(v) He has accommodated the rules of both classical and quantum mechanical physics and this thing can not be supported theoretically.

(vi) Its model only works for the hydrogen like system, just like Bohr's model.

4.7.0 DUAL NATURE OF MATTER

(مادے کی دُہری فطرت)

4.7.1 Introduction:

Luis de-Broglie in 1923, made an **analogy** (مطابقت) of matter with light and predicted (پیشن گوئی کرنا) that the particles like electrons should show the wave like property along with particle character. He suggested that wavelength of electron is inversely proportional to its momentum.

de Broglie نے کہا کہ الیکٹران ایک مادے کا ذرہ تو ہے ہی لیکن چلتے ہوئے لہریں بھی بناتا ہے۔

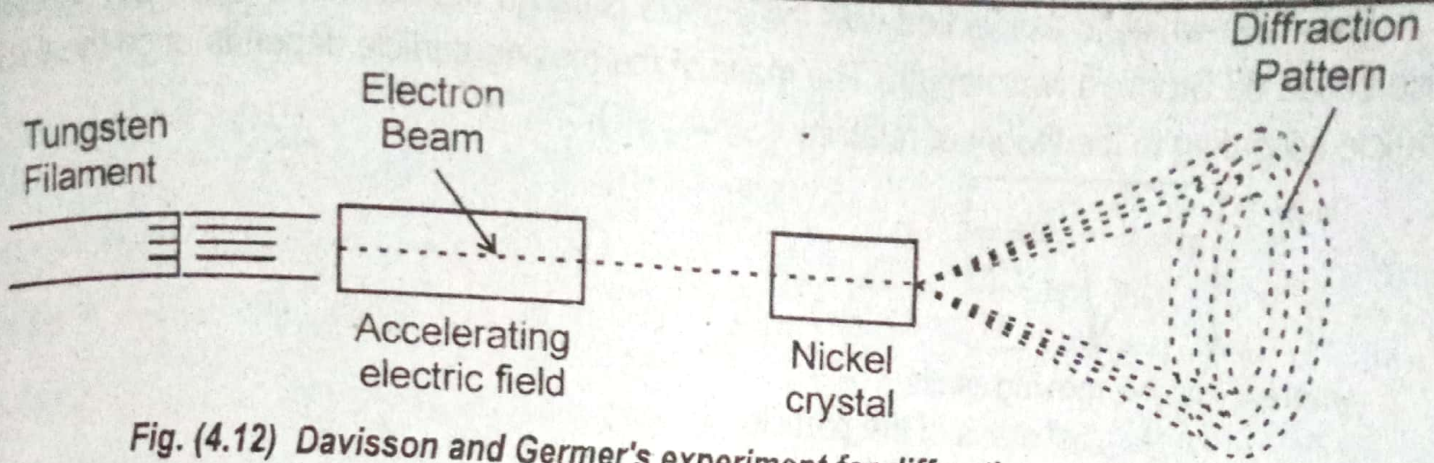

Fig. (4.12) Davisson and Germer's experiment for diffraction of electrons.

The wavelength associated with a particle of mass 'm' moving with velocity 'v' is given by the relation

$$\lambda = \frac{h}{mv} \quad \cdots\cdots \quad (1)$$

This relation is called **de-Broglie's equation** and λ is called **de-Broglie's wavelength.**

4.7.2 Derivation of de-Broglie's Equation:

This equation is derived by combining the mass and energy relationship given by Planck and Einstein. According to Einstein's law of mass energy equivalence (برابری), the photon must have a finite mass (خاص کمیت). If its mass is 'm', then

$$E = mc^2 \quad \cdots\cdots \quad (2)$$

According to quantum theory of radiation, the energy associated with one photon of light is,

$$E = h\nu \quad \cdots\cdots \quad (3)$$

Comparing equations (2) and (3),

$$mc^2 = h\nu$$

$$mc = \frac{h\nu}{c}$$

$$mc = \frac{h}{c/\nu}$$

$$mc = \frac{h}{\lambda}$$

$$\boxed{\lambda = \frac{h}{mc}} \quad \cdots\cdots \quad (4)$$

So, the wavelength of the photon of light is inversely proportional to the momentum (معیار حرکت) of the photon.

