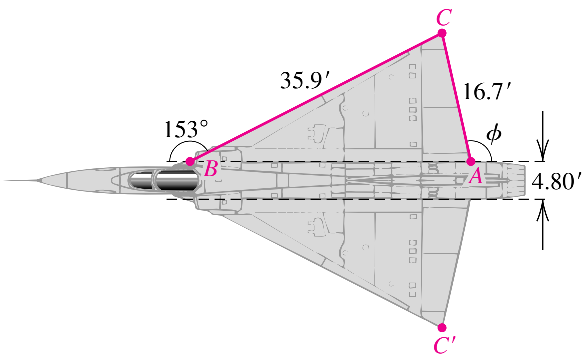
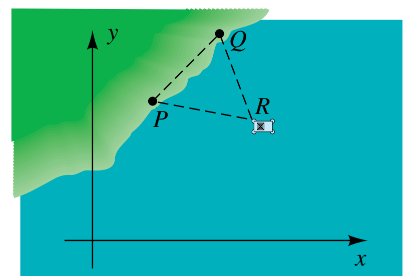
Exercise 30



31 Software for surveyors Computer software for surveyors makes use of coordinate systems to locate geographic positions. An offshore oil well at point R in the figure is viewed from points P and Q, and $\angle QPR$ and $\angle RQP$ are found to be $55°50'$ and $65°22'$, respectively. If points P and Q have coordinates $(1487.7, 3452.8)$ and $(3145.8, 5127.5)$, respectively, approximate the coordinates of R.

Exercise 31



8.2 The Law of Cosines

In the preceding section we stated that the law of sines cannot be applied directly to find the remaining parts of an oblique triangle given either of the following:

(1) two sides and the angle *between* them (SAS)

(2) three sides (SSS)

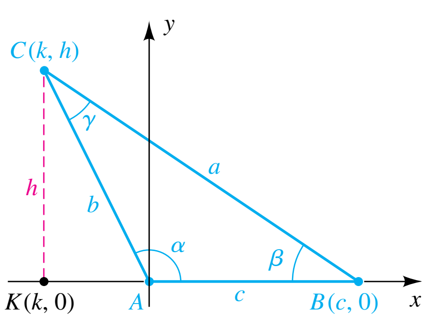
For these cases we may apply the *law of cosines,* which follows.

The Law of Cosines

If ABC is a triangle labeled in the usual manner (as in Figure 1), then

$$\textbf{(1)}\quad a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$\textbf{(2)}\quad b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$\textbf{(3)}\quad c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Figure 1



PROOF Let us prove the first formula. Given triangle ABC, place α in standard position, as illustrated in Figure 1. We have pictured α as obtuse; however, our discussion is also valid if α is acute. Consider the dashed line through C, parallel to the y-axis and intersecting the x-axis at the point $K(k, 0)$. If we let $d(C, K) = h$, then C has coordinates (k, h). By the definition of the trigonometric functions of any angle,

$$\cos\alpha = \frac{k}{b} \qquad \text{and} \qquad \sin\alpha = \frac{h}{b}.$$

Solving for k and h gives us

$$k = b\cos\alpha \qquad \text{and} \qquad h = b\sin\alpha.$$

Since the segment AB has length c, the coordinates of B are $(c, 0)$, and we obtain the following:

$$\begin{aligned} a^2 &= [d(B, C)]^2 = (k - c)^2 + (h - 0)^2 && \text{distance formula} \\ &= (b \cos \alpha - c)^2 + (b \sin \alpha)^2 && \text{substitute for } k \text{ and } h \\ &= b^2 \cos^2 \alpha - 2bc \cos \alpha + c^2 + b^2 \sin^2 \alpha && \text{square} \\ &= b^2(\cos^2 \alpha + \sin^2 \alpha) + c^2 - 2bc \cos \alpha && \text{factor the first and last terms} \\ &= b^2 + c^2 - 2bc \cos \alpha && \text{Pythagorean identity} \end{aligned}$$

Our result is the first formula stated in the law of cosines. The second and third formulas may be obtained by placing β and γ, respectively, in standard position on a coordinate system.

Note that if $\alpha = 90°$ in Figure 1, then $\cos \alpha = 0$ and the law of cosines reduces to $a^2 = b^2 + c^2$. This shows that the Pythagorean theorem is a special case of the law of cosines.

Instead of memorizing each of the three formulas of the law of cosines, it is more convenient to remember the following statement, which takes all of them into account.

