Definite integral:-

The integral $\int_a^b f(x)\,dx$ is called definite integral from a to b where "a" is called lower limit and "b" is called upper limit.

By fundamental theorem of integral calculus if $f(x)$ is continuous on the interval $a \leq x \leq b$ and $F(x)$ is integral of $f(x)$ then.

$$\int_a^b f(x)\,dx = F(x)\Big]_a^b \Rightarrow F(b) - F(a)$$

(OR)

The definite integral of a continuous function $f(x)$ over the interval a to b but $a < b$ is expressed Symbolically $\int_a^b f(x)\,dx$
where "a" is the lower limit of the integration and "b" is the upper limit of the integration. Here limit means the value of the variable at the given end of its range. Here the definite numerical value is

$$\int_a^b f(x)\,dx = F(x) + C\Big|_a^b$$

$$= F(b) + \cancel{c} - F(a) - \cancel{c} = F(b) - F(a)$$

Properties of Definite integral.

Following are the properties of definite integral are as under:

Property No1. The interchange of the limits of integration change the sign of the definite integral

i.e $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Proof: $$L.H.S = \int_a^b f(x)\,dx = F(x) + C\Big|_a^b$$

$$= F(b) + \cancel{c} - F(a) - \cancel{c} = F(b) - F(a)$$

Now $R.H.S = -\int_b^a f(x)\,dx = -\left[F(x) + c\right]_b^a$

$= -\left(F(a) + \cancel{c} - F(b) - \cancel{c}\right) = -\left(F(a) - F(b)\right)$

$= -F(a) + F(b) = F(b) - F(a)$

$L.H.S = R.H.S$ Hence proved.

Property-2: The definite integral has a value of zero when the two limits are identical (same).

(OR)

If the upper limit of integration equal to the lower limit of integration, then the value of definite integral is zero i.e $\int_a^a f(x)\,dx = F(a) - F(a) = 0$

Proof: $L.H.S = \int_a^a f(x)\,dx = F(x) + c\Big|_a^a$

$= F(a) + \cancel{c} - F(a) - \cancel{c} = F(a) - F(a) = 0$

Hence $L.H.S = R.H.S$

Ans

Property No 3: (Additive property)

A definite integral can be expressed as a sum of a finite number of definite sub integrals as follows

$$\int_a^d f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx + \int_c^d f(x)\,dx$$

$(a \leq b < c < d)$

Property No 4 :- $\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$

Proof :- $\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$

$$L.H.S = \int_a^b -f(x)\,dx = -\Big[F(x)+c\Big]_a^b$$

$$= -\Big[F(b)+\cancel{c}-F(a)-\cancel{c}\Big] = -\Big[F(b)-F(a)\Big]$$

$$= -F(b)+F(a) = F(a)-F(b)$$

Now we take R.H.S

$$R.H.S = -\int_a^b f(x)\,dx = -\Big[F(x)+c\Big]_a^b$$

$$= -\Big[F(b)+\cancel{c}-F(a)-\cancel{c}\Big] = -\Big[F(b)-F(a)\Big]$$

$$= -F(b)+F(a) = F(a)-F(b)$$

$\therefore$ L.H.S = R.H.S Hence proved.

Ans

Property No 5 :- $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$

Proof :- $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$

$$L.H.S = \int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$$

$$= k\left[F(x)+c\Big|_a^b\right] = k\left[F(b)+\cancel{c}-F(a)-\cancel{c}\right]$$

$$= k\left[F(b)-F(a)\right] = k\left(F(b)-F(a)\right)$$

Now $R.H.S = k\int_a^b f(x)\,dx = k\left[F(x)+c\Big|_a^b\right]$

$$= k\left[F(b)+\cancel{c}-F(a)-\cancel{c}\right] = k\left(F(b)-F(a)\right)$$

Hence $L.H.S = R.H.S$

Ans

Property No 6: $\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

Proof: $L.H.S = \int_a^b [f(x)+g(x)]\,dx$

$$= \int_a^b f(x)\,dx + \int_a^b g(x)\,dx = F(x)+c\Big|_a^b + G(x)+c\Big|_a^b$$

$$= F(b)+\cancel{c}-F(a)-\cancel{c} + G(b)+\cancel{c}-G(a)-\cancel{c}$$

$$= F(b)-F(a)+G(b)-G(a)$$

Now $R.H.S = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

$$= F(x)+c\Big|_a^b + G(x)+c\Big|_a^b$$

$$= F(b)+\cancel{c}-F(a)-\cancel{c}+G(b)+\cancel{c}-G(a)-\cancel{c}$$

$$= F(b)-F(a)+G(b)-G(a)$$

Hence $L.H.S = R.H.S$ Ans

Property No 7 :- (By parts)

$$\int_{x=a}^{x=b} v\,du = uv - \Big|_{x=a}^{x=b} \int_{a}^{b} u\,dv$$