College of Engineering &Technology

University of Sargodha

Department of Electrical Engineering Technology

ET-314

Telecommunication Technology

Lecture 13

Inter Symbol Interference

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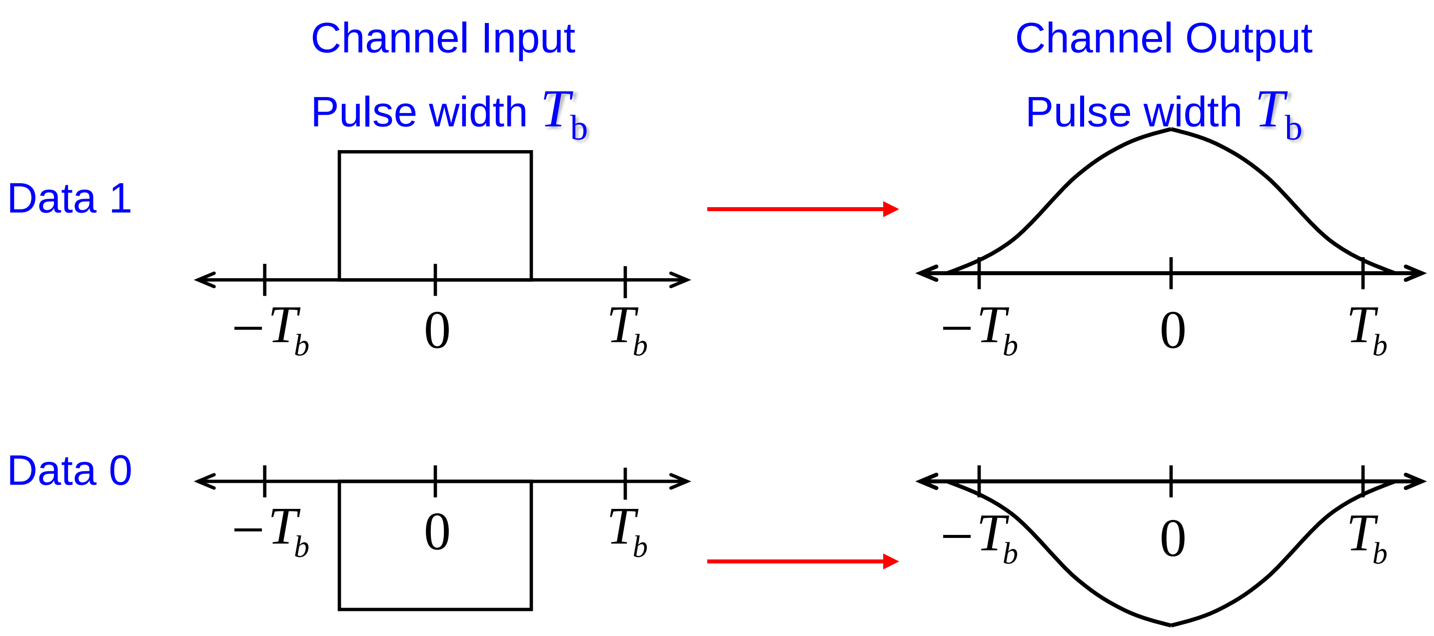
INTERSYMBOL INTERFERENCE (ISI)

- Intersymbol Interference
- ISI on Eye Patterns
- Combatting ISI
- Nyquist's First Method for zero ISI
- Raised Cosine-Rolloff Pulse Shape
- Nyquist Filter

Intersymbol Interference

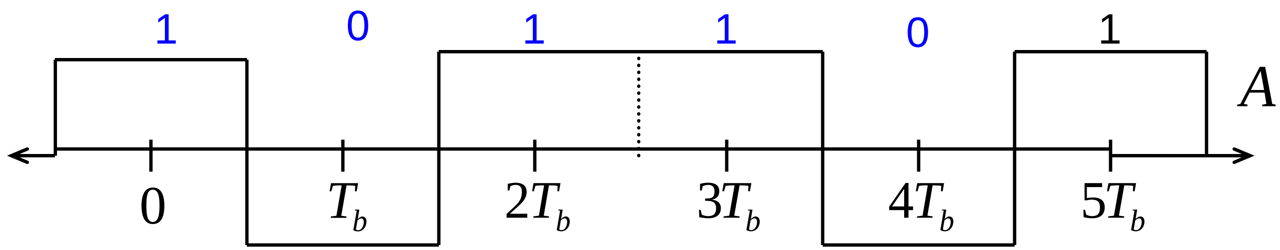
- **Intersymbol interference (ISI)** occurs when a pulse spreads out in such a way that it interferes with adjacent pulses *at the sample instant.*

- Example: assume polar NRZ line code. The channel outputs are shown as spreaded (width T_b becomes $2T_b$) pulses shown (Spreading due to bandlimited channel characteristics).