The Law of Cosines (General Form)

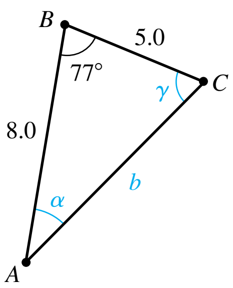
The square of the length of any side of a triangle equals the sum of the squares of the lengths of the other two sides minus twice the product of the lengths of the other two sides and the cosine of the angle between them.

Given two sides and the included angle of a triangle, we can use the law of cosines to find the third side. We may then use the law of sines to find another angle of the triangle. Whenever this procedure is followed, it is best to find the angle opposite the shortest side, since that angle is always acute. In this way, we avoid the possibility of obtaining two solutions when solving a trigonometric equation involving that angle, as illustrated in the following example.

EXAMPLE 1 **Using the law of cosines (SAS)**

Solve $\triangle ABC$, given $a = 5.0$, $c = 8.0$, and $\beta = 77°$.

Figure 2



SOLUTION The triangle is sketched in Figure 2. Since β is the angle *between* sides a and c, we begin by approximating b (the side opposite β) as follows:

$$\begin{aligned} b^2 &= a^2 + c^2 - 2ac \cos \beta && \text{law of cosines} \\ &= (5.0)^2 + (8.0)^2 - 2(5.0)(8.0) \cos 77° && \text{substitute for } a, c, \text{ and } \beta \\ &= 89 - 80 \cos 77° \approx 71.0 && \text{simplify and approximate} \\ b &\approx \sqrt{71.0} \approx 8.4 && \text{take the square root} \end{aligned}$$

(continued)

Let us find another angle of the triangle using the law of sines. In accordance with the remarks preceding this example, we will apply the law of sines and find α, since it is the angle opposite the shortest side a:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} \qquad \text{law of sines}$$

$$\sin\alpha = \frac{a\sin\beta}{b} \qquad \text{solve for } \sin\alpha$$

$$\approx \frac{5.0\sin 77^\circ}{\sqrt{71.0}} \approx 0.5782 \qquad \text{substitute and approximate}$$

Since α is acute,

$$\alpha = \sin^{-1}(0.5782) \approx 35.3^\circ \approx 35^\circ.$$

Finally, since $\alpha + \beta + \gamma = 180^\circ$, we have

$$\gamma = 180^\circ - \alpha - \beta \approx 180^\circ - 35^\circ - 77^\circ = 68^\circ.$$

Given the three sides of a triangle, we can use the law of cosines to find *any* of the three angles. We shall always find the largest angle first—that is, *the angle opposite the longest side*—since this practice will guarantee that the remaining angles are acute. We may then find another angle of the triangle by using either the law of sines or the law of cosines. Note that when an angle is found by means of the law of cosines, there is no ambiguous case, since we always obtain a unique angle between 0° and 180°.

EXAMPLE 2 Using the law of cosines (SSS)

If triangle ABC has sides $a = 90$, $b = 70$, and $c = 40$, approximate angles α, β, and γ to the nearest degree.

SOLUTION In accordance with the remarks preceding this example, we first find the angle opposite the longest side a. Thus, we choose the form of the law of cosines that involves α and proceed as follows:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \qquad \text{law of cosines}$$

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc} \qquad \text{solve for } \cos\alpha$$

$$= \frac{70^2 + 40^2 - 90^2}{2(70)(40)} = -\frac{2}{7} \qquad \text{substitute and simplify}$$

$$\alpha = \cos^{-1}\left(-\tfrac{2}{7}\right) \approx 106.6^\circ \approx 107^\circ \qquad \text{approximate } \alpha$$

We may now use either the law of sines or the law of cosines to find β. Let's use the law of cosines in this case:

$$b^2 = a^2 + c^2 - 2ac\cos\beta \qquad \text{law of cosines}$$

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac} \qquad \text{solve for } \cos\beta$$

$$= \frac{90^2 + 40^2 - 70^2}{2(90)(40)} = \frac{2}{3} \qquad \text{substitute and simplify}$$

$$\beta = \cos^{-1}\left(\tfrac{2}{3}\right) \approx 48.2° \approx 48° \qquad \text{approximate } \beta$$

At this point in the solution, we could find γ by using the relationship $\alpha + \beta + \gamma = 180°$. But if either α or β was incorrectly calculated, then γ would be incorrect. Alternatively, we can approximate γ and then check that the sum of the three angles is 180°. Thus,

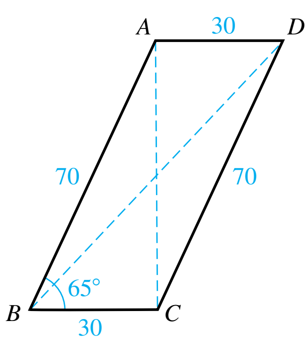
$$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}, \qquad \text{so} \qquad \gamma = \cos^{-1}\frac{90^2 + 70^2 - 40^2}{2(90)(70)} \approx 25°.$$

Note that $\alpha + \beta + \gamma = 107° + 48° + 25° = 180°$.

