INTEGRATION

CONCEPT OF INTEGRATION (UAJK: 2010, 2013)

The concept of derivative represents the rate of change of a function with respect to its independent variable. The process of integration is reverse to that of Differentiation. Thus, if the process of differentiation is reversed it is called *integration* which is shown by $F(x)$. It is also known as Integral of the function $f(x)$ or Anti-derivative of $f(x)$. It is expressed as :

$$\int f(x)\,dx = F(x) + c.$$

The symbol " $\int$ " represents integral. The above expression is read as " integral of $f(x)$ ". The function $f(x)$ is called *Integrand* while c is called the *constant of integration* which may assume any value. It means to say that it may not be finite. This is the reason that such integral is called the " *Indefinite Integral*".

ECONOMIC APPLICATION OF INDEFINITE INTEGRATION

(UAJK: 2012) (GUDIKHAN: 2014) (UOB: 2011) (UOH: 2011)

EXAMPLE 1. If $MC = \frac{dC}{dQ} = 25 + 30Q - 9Q^2$, while $TFC = 55$, find (1) TC, (2) AC and (3) AVC. (UOB: 2011) (UOPR: 2007, 2008, 2012)

(1) $TC = \int MC\,dQ = \int (25 + 30Q - 9Q^2)\,dQ = \int 25\,dQ + 30\int Q\,dQ - 9\int Q^2\,dQ$

$= 25.Q + 30.\frac{1}{1+1}\,Q^{1+1} - 9.\frac{1}{2+1}.Q^{2+1}$

$= 25.Q + 15\,Q^2 - 3Q^3 + c$

$= 25.Q + 15\,Q^2 - 3Q^3 + 55$ (as $c = TFC = 55$)

> **Self Solve:**
> Q: Given the marginal Cost function for production
> $C(x) = 10 + 24x - 3x^2$, if TC of producing one unit is 25, find TC and AC functions. (GUDIKHAN: 2014)

(2) $AC = \frac{TC}{Q} = \frac{25Q + 15\,Q^2 - 3Q^3 + 55}{Q} = 25 + 15Q - 3Q^2 + 55Q^{-1}$

(3) $AVC = \frac{TVC}{Q} = \frac{25 + 15Q^2 - 3Q^3}{Q} = 25 + 15Q - 3Q^2$

EXAMPLE 2. If MR = $60 - 2Q - 2Q^2$, find (1) TR, (2) P or $P = f(Q)$

(1) $TR = \int MR\, dQ = \int (60 - 2Q - 2Q^2)\, dQ$ (Kohat University 2014)

$$= 60Q - 2\frac{1}{1+1} Q^{1+1} - 2.\frac{1}{2+1}.Q^{2+1} = 60Q - Q^2 - \frac{2}{3} Q^3 + c$$

If $Q = 0$, $TR = 0$, then we have $0 = 60(0) - (0)^2 - \frac{2}{3}(0)^3 + c \Rightarrow c = 0$

Hence $TR = 60Q - Q^2 - \frac{2}{3} Q^3$. If $Q = 3$, $TR = 250$, then we have

$$250 = 60(3) - (3)^2 - \frac{2}{3}(3)^3 + c \Rightarrow 250 = 180 - 9 - 18 + c$$

$$250 - 153 = c \Rightarrow c = 97. \text{ Hence } TR = 60Q - Q^2 - \frac{2}{3} Q^3 + 97$$

We also find demand function.

$$TR = 60Q - Q^2 - \frac{2}{3}Q^3 \Rightarrow P = \frac{60Q - Q^2 - \frac{2}{3}Q^3}{Q} = 60 - Q - \frac{2}{3}Q^2$$

EXAMPLE 3. If $MR = 84 - 4Q - Q^2$, find TR.

Solution. (1) $TR = \int MR\, dQ = \int (84 - 4Q - Q^2)\, dQ$

$$= 84Q - 4\frac{1}{1+1}Q^{1+1} - \frac{1}{2+1}.Q^{2+1} = 84Q - 2Q^2 - \frac{1}{3}Q^3 + c$$

EXAMPLE 4. If $Y' = MPC = \frac{dC}{dY} = 0.8$, while $c = 40$ and $Y = 0$, find consumption (C).

Solution. $C = \int f'(Y)\, dY = 0.8Y + c$

The consumption will be 40 if the value of the constant (c) = 40, then the consumption function will be as $C = 0.8Y + 40$

EXAMPLE 5. If MPC = 0.6 + $\frac{0.1}{\ldots}$ find the equation of consumption. Again, if c = 45.

Thus $C = 0.6Y + 0.15 Y^{-} + 45$

EXAMPLE 6. If $MPS = \frac{dS}{dY} = 0.5 - 2Y^{-\frac{1}{2}}$,

find saving function when $S = -3.5$ and $Y = 25$.

> $MPS = S'(Y) = 0.5 - 0.1\ Y^{3/2} dY$,
> Find saving function. (UOH: 2006)

$$S = \int \left(0.5 - 2Y^{-\frac{1}{2}}\right) dY = 0.5Y - 0.2 . \frac{1}{-\frac{1}{2}+1} Y^{-\frac{1}{2}+1}$$

$$= 0.5Y - 0.2 . \frac{1}{1/2} Y^{\frac{1}{2}} = 0.5Y - 0.2.\left(\frac{2}{1}\right)Y^{\frac{1}{2}} = 0.5Y - 0.4\,Y^{\frac{1}{2}} + c$$

Using $S = -3.5$, $Y = 25$, we have

$$-3.5 = 0.5(25) - 0.4\,(25)^{\frac{1}{2}} + c \Rightarrow -3.5 = 12.5 - 0.4\,(5) + c$$

$$-3.5 = 12.5 - 2 + c \Rightarrow c = -3.5 - 12.5 + 2 = -14$$

Thus $S = 0.5Y - 0.4\,Y^{\frac{1}{2}} - 14$

EXAMPLE 7. The rate of investment is shown by I (t) and its specific equation is $I(t) = 140t^{\frac{3}{4}}$ and the initial stock of capital is 150 at t = 0, find the capital stock function.

$$K = \int 140\, t^{\frac{3}{4}}\, dt = 140 \times \frac{1}{\frac{3}{4}+1}\, t^{\frac{3}{4}+1} = 140 \times \left(\frac{4}{7}\right) t^{\frac{7}{4}}$$

$K = 80\, t^{\frac{7}{4}} + c \ldots \quad (1).$ Using K = 150, t = 0 in (1), we get

$150 = 80\,(0)^{\frac{7}{4}} + c \Rightarrow c = 150$, putting in (1) gives $K = 80\, t^{\frac{7}{4}} + 150$

EXAMPLE 8. If $MC = 12e^{0.5Q}$ while TFC = 36, find TC.

$$TC = \int 12e^{0.5Q}\, dQ = 12 \times \frac{1}{0.5} \cdot e^{0.5Q} = 24\, e^{0.5Q} + c$$

Using TFC = 36, Q = 0, we get

$$36 = 24\, e^{0.5(0)} + c \Rightarrow 36 = 24 + c \Rightarrow c = 12$$

Hence $TC = 24\, e^{0.5Q} + 12$

DEFINITE INTEGRAL (UAJK: 2010)