Let us represent the momentum by P.

So, $$\boxed{\lambda = \frac{h}{P}} \quad \cdots\cdots \quad (5)$$

The French scientist Luis de-Broglie extended (بڑھاوا دینا) this idea to all other particles travelling with a finite velocity. This idea helped the scientists to interpret a number of phenomenon in the microscopic world (خوردنی ذرات کی دنیا), so he said that the wavelength of the particle of mass 'm' moving with a velocity 'v' is given by the equation,

$$\lambda = \frac{h}{mv} \quad \cdots\cdots \quad (1)$$

4.8.0 HEISENBERG'S UNCERTAINITY PRINCIPLE

(ہائیزن برگ کا غیر یقینی پن کا اصول)

4.8.1 Introduction:

When we are studying a large moving object say a planet, then we can follow its definite ... on which it travels. If we know its initial position and momentum, then we can predict its position ... momentum at any other time. But this is not possible for electron, proton and neutron which ... microscopic particles. Heisenberg has given a principle in this connection. He says that ... **impossible to measure simultaneously both the position and momentum of a microsc... particle with accuracy or certainity.**

Mathematically this principle can be put as follows:

$$\Delta X \times \Delta P \geq \frac{h}{4\pi}$$

ΔX = Uncertainity in the position

ΔP = Uncertainity in the momentum

These two uncertainities are **inversely proportional to each other**. So, if position of ... microscopic particle is known with more accuracy, then there will be more uncertainity in its moment... and vice versa.

4.8.2 Physical Concept of Uncertainity Principle (اصول غیر یقینی کا طبعی ادراک):

In order to know the position of an object, we throw the photons of light upon them. If we wa... to have the idea for the position of electron, then the photons of X-rays region have to be used becau... their wavelengths are very small and the possibility for the hitting of electron is there. During this hittin... the photon transfers (منتقل کرتا ہے) some of its energy to the electron. Therefore, the velocity and hen... the momentum of electron changes.

If we use the photons of longer wavelength say of visible region, the velocity and th... momentum will not change appreciably (کسی خاص حد تک) because longer wavelengths rarely find th... chance to hit the electron. But its position can not be determined because object will not be visible.

Keep it in mind that, the uncertainity is not due to lack (کمی) of better techniques (بہتر ہنر مندی) f... the measurement of position and momentum. **It is due to the reason that we cannot observe th... microscopic objects without disturbing them.** Uncertainity principle is not applicable to stationar... electron (ساکن الیکٹران) because in stationary state the velocity of an electron is zero. As a result, positio... of electron can be accurately determined. But both positions and velocities of electron cannot b... determined accurately.

4.8.3 Mathematical Form of Uncertainity Principle:

We have to consider an hypothetical (فرضی) experiment in which we can measure the position and velocity of an electron. Following diagram (4.14) shows an arrangement in this respect.