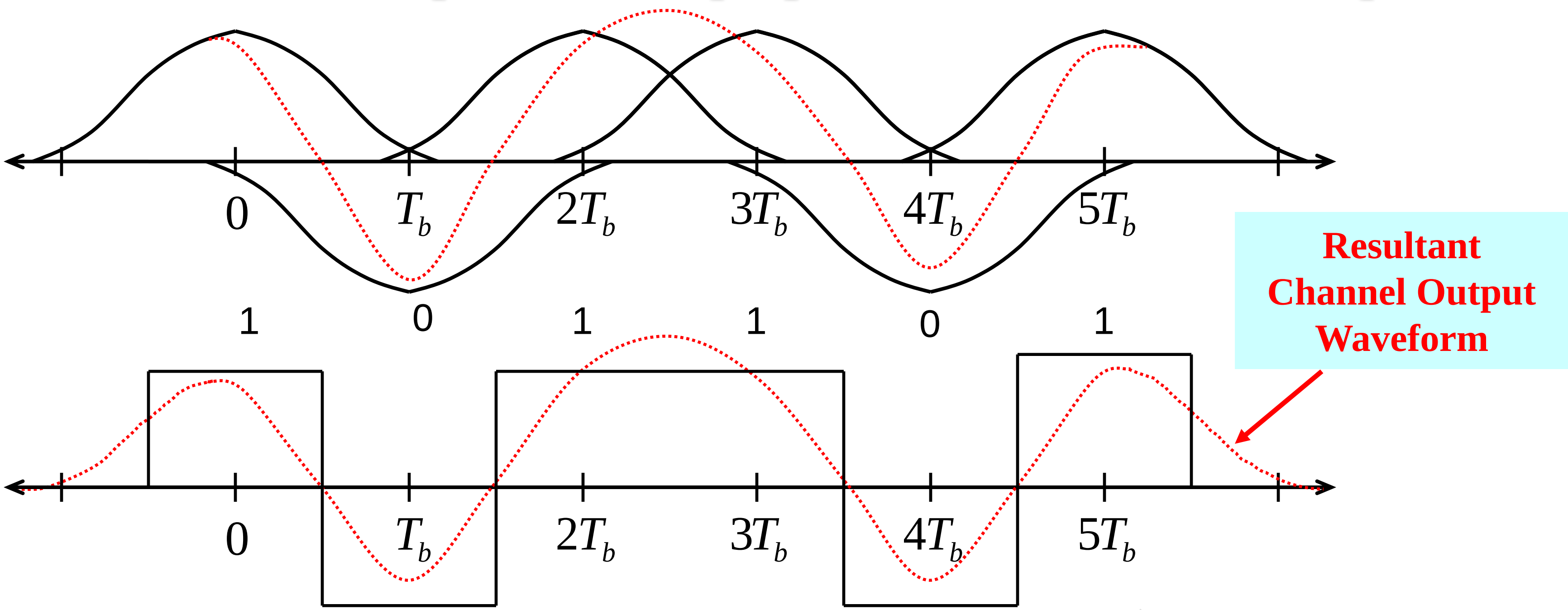


Intersymbol Interference

- For the input data stream:

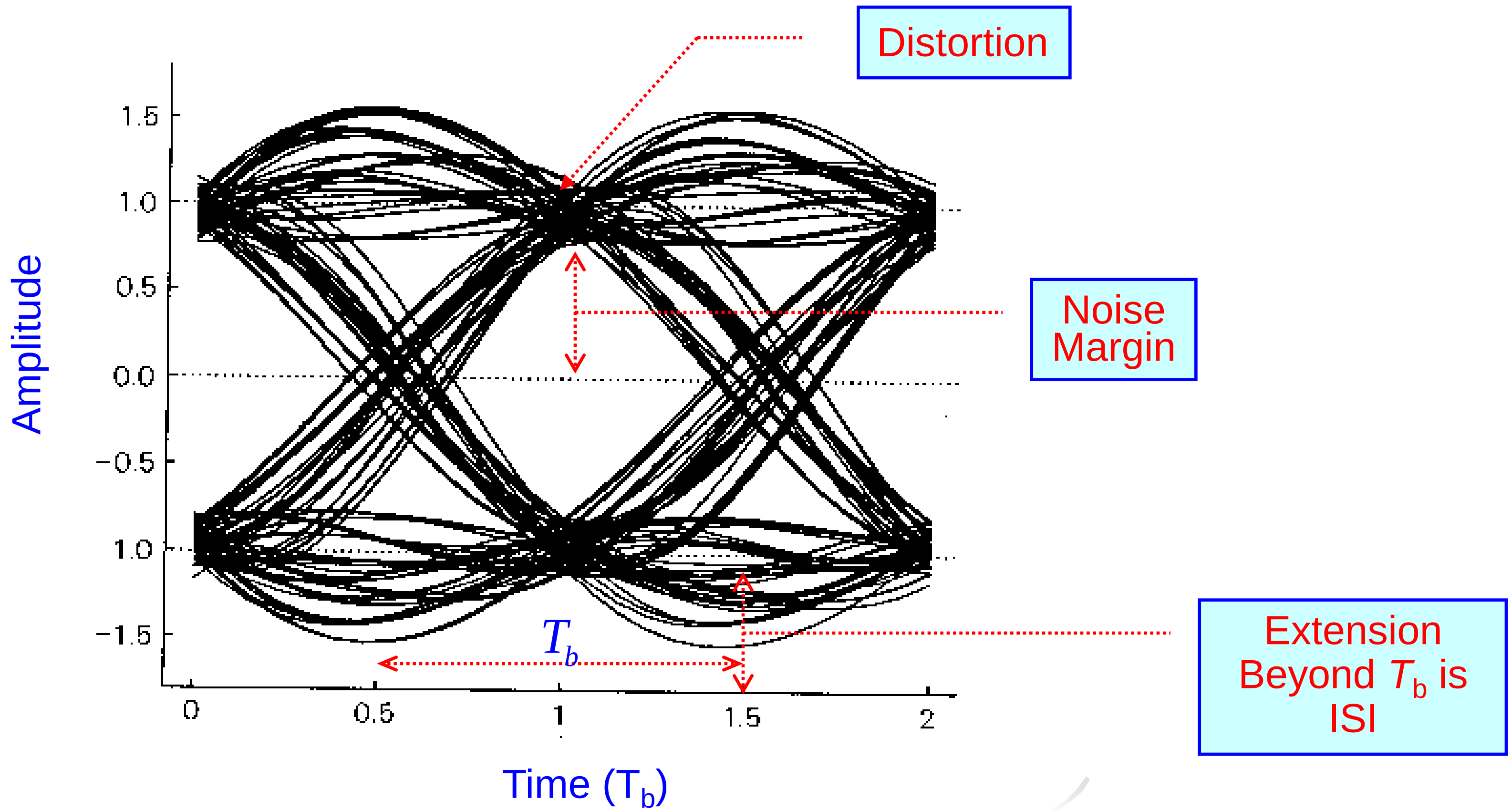


- The channel output is the superposition of each bit's output:



ISI on Eye Patterns

- The amount of ISI can be seen on an oscilloscope using an ***Eye Diagram*** or ***Eye pattern***.



Intersymbol Interference

➢ If the rectangular multilevel pulses are filtered improperly as they pass through a communications system, they will spread in time, and the pulse for each symbol may be smeared into adjacent time slots and cause ***Intersymbol Interference.***