EXAMPLE 3 **Approximating the diagonals of a parallelogram**

A parallelogram has sides of lengths 30 centimeters and 70 centimeters and one angle of measure 65°. Approximate the length of each diagonal to the nearest centimeter.

Figure 3



SOLUTION The parallelogram $ABCD$ and its diagonals AC and BD are shown in Figure 3. Using triangle ABC with $\angle ABC = 65°$, we may approximate AC as follows:

$$(AC)^2 = 30^2 + 70^2 - 2(30)(70)\cos 65° \qquad \text{law of cosines}$$
$$\approx 900 + 4900 - 1775 = 4025 \qquad \text{approximate}$$
$$AC \approx \sqrt{4025} \approx 63 \text{ cm} \qquad \text{take the square root}$$

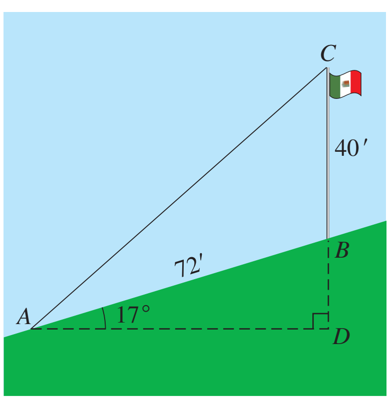
Similarly, using triangle BAD and $\angle BAD = 180° - 65° = 115°$, we may approximate BD as follows:

$$(BD)^2 = 30^2 + 70^2 - 2(30)(70)\cos 115° \approx 7575 \qquad \text{law of cosines}$$
$$BD \approx \sqrt{7575} \approx 87 \text{ cm} \qquad \text{take the square root}$$

EXAMPLE 4 **Finding the length of a cable**

A vertical pole 40 feet tall stands on a hillside that makes an angle of 17° with the horizontal. Approximate the minimal length of cable that will reach from the top of the pole to a point 72 feet downhill from the base of the pole.

Figure 4



SOLUTION The sketch in Figure 4 depicts the given data. We wish to find AC. Referring to the figure, we see that

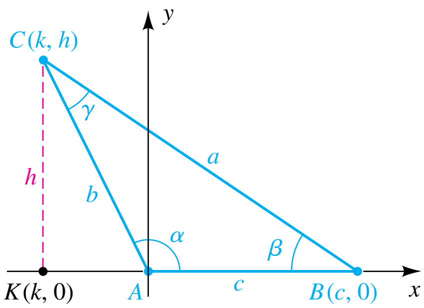
$$\angle ABD = 90° - 17° = 73° \qquad \text{and} \qquad \angle ABC = 180° - 73° = 107°.$$

Using triangle ABC, we may approximate AC as follows:

$$(AC)^2 = 72^2 + 40^2 - 2(72)(40)\cos 107° \approx 8468 \qquad \text{law of cosines}$$
$$AC \approx \sqrt{8468} \approx 92 \text{ ft} \qquad \text{take the square root}$$

The law of cosines can be used to derive a formula for the area of a triangle. Let us first prove a preliminary result.

Figure 5



Given triangle ABC, place angle α in standard position (see Figure 5). As shown in the proof of the law of cosines, the altitude h from vertex C is $h = b \sin \alpha$. Since the area $\mathcal{A}$ of the triangle is given by $\mathcal{A} = \frac{1}{2}ch$, we see that

$$\mathcal{A} = \tfrac{1}{2}bc \sin \alpha.$$

Our argument is independent of the specific angle that is placed in standard position. By taking β and γ in standard position, we obtain the formulas

$$\mathcal{A} = \tfrac{1}{2}ac \sin \beta \qquad \text{and} \qquad \mathcal{A} = \tfrac{1}{2}ab \sin \gamma.$$

All three formulas are covered in the following statement.

Area of a Triangle	The area of a triangle equals one-half the product of the lengths of any two sides and the sine of the angle between them.

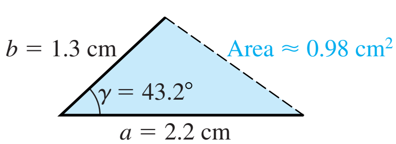
The next two examples illustrate uses of this result.