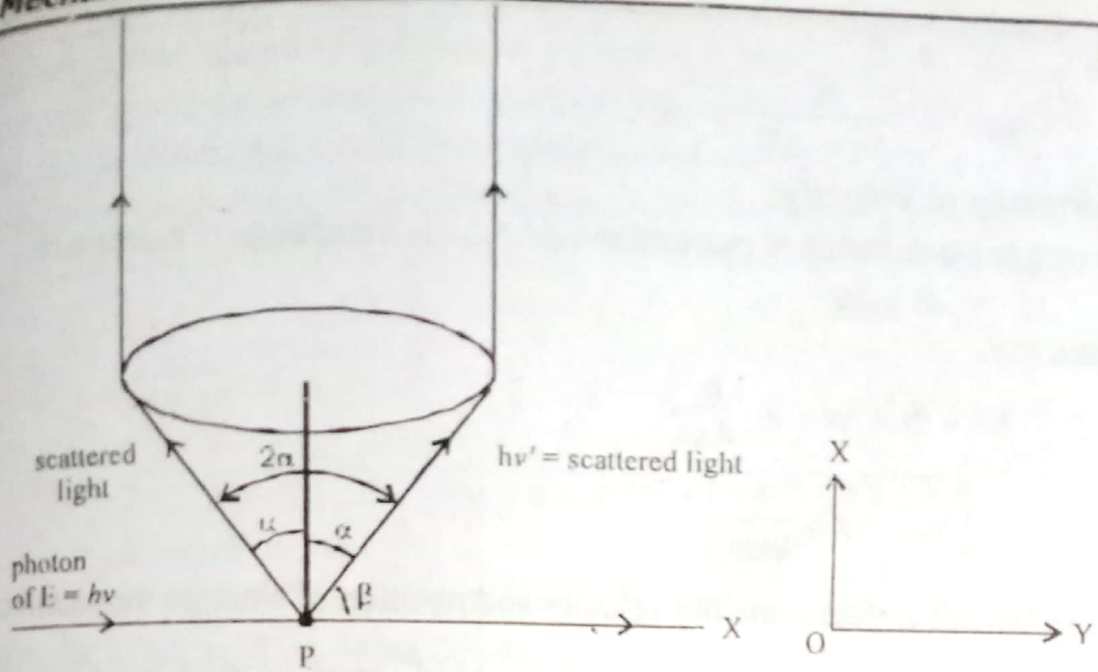

Fig. (4.14) Microscope to study the uncertainity principle.

A photon from a source of γ-rays or X-rays with energy 'hυ' strikes the electron at the point P. When the electron scatters (بکھیرتا ہے) this photon into the microscope in a direction making an angle 'β' with the x-axis, the electron will receive some momentum (معیار حرکت), from the photon along x-axis. Scattered photon can enter the microscope anywhere within the angle '2α'. Its contribution (حصہ ادا کرنا) to x-component of the momentum of electron is,

$$\Delta P_x = 2P \sin\alpha = \frac{2h}{\lambda} \sin\alpha \qquad \cdots\cdots \quad (1)$$

$$\left(\text{because } \lambda = \frac{h}{mv}\right)$$

Rayleigh's equation for the resolving power (اس کی تحلیل کرنے کی صلاحیت) can be used to find the accuracy (ٹھیک) with which an object can be located (جگہ معلوم کرنا) by a microscope and is given by,

$$\Delta X = \frac{\lambda}{2\sin\alpha} \qquad \cdots\cdots \quad (2)$$

ΔX = Distnace between two points which can just be resolved (تحلیل کرنا) by the microscope

λ = Wavelength of photon

Multiplying equation (1) with (2),

$$\Delta X \,.\, \Delta P_x = \frac{\lambda}{2\sin\alpha} \cdot \frac{2h}{\lambda} \sin\alpha \approx h \qquad \cdots\cdots \quad (3)$$

According to the equation (3), the **product of two uncertainities lies in the range of 'h'**. If the calculation is done more carefully, then we come to know that,

$$\Delta X \,.\, \Delta P_x \geq \frac{h}{4\pi} \qquad \cdots\cdots \quad (4)$$

This equation indicates that greater the accuracy in determining the position, greater the uncertainity in determining the momentum. Thus a certainity in one quantity introduces an uncertainity in its conjugate (مواصلات) quantity. In other words, **if one quantity is known free from error, then the error in the other quantity becomes infinity.**

When $\Delta X = 0$

$$\Delta P = \frac{h}{4\pi \times \Delta X} = \infty$$

.8.4 Uncertainity of Velocity:

According to the definition of momentum (معیارِ حرکت), and change of momentum,

$$\Delta P = m \times \Delta v$$

It means that,

$$\Delta X \times m \times \Delta v \geq \frac{h}{4\pi}$$

$$\boxed{\Delta v \times \Delta X \geq \frac{h}{4\pi m}}$$

So, **it is difficult to determine the velocity and position of electron simultaneously.**

رفتار اور الیکٹران کی جگہ ایک ہی وقت میں معلوم کرنا مشکل ہے۔

Uncertainity principle can also be applied for another conjugate pair i.e. energy and time.

Since,

$$\Delta \upsilon = \frac{1}{\Delta t}$$

As, $$\Delta E = h \times \Delta \upsilon$$

So, $$\Delta E = \frac{h}{\Delta t}$$

$$\Delta E \times \Delta t = h$$

More realistic treatment shows that,

$$\boxed{\Delta E \times \Delta t = \frac{h}{2\pi}}$$

It means that **it is difficult to determine the energy and time for the particle simultaneously.**

So, Heisenberg uncertainity principle is applicable to any **conjugate pairs** (مواصلاتی جوڑا) of variables (قابلِ تغیر) and we reach the conclusion that the product of uncertainities of any two conjugate variable is always constant and its value range between $\frac{h}{2\pi}$ and $\frac{h}{4\pi}$.