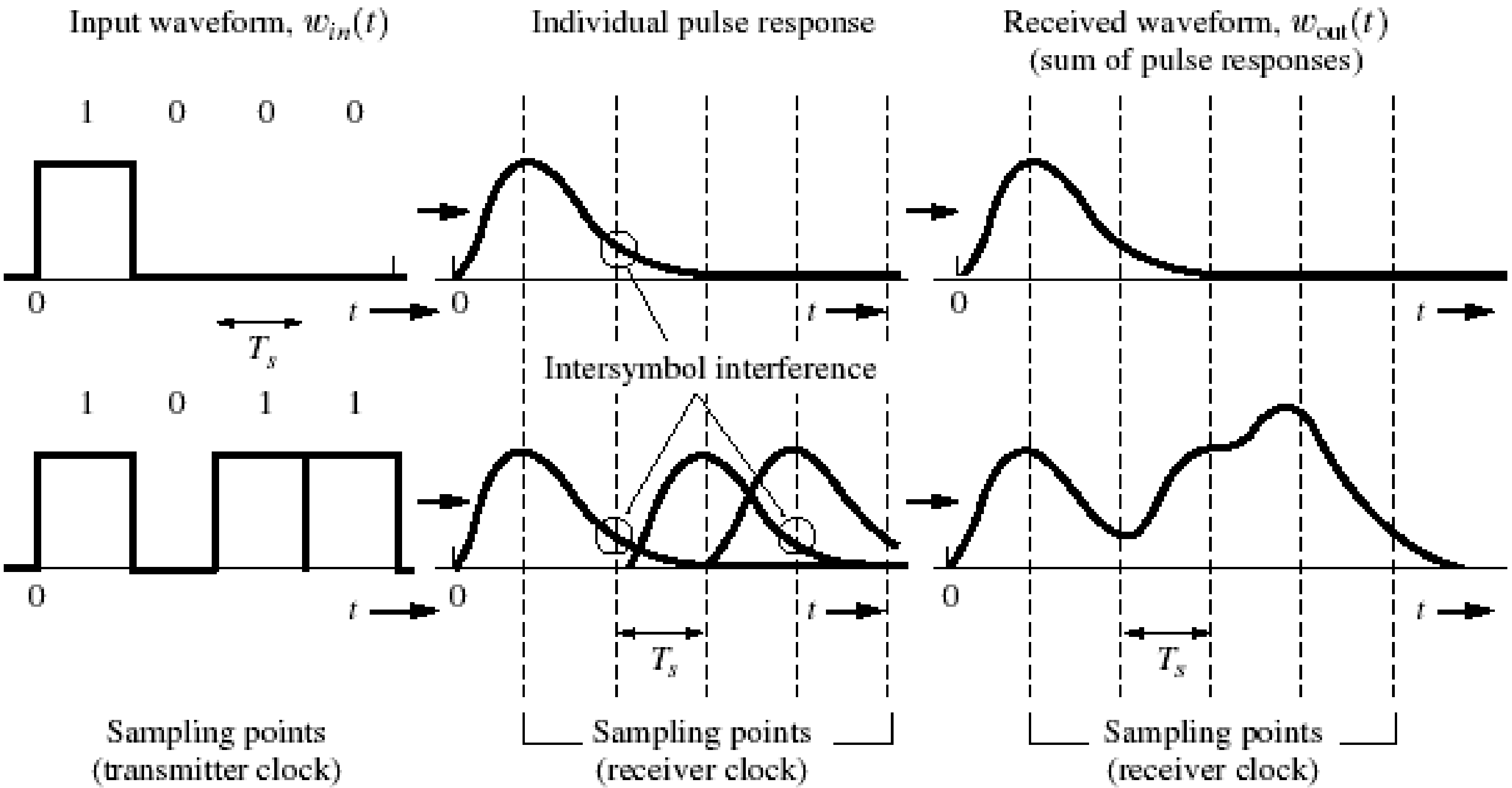


Figure 3–23 Examples of ISI on received pulses in a binary communication system.

➢ **How can we restrict BW and at the same time not introduce ISI? 3 Techniques.**

Intersymbol Interference

- Flat-topped multilevel input signal having pulse shape $h(t)$ and values a_k:

$$w_{in}(t) = \sum a_n h(t - nT_s) = \sum_n a_n h(t) * \delta(t - nT_s) = \left[\sum_n a_n \delta(t - nT_s) \right] * h(t)$$

Where $h(t) = \prod\left(\frac{t}{T_s}\right)$ Where $D = \frac{1}{T_s}$ pulses/s

$$w_{out}(t) = \left[\sum_n a_n \delta(t - nT_s) \right] * h_e(t) = \sum_n a_n h_e(t - nT_s)$$

Equivalent impulse response: $h_e(t) = h(t) * h_T(t) * h_C(t) * h_R(t)$

- $h_e(t)$ is the pulse shape that will appear at the output of the receiver filter.

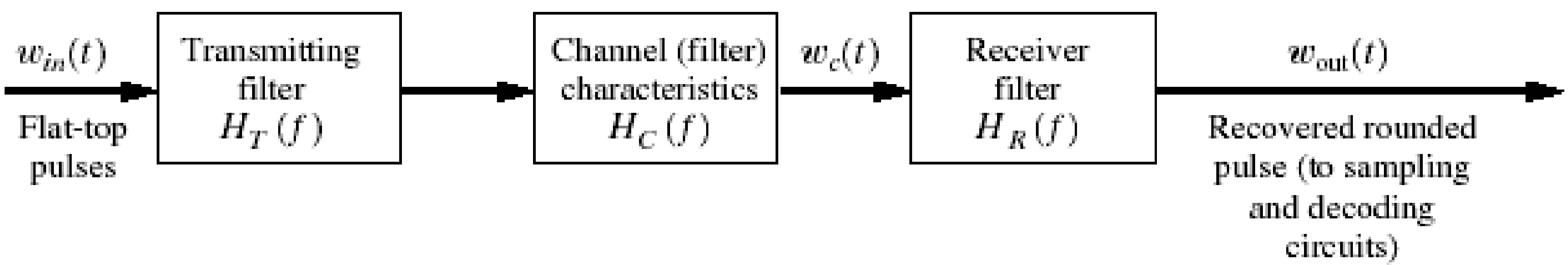


Figure 3–24 Baseband pulse-transmission system.

Intersymbol Interference

- Equivalent Impulse Response $h_e(t)$:

$$h_e(t) = h(t) * h_T(t) * h_C(t) * h_R(t)$$

- Equivalent transfer function:

$$\mathrm{H}_e(f) = H(f) H_T(f) H_C(f) H_R(f) \quad \text{Where} \quad \mathrm{H}(f) = F\left[\prod\left(\frac{t}{T_s}\right)\right] = T_s\left(\frac{\sin \pi T_s f}{\pi T_s f}\right)$$

- Receiving filter can be designed to produce a needed $H_e(f)$ in terms of $H_T(f)$ and $H_C(f)$:

$$\mathrm{H}_R(f) = \frac{H_e(f)}{H(f) H_T(f) H_C(f)}$$

- Output signal can be rewritten as:

$$w_{out}(t) = \sum_n a_n h_e(t - nT_s)$$

- $H_e(f)$, chosen such to minimize ISI is called EQUALIZING FILTER)

Combating ISI

- Three strategies for eliminating ISI:
  - Use a line code that is absolutely bandlimited.
    - Would require Sinc pulse shape.
    - Can't actually do this (but can approximate).
  - Use a line code that is zero during adjacent sample instants.
    - It's okay for pulses to overlap somewhat, as long as there is no overlap at the sample instants.
    - Can come up with pulse shapes that don't overlap during adjacent sample instants.
      - Raised-Cosine Rolloff pulse shaping
  - Use a filter at the receiver to "undo" the distortion introduced by the channel.
    - Equalizer.