EXAMPLE 5 **Approximating the area of a triangle**

Approximate the area of triangle ABC if $a = 2.20$ cm, $b = 1.30$ cm, and $\gamma = 43.2°$.

SOLUTION Since γ is the angle between sides a and b as shown in Figure 6, we may use the preceding result directly, as follows:

Figure 6



$$\begin{aligned} \mathcal{A} &= \tfrac{1}{2}ab \sin \gamma && \text{area of a triangle formula} \\ &= \tfrac{1}{2}(2.20)(1.30) \sin 43.2° \approx 0.98 \text{ cm}^2 && \text{substitute and approximate} \end{aligned}$$

EXAMPLE 6 **Approximating the area of a triangle**

Approximate the area of triangle ABC if $a = 5.0$ cm, $b = 3.0$ cm, and $\alpha = 37°$.

SOLUTION To apply the formula for the area of a triangle, we must find the angle γ between known sides a and b. Since we are given a, b, and α, let us first find β as follows:

$$\begin{aligned} \frac{\sin \beta}{b} &= \frac{\sin \alpha}{a} && \text{law of sines} \\ \sin \beta &= \frac{b \sin \alpha}{a} && \text{solve for } \sin \beta \\ &= \frac{3.0 \sin 37°}{5.0} && \text{substitute for } b, \alpha, \text{ and } a \\ \beta_R &= \sin^{-1}\left(\frac{3.0 \sin 37°}{5.0}\right) \approx 21° && \text{reference angle for } \beta \\ \beta \approx 21° \quad &\text{or} \quad \beta \approx 159° && \beta_R \text{ or } 180° - \beta_R \end{aligned}$$

We reject $\beta \approx 159^\circ$, because then $\alpha + \beta = 196^\circ \geq 180^\circ$. Hence, $\beta \approx 21^\circ$ and

$$\gamma = 180^\circ - \alpha - \beta \approx 180^\circ - 37^\circ - 21^\circ = 122^\circ.$$

Finally, we approximate the area of the triangle as follows:

$$\begin{aligned} \mathscr{A} &= \tfrac{1}{2}ab \sin \gamma && \text{area of a triangle formula} \\ &\approx \tfrac{1}{2}(5.0)(3.0) \sin 122^\circ \approx 6.4 \text{ cm}^2 && \text{substitute and approximate} \end{aligned}$$

We will use the preceding result for the area of a triangle to derive *Heron's formula,* which expresses the area of a triangle in terms of the lengths of its sides.

Heron's Formula

The area $\mathscr{A}$ of a triangle with sides a, b, and c is given by

$$\mathscr{A} = \sqrt{s(s-a)(s-b)(s-c)},$$

where s is one-half the perimeter; that is, $s = \frac{1}{2}(a + b + c)$.

PROOF The following equations are equivalent:

$$\begin{aligned} \mathscr{A} &= \tfrac{1}{2}bc \sin \alpha \\ &= \sqrt{\tfrac{1}{4}b^2c^2 \sin^2 \alpha} \\ &= \sqrt{\tfrac{1}{4}b^2c^2(1 - \cos^2 \alpha)} \\ &= \sqrt{\tfrac{1}{2}bc(1 + \cos \alpha) \cdot \tfrac{1}{2}bc(1 - \cos \alpha)} \end{aligned}$$

We shall obtain Heron's formula by replacing the expressions under the final radical sign by expressions involving only a, b, and c. We solve formula 1 of the law of cosines for $\cos \alpha$ and then substitute, as follows:

$$\begin{aligned} \frac{1}{2}bc(1 + \cos \alpha) &= \frac{1}{2}bc\left(1 + \frac{b^2 + c^2 - a^2}{2bc}\right) \\ &= \frac{1}{2}bc\left(\frac{2bc + b^2 + c^2 - a^2}{2bc}\right) \\ &= \frac{2bc + b^2 + c^2 - a^2}{4} \\ &= \frac{(b + c)^2 - a^2}{4} \\ &= \frac{(b + c) + a}{2} \cdot \frac{(b + c) - a}{2} \end{aligned}$$

(continued)

We use the same type of manipulations on the second expression under the radical sign:

$$\frac{1}{2}bc(1 - \cos\alpha) = \frac{a - b + c}{2} \cdot \frac{a + b - c}{2}$$

If we now substitute for the expressions under the radical sign, we obtain

$$\mathcal{A} = \sqrt{\frac{b + c + a}{2} \cdot \frac{b + c - a}{2} \cdot \frac{a - b + c}{2} \cdot \frac{a + b - c}{2}}.$$

Letting $s = \frac{1}{2}(a + b + c)$, we see that

$$s - a = \frac{b + c - a}{2}, \quad s - b = \frac{a - b + c}{2}, \quad s - c = \frac{a + b - c}{2}.$$

Substitution in the above formula for $\mathcal{A}$ gives us Heron's formula.

