:-** differential equations are involved derivatives or differentials. they expressed rate of change of continuous functions over time. The objective is working with differential equations is to find a function without derivatives or differential which satisfies the differential equations such that a function is called the solution or integrals of the equation

The order of a differential equations is the order of the highest derivative in the equation. The degree of a differential equation is the highest power to which the derivative of the highest order is raised.

EXAMPLE: Determining the order and degree of differential equations.

(i) $\frac{dy}{dx} = (2x+6)$ (First order, First degree)

(2) $\left(\frac{dy}{dx}\right)^4 = 5x^5$ (First order, First degree)

(3) $\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^3 + x = 0$

(Second order, First degree)

FIRST ORDER DIFFERETIAL EQUATIONS:

A linear first order differential equation will generally take the form

$$\frac{dy}{dt} + u(t)y = w(t)$$

Where u and w are the two functions of "t" as is y.

THE HOMOGENIOUS CASE:

If u and w are constant functions and if w happens to be identically zero. Then

$$dy/dt = -ay \Rightarrow 1/y$$

Where "a" is any constant. The differential equation is said to be homogenious. The above differential equation can be written as.

$$dy/dt = -ay \Rightarrow \frac{1}{y}\, dy/dt = -a$$

Integrating w.r.to dt we get.

$$\int \frac{1}{y} \frac{dy}{dt} dt = \int -a\,dt$$

$$\int \frac{1}{y} dy = -a \int 1 . dt \Rightarrow \ln y + c_1 = -at + c_2$$

$$\ln y = -at + c_2 - c_1$$

taking antilog on bothsides we have.

$$e^{\ln y} = e^{-at} . e^{c_2 - c_1}$$

Let

$$y = Ae^{-at} \qquad \left(\because e^{\ln y} = y, \; \because e^{c_2 - c_1} = A \right)$$

Therefore, the general solution is $\boxed{y(t) = Ae^{-at}}$

Now putting / substituting $t=0$ then, we get definite Solution.

$$y(0) = Ae^{-a(0)} \Rightarrow y(0) = Ae^{0}$$

$$y(0) = A \qquad (\because e^{0} = 1)$$

Therefore, the difinite Solution is

$$\boxed{y(t) = y(0)e^{-at}}$$

Where A is an orbitrary constant when any particular value is substituted for A, then the Solution becomes a particular solution.

The two points should be noted here.

(i) The Solution is not a numerial value but rather a function $y(t)$ a time path.

(2) The Solution is free of any derivative or differential expression.

NON HOMOGENOUS CASE:

When a non-zero constant takes place, we have a non homogenous linear differential equation.

$$dy/dt + ay = b$$

The Solution of this equation will consist of the sum of two terms.

(1) Complementary function (y_c).

(2) Particular Integral (y_p).

(1) COMPLEMENTARY FUNCTION:

The completely function is the general solution of the reduced equation and denoted by Y_c.

(2) PARTICULAR INTEGRAL: (Y_p)

The particular Integral (Y_p) is any particular Solution equilibrium level of $Y(t)$. Since the particular integral is any particular Solution of the complete equation.

CASE-1: If $Y = K$ is any constant.

then $dy/dt = 0$

$$\left(\because dy/dt + ay = b\right)$$

So the above equation will become.

$0 + ay = b \Rightarrow ay = b \qquad (\because dy/dt = 0)$

$ak = b \Rightarrow k = b/a \qquad \because y = k$

So $\boxed{y_p = b/a}$

The general solution of the reduced equation is

$$y_c = A e^{-at}$$

The sum of complementary function and particular integral is

$$y(t) = y_c + y_p \qquad \left(\begin{array}{l} \because y_c = Ae^{-at} \\ \because y_p = b/a \end{array}\right)$$

$$y(t) = Ae^{-at} + b/a$$

$$\boxed{y(t) = Ae^{-at} \pm b/a}$$

hence to get the definite solution is

Suppose $y(0)$ when put $t=0$

$$y(0) = Ae^{-a(0)} + b/a \Rightarrow y(0) = Ae^{0} + b/a$$

$$y(0) = A + b/a \Rightarrow A = y(0) - b/a$$

The above relation can be written as

$$\boxed{y(t) = \left[y(0) - b/a\right] e^{-at} + b/a}$$

EXAMPLE:-

Solve the equation $\frac{dy}{dt} + 2y = 6$ with the initial condition $y(0) = 10$:

Solution. Here $a = 2$, $b = 6$

So the particular integral is

$$y_p = b/a = 6/2 = 3$$

or Let $y = k \Rightarrow dy/dt = 0$

$$2y = 6 \Rightarrow y_p = 6/2 = 3$$

the complementary function is

$$y_c = Ae^{-at} \Rightarrow y_c = Ae^{-2t}$$

therefore, the general Solution is

$$y(t) = y_c + y_p$$

$$\boxed{\therefore y(t) = Ae^{-2t} + 3}$$

To obtained definite solution putting $t = 0$

$$y(0) = Ae^{0} + 3$$

$$10 = A(1) + 3$$

$$10 = A + 3 \Rightarrow A = 10 - 3$$

$$\boxed{A = 7}$$

therefore, the definite Solution is

$$\boxed{\therefore y(t) = 7e^{-2t} + 3}$$

Ans

Example: No: 2:- Solve the equation $\frac{dY}{dt} + 4 = 0$ with the initial condition

$$y(0) = 1$$

Solution:- Given

$$dy/dt + 4 = 0 \quad \text{and} \quad y(0) = 1$$

Now here $a = 4$ and $b = 0$

The particular integral is

$$y_p = b/a = 0/4 = 0$$

The complementary function is

$$y_c = Ae^{-at} \Rightarrow y_c = Ae^{-4t}$$

The general Solution is

$$y(t) = y_c + y_p$$

$$y(t) = Ae^{-4t} + 0 \Rightarrow \boxed{y(t) = Ae^{-4t}}$$

Now to get the definite Solution is put $t = 0$

$$y(0) = Ae^{-4(0)} \Rightarrow y(0) = Ae^{0}$$

$$y(0) = A \qquad (\because e^{0} = 1)$$

$$1 = A \qquad (\because y(0) = 1)$$

$$\boxed{A = 1}$$

Therefore, the definite Solution is

$$\boxed{\therefore y(t) = e^{-4t}} \quad \text{Ans}$$

CASE: NO: II:-

If $a=0$ and $y = kt$ then $dy/dt = k$. The complete equation $\frac{dy}{dt} + ay = b$ will reduce to $dy/dt = b$ and particular Integral is

$$y_p = bt$$

taking Integral on bothsides we have.

$$\int dy/dt = \int b\,dt \Rightarrow y = b\int 1 \cdot dt$$

$$y = bt \Rightarrow \boxed{y_p = bt} \rightarrow \text{particular Integral}$$

The complementry Solution is

$$y_c = Ae^{-at}$$

$$y_c = Ae^{-0t} \Rightarrow y_c = Ae^{0} \quad (\because a = 0)$$

$$(\because e^{0} = 1)$$

$$\boxed{\therefore y_c = A}$$

Therefore, the general solution is

$$y(t) = y_c + y_p$$

$$\boxed{y(t) = A + bt}$$

for definit Solution is putting $t=0$

$$y(0) = A + b(0) \Rightarrow y(0) = A$$

So the definite Solution is

$$\boxed{\therefore y(t) = y(0) + bt}$$

Example: Solve the equation is $dy/dt = 2$ with initial condition $y(0)=5$

Solution. Given

$$dy/dt = 2 \quad \text{and} \quad y(0) = 5$$

Here $a=0$, $b=2$

The particular Integral is

$$y_p = bt \Rightarrow \boxed{y_p = 2t}$$

The complementary function is

$$y_c = Ae^{-at} \Rightarrow y_c = Ae^{-0(t)} = Ae^0$$

$$\boxed{y_c = A}$$

Therefore, the general solution is

$$Y(t) = Y_C + Y_P \Rightarrow Y(t) = A + 2t$$

$$\boxed{Y(t) = A + 2t}$$

Now to get the definite Solution is putting $t=0$

$$Y(0) = A + 2(0) \Rightarrow Y(0) = A$$

$$\boxed{5 = A} \qquad (\because Y(0) = 5)$$

hence the definite Solution is

$$\boxed{\therefore Y(t) = 5 + 2t} \quad \text{Ans}$$

Verification of the Solution:-

Since we know that about this term.

$$y(t) = \left[y(0) - b/a\right] e^{-at} + b/a$$