Nyquist's First Method for Zero ISI

- ISI can be eliminated by using an equivalent transfer function, $H_e(f)$, such that the impulse response satisfies the condition:

$$h_e(kT_s+\tau)=\begin{cases} C, & k=0 \\ 0, & k\neq 0 \end{cases}$$

k is an integer, T_s is the symbol (sample) period

τ is the offset in the receiver sampling clock times

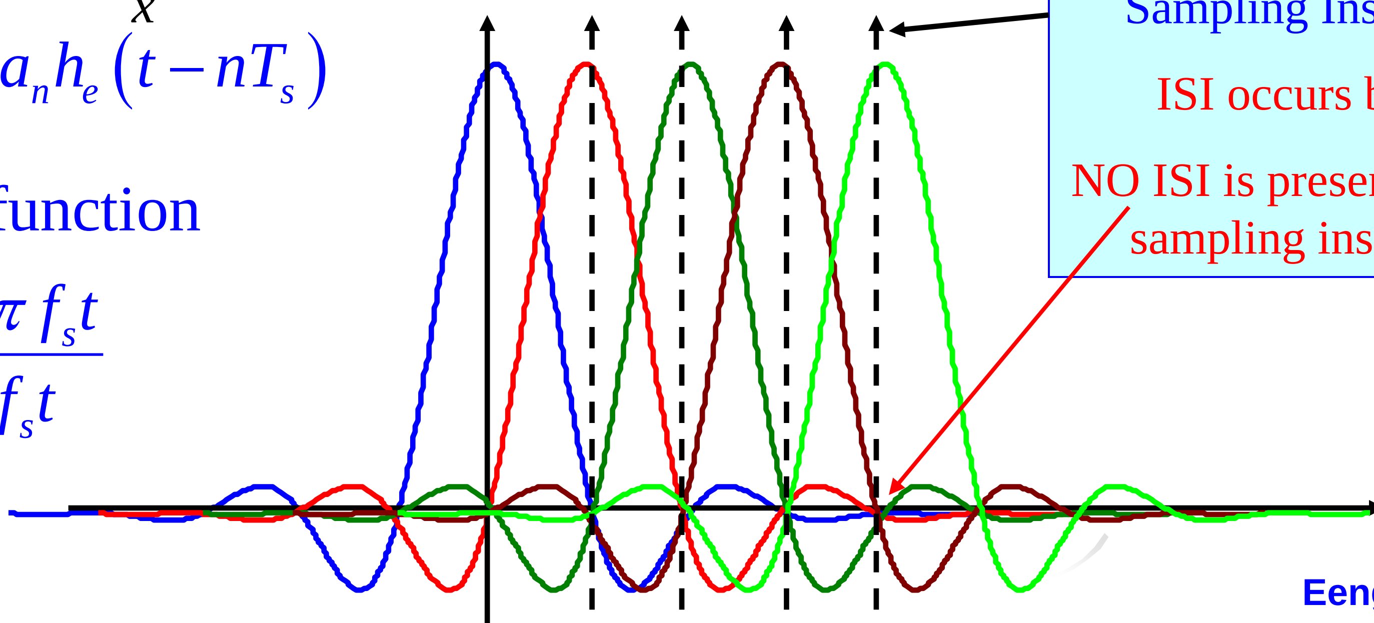
C is a nonzero constant

Now choose the $\dfrac{\sin x}{x}$ function for $h_e(t)$

$$w_{out}(t)=\sum_n a_n h_e(t-nT_s)$$

h_e is a Sa function

$$h_e(t)=\frac{\sin \pi f_s t}{\pi f_s t}$$

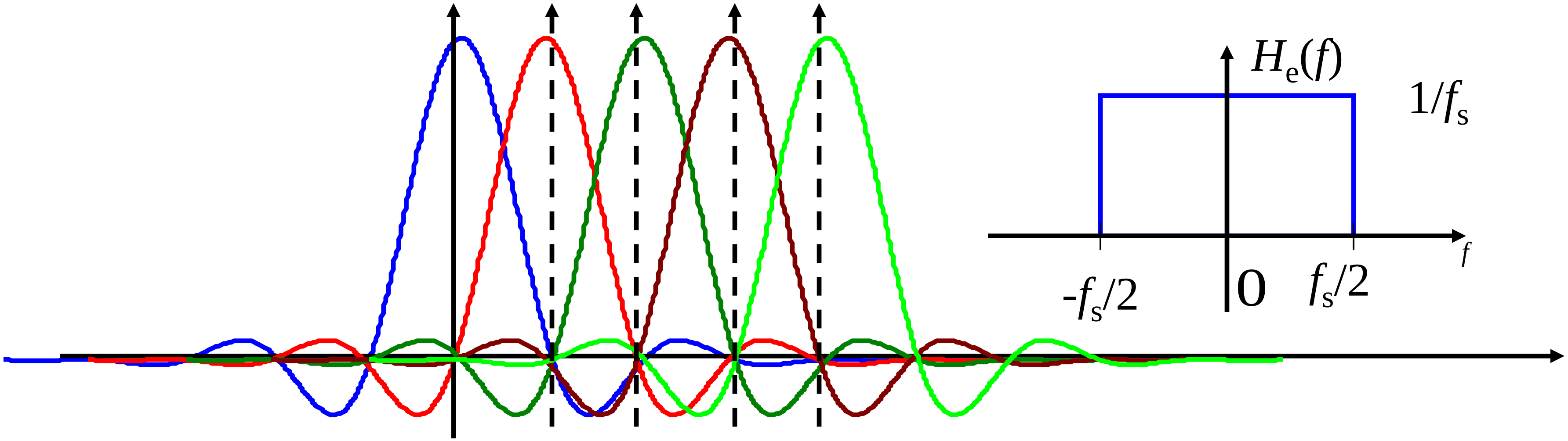


Nyquist's First Method for Zero ISI

➢ There will be **NO ISI and the bandwidth requirement will be minimum (Optimum Filtering)** if the transmit and receive filters are designed so that the overall transfer function $H_e(f)$ is:

$$H_e(f) = \frac{1}{f_s}\prod\left(\frac{f}{f_s}\right) \quad h_e(t) = \frac{\sin \pi f_s t}{\pi f_s t} \quad \text{Where} \quad f_s = \frac{1}{T_s}$$

➢ This type of pulse will allow signalling at a baud rate of $D=1/T_s=2B$ (for Binary $R=1/T_s=2B$) where B is the absolute bandwidth of the system.