EXAMPLE 7 Using Heron's formula

A triangular field has sides of lengths 125 yards, 160 yards, and 225 yards. Approximate the number of acres in the field. (One acre is equivalent to 4840 square yards.)

SOLUTION We first find one-half the perimeter of the field with $a = 125$, $b = 160$, and $c = 225$, as well as the values of $s - a$, $s - b$, and $s - c$:

$$s = \tfrac{1}{2}(125 + 160 + 225) = \tfrac{1}{2}(510) = 255$$
$$s - a = 255 - 125 = 130$$
$$s - b = 255 - 160 = 95$$
$$s - c = 255 - 225 = 30$$

Substituting in Heron's formula gives us

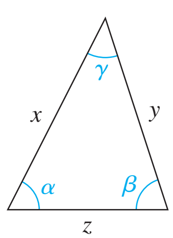
$$\mathcal{A} = \sqrt{(255)(130)(95)(30)} \approx 9720 \text{ yd}^2.$$

Since there are 4840 square yards in one acre, the number of acres is $\frac{9720}{4840}$, or approximately 2.

8.2 Exercises

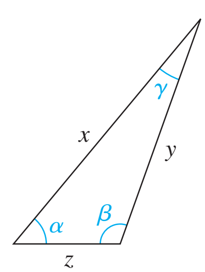
Exer. 1–2: Use common sense to match the variables and the values. (The triangles are drawn to scale, and the angles are measured in radians.)

1



(a) α (A) 12.60
(b) β (B) 1.10
(c) γ (C) 10
(d) x (D) 0.79
(e) y (E) 13.45
(f) z (F) 1.26

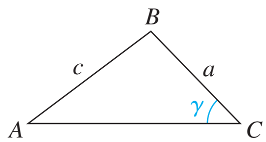
2



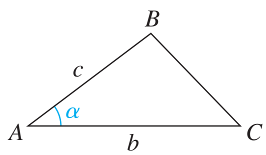
(a) α (A) 3
(b) β (B) 0.87
(c) γ (C) 8.24
(d) x (D) 1.92
(e) y (E) 6.72
(f) z (F) 0.35

Exer. 3–4: Given the indicated parts of $\triangle ABC$, what angle (α, β, or γ) or side (a, b, or c) would you find next, and what would you use to find it?

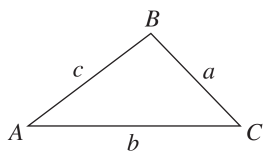
3 (a)



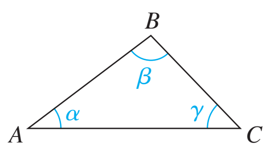
(b)



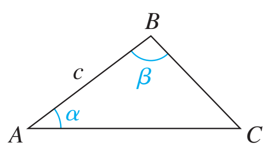
(c)



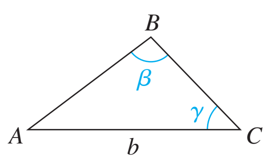
(d)



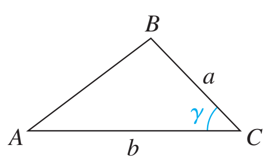
(e)



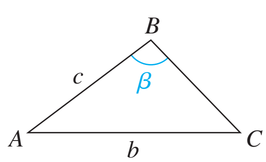
(f)



4 (a)



(b)



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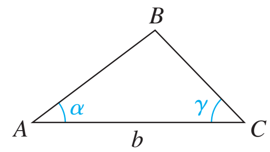

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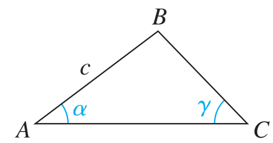

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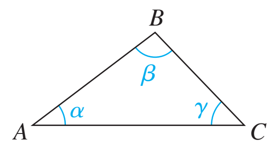
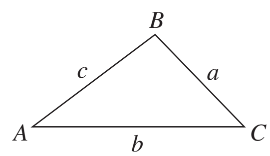


(f)



Exer. 5–14: Solve $\triangle ABC$.