Born	Werner Karl Heisenberg, 5 December 1901, Würzburg, Bavaria, G...

(iv) Ψ must be ...

(v) 'Ψ' should have a definite value over the space dτ, that ...

The above discussion shows that only acceptable values (قابلِ قبول قیمتیں) of 'Ψ' should have significance (اہمیت). These significant (معنی خیز) values of 'Ψ' are called functions or wave functions (ویو فنکشنز). This eigen function gives significant values of total energy of electron called eigen values.

4.12.0 MOTION OF PARTICLE IN ONE-DIMENSIONAL BOX

(ایک سائڈ والے باکس میں ذرے کی حرکت)

This is one of the best applications of Schrodinger wave equation and is simplest way to apply the equation.

(اس آرٹیکل میں ہم شروڈنگر کی لہری مساوات کی ایک Application پڑھنے لگے ہیں۔)

Consider a particle say an electron of mass 'm' moving in a one dimensional box of width 'a', along x-axis as shown in diagram (4.15).

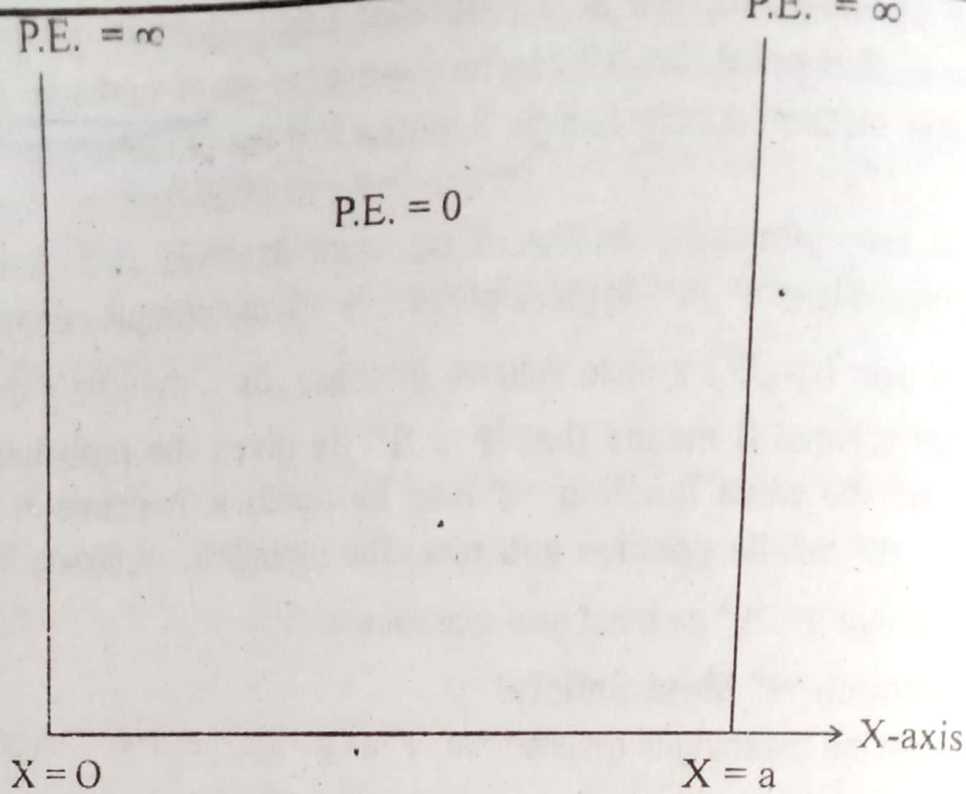

Fig. (4.15) Motion of a particle in one dimensional box, i.e., along x-axis.