Absolute bandwidth is: $B = \frac{f_s}{2}$ MINIMUM BANDWIDTH

Signalling Rate is: $D=1/T_s = 2B$ Pulses/sec

Nyquist's First Method for Zero ISI

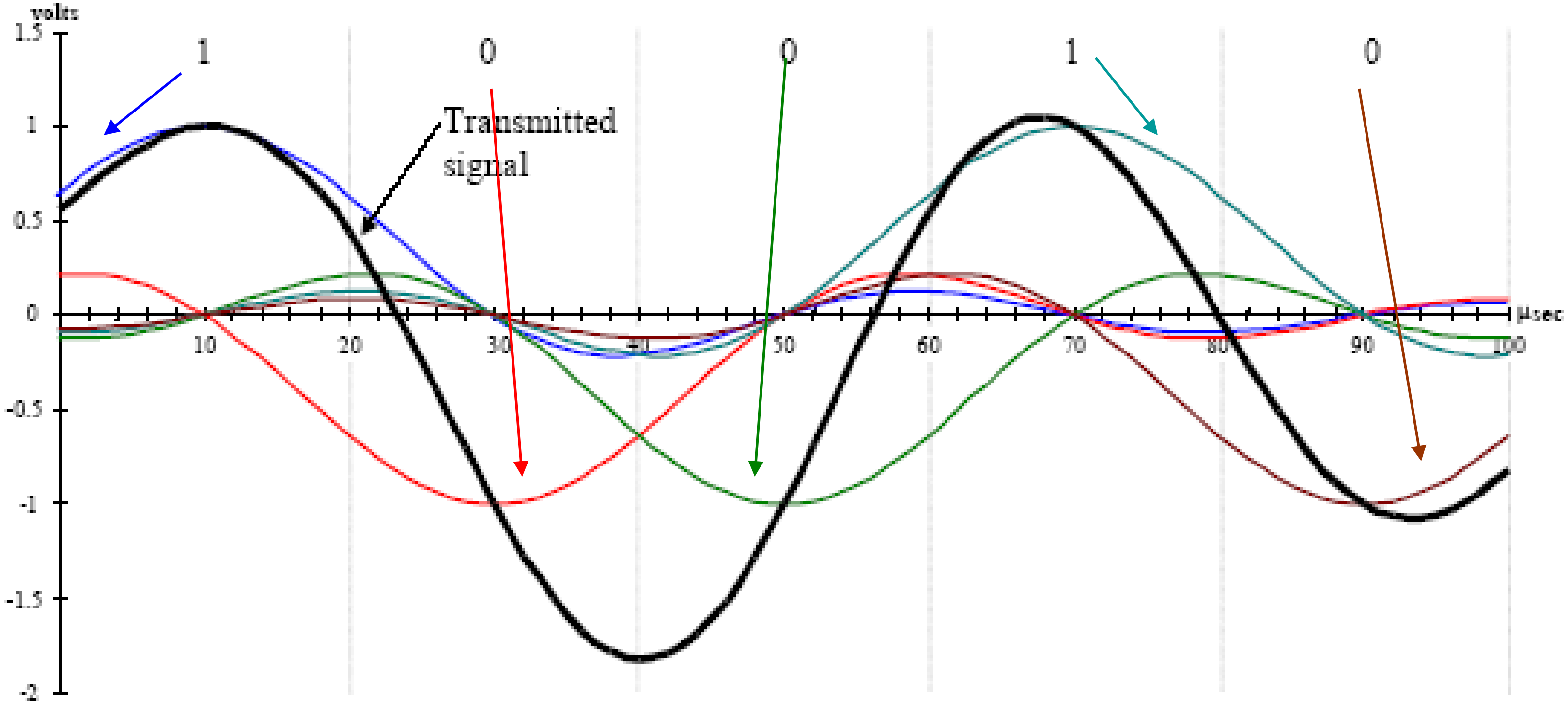
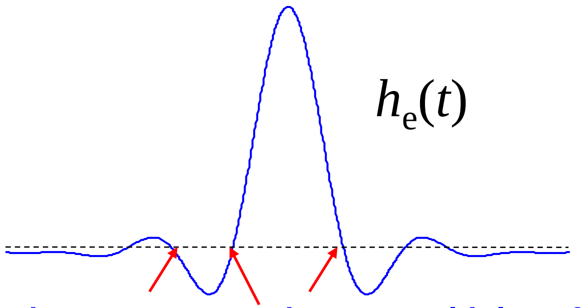
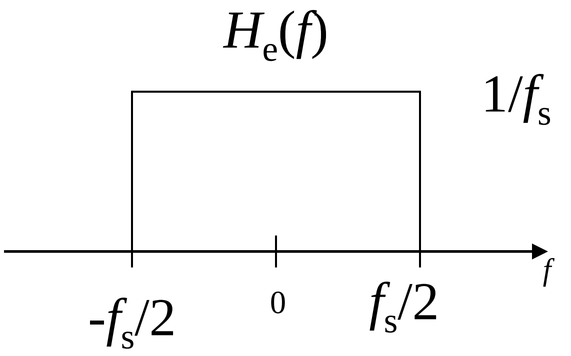


Figure 3.28 Transmitted binary PAM waveform for the data sequence "10010" using sinc-shaped pulses (raised cosine pulse shaping, $\alpha = 0$) at a transmission speed of 50,000 bits/sec. Note that this is the same plot as Figure 3.20a.

Nyquist's First Method for Zero ISI



Zero crossings at non-zero integer multiples of the bit period



- Since pulses are not possible to create due to:
  - Infinite time duration.
  - Sharp transition band in the frequency domain.

- The Sinc pulse shape can cause significant ISI in the presence of timing errors.
  - If the received signal is not sampled at *exactly* the bit instant (Synchronization Errors), then ISI will occur.

- We seek a pulse shape that:
  - Has a more gradual transition in the frequency domain.
  - Is more robust to timing errors.
  - Yet still satisfies Nyquist's first method for zero ISI.