5 $\alpha = 60°$, $b = 20$, $c = 30$

6 $\gamma = 45°$, $b = 10.0$, $a = 15.0$

7 $\beta = 150°$, $a = 150$, $c = 30$

8 $\beta = 73°50'$, $c = 14.0$, $a = 87.0$

9 $\gamma = 115°10'$, $a = 1.10$, $b = 2.10$

10 $\alpha = 23°40'$, $c = 4.30$, $b = 70.0$

11 $a = 2.0$, $b = 3.0$, $c = 4.0$

12 $a = 10$, $b = 15$, $c = 12$

13 $a = 25.0$, $b = 80.0$, $c = 60.0$

14 $a = 20.0$, $b = 20.0$, $c = 10.0$

15 **Dimensions of a triangular plot** The angle at one corner of a triangular plot of ground is $73°40'$, and the sides that meet at this corner are 175 feet and 150 feet long. Approximate the length of the third side.

16 **Surveying** To find the distance between two points A and B, a surveyor chooses a point C that is 420 yards from A and 540 yards from B. If angle ACB has measure $63°10'$, approximate the distance between A and B.

17 **Distance between automobiles** Two automobiles leave a city at the same time and travel along straight highways that differ in direction by 84°. If their speeds are 60 mi/hr and 45 mi/hr, respectively, approximately how far apart are the cars at the end of 20 minutes?

18 **Angles of a triangular plot** A triangular plot of land has sides of lengths 420 feet, 350 feet, and 180 feet. Approximate the smallest angle between the sides.

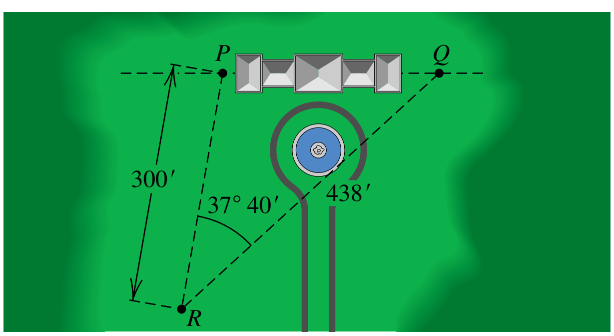
19 **Distance between ships** A ship leaves port at 1:00 P.M. and travels S35°E at the rate of 24 mi/hr. Another ship leaves the same port at 1:30 P.M. and travels S20°W at 18 mi/hr. Approximately how far apart are the ships at 3:00 P.M.?

20 **Flight distance** An airplane flies 165 miles from point A in the direction 130° and then travels in the direction 245° for 80 miles. Approximately how far is the airplane from A?

21 **Jogger's course** A jogger runs at a constant speed of one mile every 8 minutes in the direction S40°E for 20 minutes and then in the direction N20°E for the next 16 minutes. Approximate, to the nearest tenth of a mile, the straight-line distance from the endpoint to the starting point of the jogger's course.

22 **Surveying** Two points P and Q on level ground are on opposite sides of a building. To find the distance between the points, a surveyor chooses a point R that is 300 feet from P and 438 feet from Q and then determines that angle PRQ has measure $37°40'$ (see the figure). Approximate the distance between P and Q.

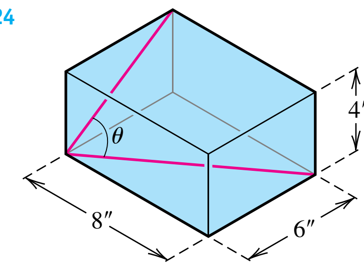
Exercise 22



23 **Motorboat's course** A motorboat traveled along a triangular course having sides of lengths 2 kilometers, 4 kilometers, and 3 kilometers, respectively. The first side was traversed in the direction N20°W and the second in a direction Sθ°W, where $\theta°$ is the degree measure of an acute angle. Approximate, to the nearest minute, the direction in which the third side was traversed.

24 **Angle of a box** The rectangular box shown in the figure has dimensions $8'' \times 6'' \times 4''$. Approximate the angle θ formed by a diagonal of the base and a diagonal of the $6'' \times 4''$ side.

