Integration by Parts

① $\int x(x+1)^{1/2}\,dx$ ② $\int x \ln x\,dx$

③ $\int x e^x\,dx$.

— x —

Exp! $\int x(x+1)^{1/2}\,dx$

$= x \cdot \int (x+1)^{1/2}\,dx - \int \left(\int (x+1)^{1/2}\,dx\right) \frac{d}{dx}(x)\cdot dx$

$= x \cdot \frac{(x+1)^{1/2+1}}{1/2+1} - \int \frac{(x+1)^{1/2+1}}{\frac{1}{2}+1} \cdot 1\,dx$

$= x \cdot \frac{(x+1)^{3/2}}{3/2} - \int \frac{(x+1)^{3/2}}{3/2}\,dx$

$= \frac{2}{3} x (x+1)^{3/2} - \frac{2}{3}\int (x+1)^{3/2}\,dx$

$= \frac{2}{3} x (x+1)^{3/2} - \frac{2}{3} \cdot \frac{(x+1)^{3/2+1}}{3/2+1} + C$

$= \frac{2}{3} x (x+1)^{3/2} - \frac{2}{3} \frac{(x+1)^{5/2}}{5/2} + C$

$= \frac{2}{3} x (x+1)^{3/2} - \frac{4}{15} (x+1)^{5/2} + C$

— x

Exp :- $\int x \ln x \, dx$

$= \ln x \cdot \int x \, dx - \int \left(\int x \, dx\right) \cdot \frac{d}{dx} \ln x \cdot dx$

$= \ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} \, dx$

$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x^{2-1} \, dx$

$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x \, dx$

$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \cdot \frac{x^2}{2} + c$

$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + c$

Exp $\int x e^{2x} \, dx$

$= x \cdot \int e^{2x} \, dx - \int \left(\int e^{2x} \, dx\right) \cdot \frac{d}{dx} x \cdot dx$

$= x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} \cdot 1 \, dx$

$= \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} \, dx$

$= \frac{1}{2} x e^{2x} - \frac{1}{2} \frac{e^{2x}}{2} + c$

$= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + c$