Raised Cosine-Rolloff Nyquist Filtering

- **Because of the difficulties caused by the Sa type pulse shape, consider other pulse shapes which require more bandwidth such as the Raised Cosine-rolloff Nyquist filter but they are less affected by synchrfonization errors.**

- **The Raised Cosine Nyquist filter is defined by its rollof factor number $r=f_\Delta/f_o$.**

$$H_e(f)=\begin{cases}1, & |f|<f_1\\ \frac{1}{2}\left\{1+\cos\left[\frac{\pi(|f|-f_1)}{2f_\Delta}\right]\right\}, & f_1<|f|<B\\ 0, & |f|>B\end{cases}$$

B is the Absolute Bandwidth

$f_\Delta = B - f_0 \quad f_1 \equiv f_0 - f_\Delta$ Where f_o is the 6-dB bandwidth of the filter

Rolloff factor: $r=\frac{f_\Delta}{f_0}$ Bandwidth: $\mathrm{B}=\frac{R_b}{2}(1+r)$

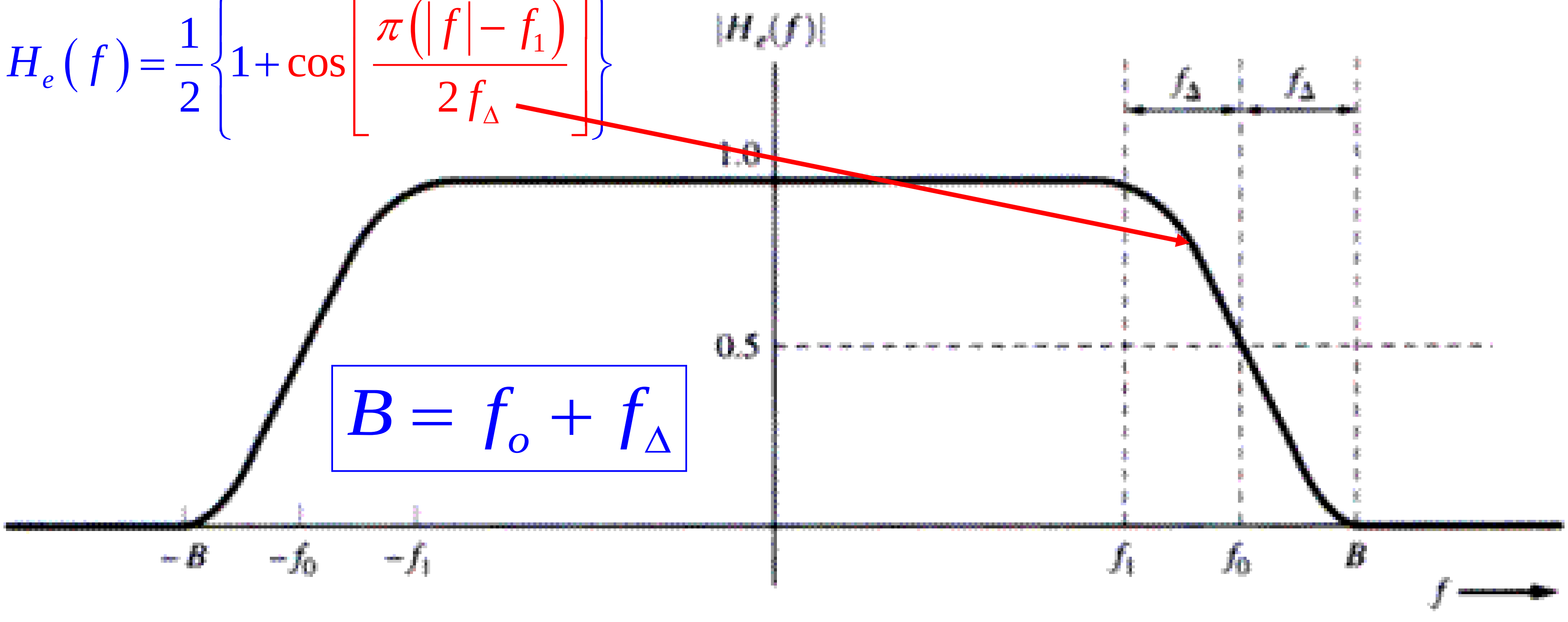
$$h_e(t)=F^{-1}\left[H_e(f)\right]=2f_0\left(\frac{\sin 2\pi f_0 t}{2\pi f_0 t}\right)\left[\frac{\cos 2\pi f_\Delta t}{1-(4f_\Delta t)^2}\right]$$

Rolloff factor: $r=\frac{f_\Delta}{f_0}$ Bandwidth: $\mathrm{B}=\frac{R_b}{2}(1+r)$

Raised Cosine-Rolloff Nyquist Filtering

- Now filtering requirements are relaxed because absolute bandwidth is increased.
- Clock timing requirements are also relaxed.
- The r=0 case corresponds to the previous Minimum bandwidth case.

$$H_e(f) = \frac{1}{2}\left\{1 + \cos\left[\frac{\pi\left(|f| - f_1\right)}{2 f_\Delta}\right]\right\}$$



$$B = f_o + f_\Delta$$

Figure 3–25 Raised cosine-rolloff Nyquist filter characteristics.

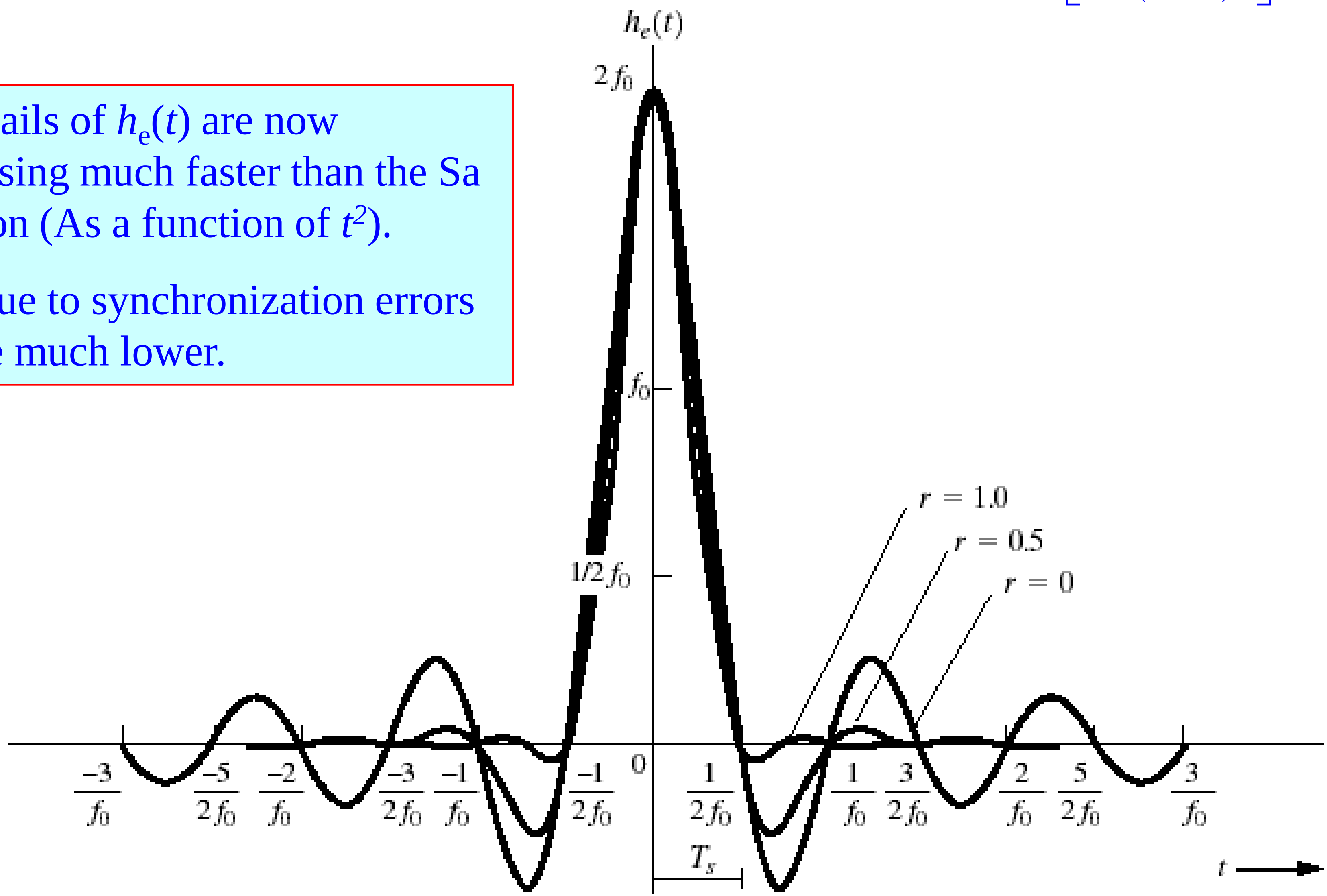
Rolloff factor: $r = \frac{f_\Delta}{f_0}$ Bandwidth: $B = \frac{R}{2}(1+r) = \frac{D}{2}(1+r)$