Exercise 24

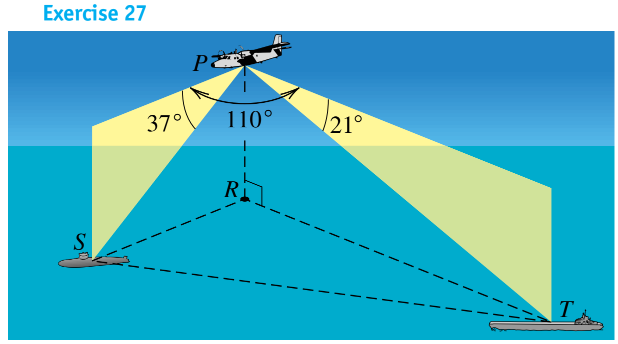


25 **Distances in a baseball diamond** A baseball diamond has four bases (forming a square) that are 90 feet apart; the pitcher's mound is 60.5 feet from home plate. Approximate the distance from the pitcher's mound to each of the other three bases.

26 A rhombus has sides of length 100 centimeters, and the angle at one of the vertices is 70°. Approximate the lengths of the diagonals to the nearest tenth of a centimeter.

27 **Reconnaissance** A reconnaissance airplane P, flying at 10,000 feet above a point R on the surface of the water, spots a submarine S at an angle of depression of 37° and a tanker T at an angle of depression of 21°, as shown in the figure. In addition, $\angle SPT$ is found to be 110°. Approximate the distance between the submarine and the tanker.

Exercise 27

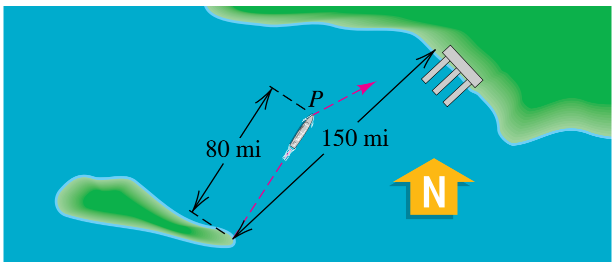


28 Correcting a ship's course A cruise ship sets a course N47°E from an island to a port on the mainland, which is 150 miles away. After moving through strong currents, the ship is off course at a position P that is N33°E and 80 miles from the island, as illustrated in the figure.

(a) Approximately how far is the ship from the port?

(b) In what direction should the ship head to correct its course?

Exercise 28

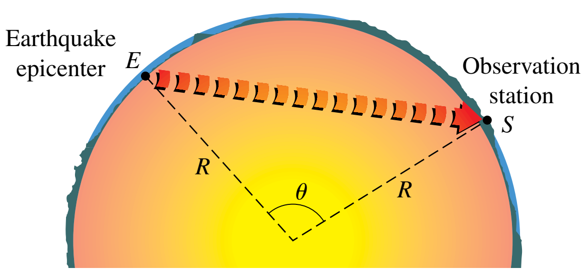


29 Seismology Seismologists investigate the structure of Earth's interior by analyzing seismic waves caused by earthquakes. If the interior of Earth is assumed to be homogeneous, then these waves will travel in straight lines at a constant velocity v. The figure shows a cross-sectional view of Earth, with the epicenter at E and an observation station at S. Use the law of cosines to show that the time t for a wave to travel through Earth's interior from E to S is given by

$$t = \frac{2R}{v}\sin\frac{\theta}{2},$$

where R is the radius of Earth and θ is the indicated angle with vertex at the center of Earth.

Exercise 29

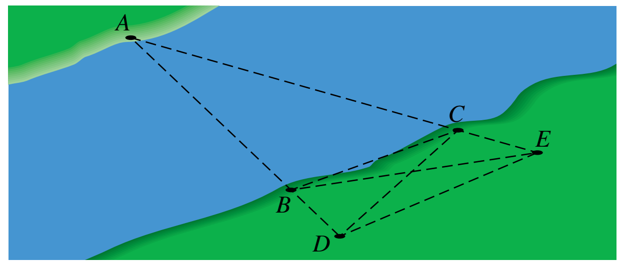


30 Calculating distances The distance across the river shown in the figure can be found without measuring angles. Two points B and C on the opposite shore are selected, and line segments AB and AC are extended as shown. Points D and E are chosen as indicated, and distances BC, BD, BE, CD, and CE are then measured. Suppose that $BC = 184$ ft, $BD = 102$ ft, $BE = 218$ ft, $CD = 236$ ft, and $CE = 80$ ft.

(a) Approximate the distances AB and AC.

(b) Approximate the shortest distance across the river from point A.