The boundaries (حدود) of the box are x = 0 and x = a. The height of the walls at +x = 0 a x = a are infinite (لامتناہی). The potential energy 'P' inside the box is zero. So, the electron can mo without any restriction (پابندی) inside the box. Anyhow, the potential energy at the walls and outside t box is at infinity. It means that the particle is fully confined (محصور ہے) within the box and it cannot escap from the box by crossing (عبور کرنا) the walls of infinite height (لامتناہی بلندی).

Now, let us apply the Schrodinger wave equation to understand the motion of the particle this box.

Schrodinger equation in three dimensional motion is as follows:

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2} + \frac{8\pi^2 m}{h^2}(E - P)\Psi = 0 \quad \cdots\cdots (1)$$

In one dimension box the particle is not moving along y- and z-axis in one dimensional box. So, the derivative of "Ψ" with respect to y and z, is zero. Moreover, the potential energy 'P' is zero within the box. So, equation (1) becomes

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E)\Psi = 0 \quad \cdots\cdots (2)$$

For the given state of the system, the energy 'E' is constant, which is one of the postulates (مفروضہ جات) of quantum mechanics (ایسی میکانکس جو خورد بینی ذرات کی حرکات کو سمجھنے کے لیے لاگو ہوتی ہے۔). Now put

$$\frac{8\pi^2 m}{h^2} E = k^2 \quad \cdots\cdots (3)$$

'k' is constant and is independent of 'x'.

Equation (2) can be written as

$$\frac{d^2\Psi}{dx^2} + k^2\Psi = 0 \quad \cdots\cdots (4)$$

(مساوات (3) میں جو k رکھا ہے وہ صرف مساوات کو (4) کی شکل دیگر سادے طریقے سے لکھنے کے لئے کیا ہے۔)

This is a second order differential equation has the following solution.

$$\Psi = A \sin(kx) + B \cos(kx) \quad \cdots\cdots (5)$$

(یاد رکھیں Calculus میں کسی differential equation کو حل کرتے ہوئے ایک standard طریقے سے اس کا حل تلاش کیا جاتا ہے۔)

Here 'A', 'B' and 'k' are the arbitrary constants. Let us differentiate the equation (5), twice to get the value of $\frac{d^2\Psi}{dx^2}$.

$$\frac{d^2\Psi}{dx^2} = -k^2 [A \sin (kx) + B \cos (kx)] = -k^2\Psi \quad \ldots\ldots (6)$$

Putting in equation (4)

$$-k^2\psi + k^2\psi = 0$$

$$0 = 0$$

(لہٰذا ثابت ہو گیا کہ جو مساوات (5) ہم نے سولیوشن کے طور پر لی تھی وہ ٹھیک ہے۔)

In order to determine the value of constant 'k', let us apply the boundary conditions, (حدود والی شرائط) in equation (5)

(i) When $x = 0$, $\Psi = 0$ (First boundary condition)

It means that selection of equation (5) as the solution of (4) is correct.

$$0 = A \sin (k \times 0) + B \cos (k \times 0) = A \sin 0 + B \cos 0$$

$$0 = 0 + B \times 1$$

or, $B = 0$

Putting this condition that $B = 0$ in equation (5), we get

اس بونڈری کی شرط سے ہمیں B کی قیمت مل گئی ہے۔

$$\Psi = A \sin (kx) \quad \ldots\ldots (7)$$

(ii) At $x = a$, $\Psi = 0$ (Second boundary condition

$$0 = A \sin (k \times a) + 0 \cos (k \times a) = A \sin (k \times a)$$

'A' can not be zero. If A = 0, then it will lead to $\Psi = 0$ for any value of 'x'.

It means that the particle does not exist in the box, which is not acceptable.

Hence, $\sin (k \times a) = 0 = \sin (n\pi)$ when $n = 0, 1, 2, 3, \ldots\ldots$

So, $(k \times a) = n\pi$

$$k = \frac{n\pi}{a} \quad \ldots\ldots (8)$$

Putting this value of 'k' in equation (7). (ہمیں k کی قیمت مل گئی ہے)

$$\Psi = A \sin \left(\frac{n\pi x}{a}\right) \quad \ldots\ldots (9)$$

Where 'n' = quantum number.

Though the zero value of 'n' is permitted, so, n = 0, $\Psi = 0$, everywhere within the box.

Hence n = 0, is not acceptable.