Raised Cosine-Rolloff Nyquist Filtering

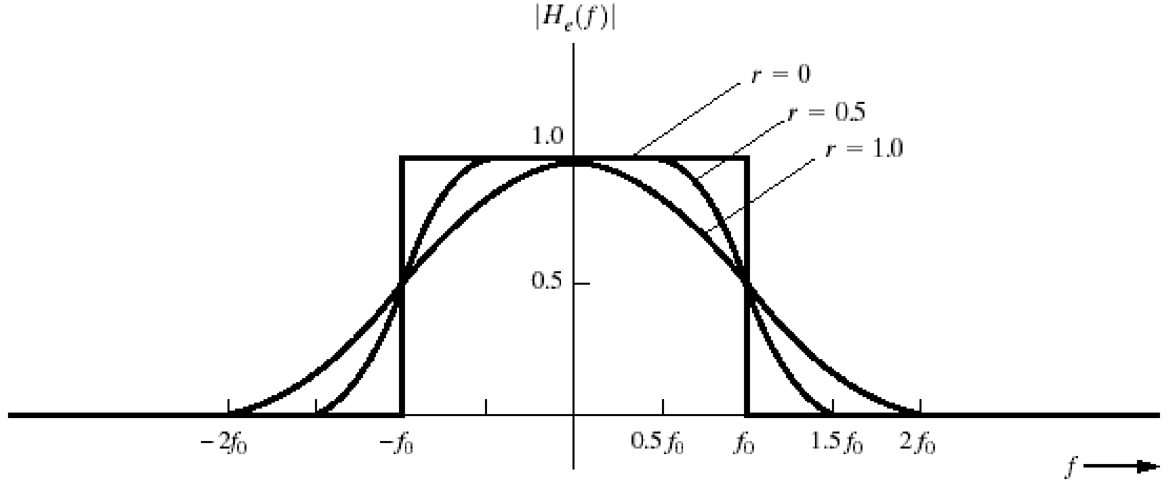
- Impulse response is given by: $h_e(t) = F^{-1}\left[H_e(f)\right] = 2f_0\left(\dfrac{\sin 2\pi f_0 t}{2\pi f_0 t}\right)\left[\dfrac{\cos 2\pi f_\Delta t}{1-(4f_\Delta t)^2}\right]$

- The tails of $h_e(t)$ are now decreasing much faster than the Sa function (As a function of t^2).

- ISI due to synchronization errors will be much lower.

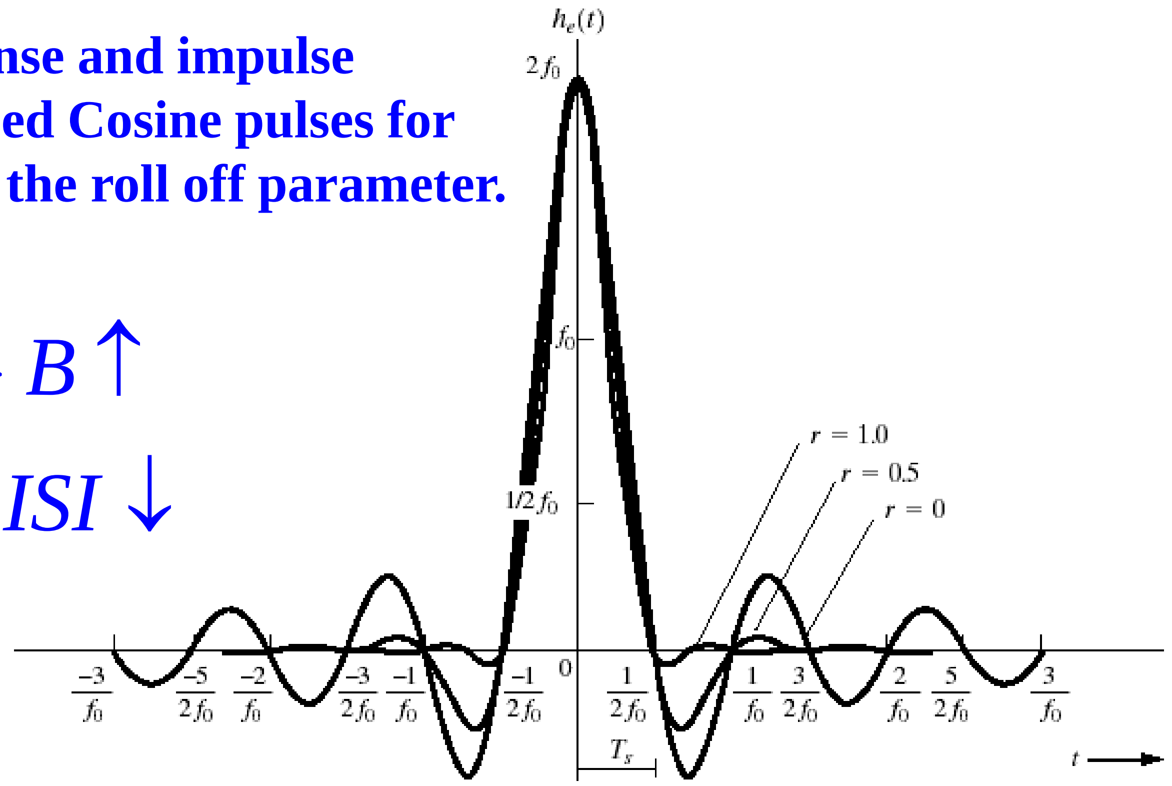


Raised Cosine-Rolloff Nyquist Filtering



(a) Magnitude Frequency Response

Frequency response and impulse responses of Raised Cosine pulses for various values of the roll off parameter.