Exercise 30



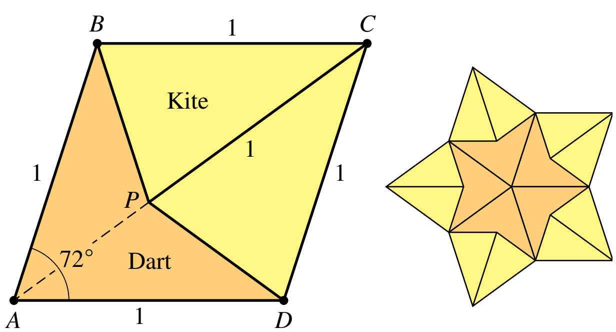
31 Penrose tiles Penrose tiles are formed from a rhombus $ABCD$ having sides of length 1 and an interior angle of 72°. First a point P is located that lies on the diagonal AC and is a distance 1 from vertex C, and then segments PB and PD are drawn to the other vertices of the diagonal, as shown in the figure. The two tiles formed are called a dart and a kite. Three-dimensional counterparts of these tiles have been applied in molecular chemistry.

(a) Find the degree measures of $\angle BPC$, $\angle APB$, and $\angle ABP$.

(b) Approximate, to the nearest 0.01, the length of segment BP.

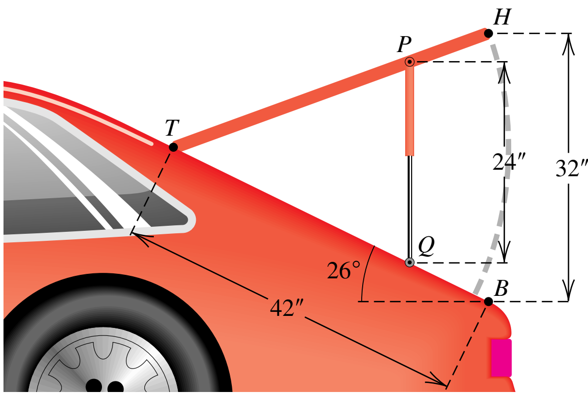
(c) Approximate, to the nearest 0.01, the area of a kite and the area of a dart.

Exercise 31



32 Automotive design The rear hatchback door of an automobile is 42 inches long. A strut with a fully extended length of 24 inches is to be attached to the door and the body of the car so that when the door is opened completely, the strut is vertical and the rear clearance is 32 inches, as shown in the figure. Approximate the lengths of segments TQ and TP.

Exercise 32



Exer. 33–40: Approximate the area of triangle *ABC*.

33 $\alpha = 60°$, $b = 20$, $c = 30$

34 $\gamma = 45°$, $b = 10.0$, $a = 15.0$

35 $\alpha = 40.3°$, $\beta = 62.9°$, $b = 5.63$

36 $\alpha = 35.7°$, $\gamma = 105.2°$, $b = 17.2$

37 $\alpha = 80.1°$, $a = 8.0$, $b = 3.4$

38 $\gamma = 32.1°$, $a = 14.6$, $c = 15.8$

39 $a = 25.0$, $b = 80.0$, $c = 60.0$

40 $a = 20.0$, $b = 20.0$, $c = 10.0$

Exer. 41–42: A triangular field has sides of lengths a, b, and c (in yards). Approximate the number of acres in the field (1 acre = 4840 yd^2).

41 $a = 115$, $b = 140$, $c = 200$

42 $a = 320$, $b = 350$, $c = 500$

Exer. 43–44: Approximate the area of a parallelogram that has sides of lengths a and b (in feet) if one angle at a vertex has measure θ.

43 $a = 12.0$, $b = 16.0$, $\theta = 40°$

44 $a = 40.3$, $b = 52.6$, $\theta = 100°$

8.3 Vectors

Quantities such as area, volume, length, temperature, and time have magnitude only and can be completely characterized by a single real number (with an appropriate unit of measurement such as in^2, ft^3, cm, deg, or sec). A quantity of this type is a **scalar quantity,** and the corresponding real number is a **scalar.** A concept such as velocity or force has both magnitude and direction and is often represented by a **directed line segment**—that is, a line segment to which a direction has been assigned. Another name for a directed line segment is a **vector.**

As shown in Figure 1, we use $\overrightarrow{PQ}$ to denote the vector with **initial point** P and **terminal point** Q, and we indicate the direction of the vector by placing the arrowhead at Q. The **magnitude** of $\overrightarrow{PQ}$ is the length of the segment PQ and is denoted by $\|\overrightarrow{PQ}\|$. As in the figure, we use boldface letters such as **u** and **v** to denote vectors whose endpoints are not specified. In handwritten work, a notation such as $\vec{u}$ or $\vec{v}$ is often used.

Vectors that have the same magnitude and direction are said to be **equivalent.** In mathematics, a vector is determined only by its magnitude and direc-