In other words, values of 'n' which are acceptable in the equation (9) are

n = 1, 2, 3, ...

The expression for the eign value 'E' can be obtained as follows.

$$k^2 = \frac{8\pi^2 mE}{h^2}$$

$$E = \frac{k^2h^2}{8\pi^2 m} \qquad \cdots\cdots (10)$$

Putting equation (8) in (10)

$$E = \left(\frac{n^2\pi^2}{a^2}\right)\frac{h^2}{8\pi^2 m} = \frac{n^2h^2}{8ma^2}$$

Hence, $$\boxed{E = \frac{n^2h^2}{8m\,a^2}} \qquad \cdots\cdots (11)$$

Equation (11) gives the values of the energies of the moving electron in one dimensional b[ox]. These permitted values of energy are called **eigen values**. When we put the values of n = 1, 2, 3, ... then we get the energies associated with that particle in one dimensional box.

The diagram for energy levels is shown in Fig (4.16)

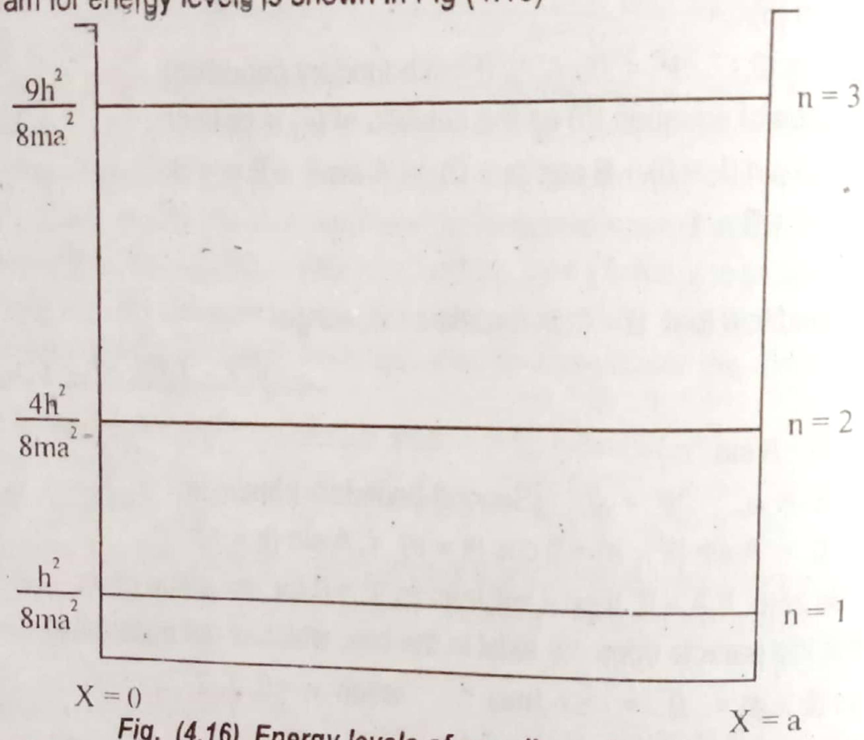

Fig. (4.16) Energy levels of one-dimensional box.

(ہم نے اس box میں انرجی لیولز اوپر نیچے سیڑی کی طرح بنائے ہیں۔ حالانکہ الیکٹران صرف x axis پر x = 0 اور x = a کے درمیان حرکت پذیر ہے۔ لہٰذا اسیٹری کی مانند بنائے ہوئے انرجی لیولز سے یہ تاثر نہ لیں کہ الیکٹران نے دوسرے dimension میں بھی حرکت شروع کر دی ہے۔ یہ تو اس کی انرجی کی قیمتوں کو ذہن میں بٹھانے کا ایک طریقہ اختیار کیا ہے۔ الیکٹران اسی x axis والی لائن میں حرکت کر رہا ہے۔)

This diagram shows that energy levels are not equally spaced (انرجی کے لیولز کا فرق ایک جتنا نہیں ہے). The energy gap between adjacent levels goes on increasing (یہ فرق بڑھتا جاتا ہے). In other words, th[e] energy levels become widely spaced Fig. (4.17).

Conclusions:

The formulae of eigen values shows that:

(i) $E \propto \frac{1}{m}$