$$r\uparrow \rightarrow B\uparrow$$

$$r\uparrow \rightarrow ISI\downarrow$$

Raised Cosine-Rolloff Nyquist Filtering

- **Illustrating the received bit stream of Raised Cosine pulse shaped transmission corresponding to the binary stream of 1 0 0 1 0 for 3 different values of *r*=0, 0.5, 1.**

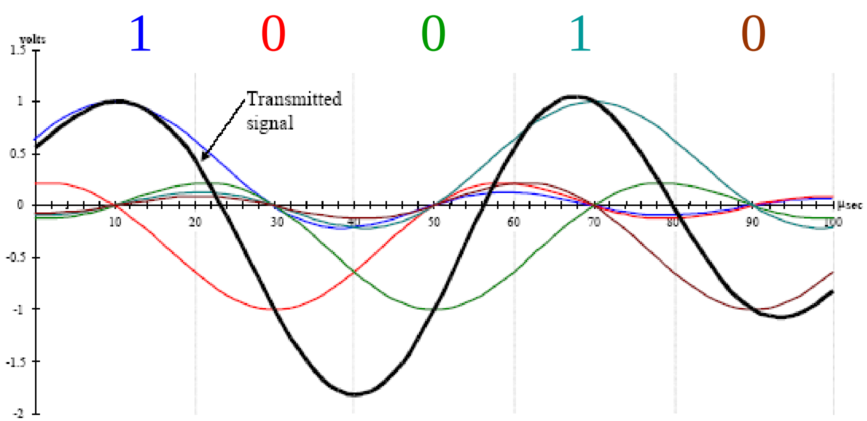


Figure 3.28 Transmitted binary PAM waveform for the data sequence "10010" using sinc-shaped pulses (raised cosine pulse shaping, $\alpha = 0$) at a transmission speed of 50,000 bits/sec. Note that this is the same plot as Figure 3.20a.

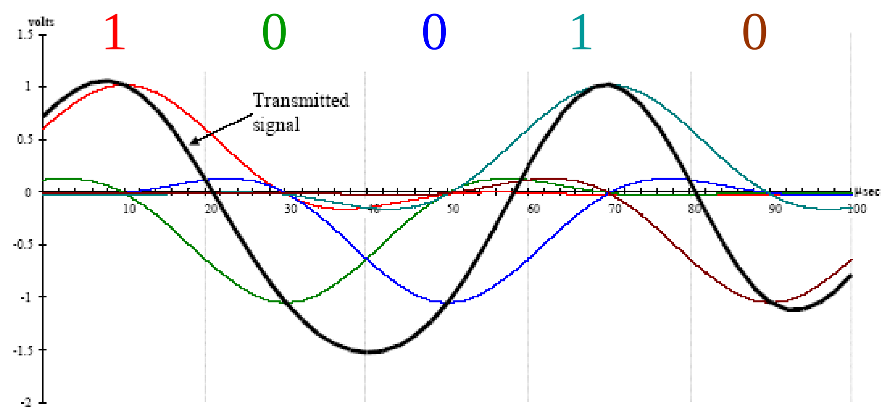


Figure 3.30 "10010" with raised cosine pulse shaping, $\alpha = 0.5$

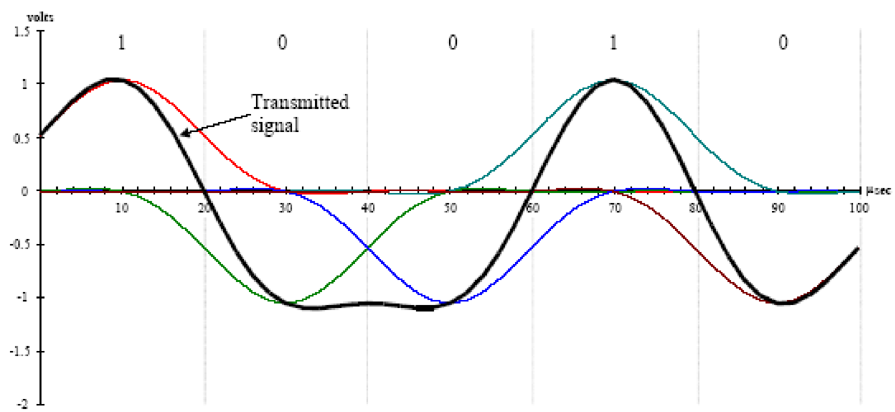


Figure 3.32 "10010" with raised cosine pulse shaping, $\alpha = 1$

Bandwidth for Raised Cosine Nyquist Filtering

- The bandwidth of a Raised-cosine (RC) rolloff pulse shape is a function of the bit rate and the rolloff factor:

$$B = f_o + f_\Delta = f_o\left(1 + \frac{f_\Delta}{f_o}\right) = f_o(1+r)$$

$$B = \frac{R}{2}(1+r)$$

$$B = \frac{D}{2}(1+r) \qquad \text{Multilevel Signalling}$$

- Or solving for bit rate yields the expression:

$$R = \frac{2B}{1+r}$$

  - This is the maximum transmitted bit rate when a RC-rolloff pulse shape with Rolloff factor *r* is transmitted over a baseband channel with bandwidth *B*.

Nyquist Filter

- **Raised Cosine Filter is also called a NYQUIST FILTER.**
- **NYQUIST FILTERS refer to a general class of filters that satisfy the NYQUIST's First Criterion.**
- **Theorem:** A filter is said to be a Nyquist filter if the effective transfer function is :

$$H_e(f) = \begin{cases} \Pi\left(\dfrac{f}{2f_0}\right) + Y(f), & |f| < f_0 \\ 0, & f \quad \text{Elsewhere} \end{cases}$$

$Y(f)$ is a real function and even symmetric about $f = 0$:

$$Y(-f) = Y(f), \qquad |f| < 2f_0$$

Y is odd symmetric about $f = f_0$:

$$Y(-f + f_0) = -Y(f + f_0), \qquad |f| < f_0$$

- There will be no intersymbol interference at the system output if the symbol rate is

$$D = f_s = 2f_0$$

Nyquist Filter Characteristics

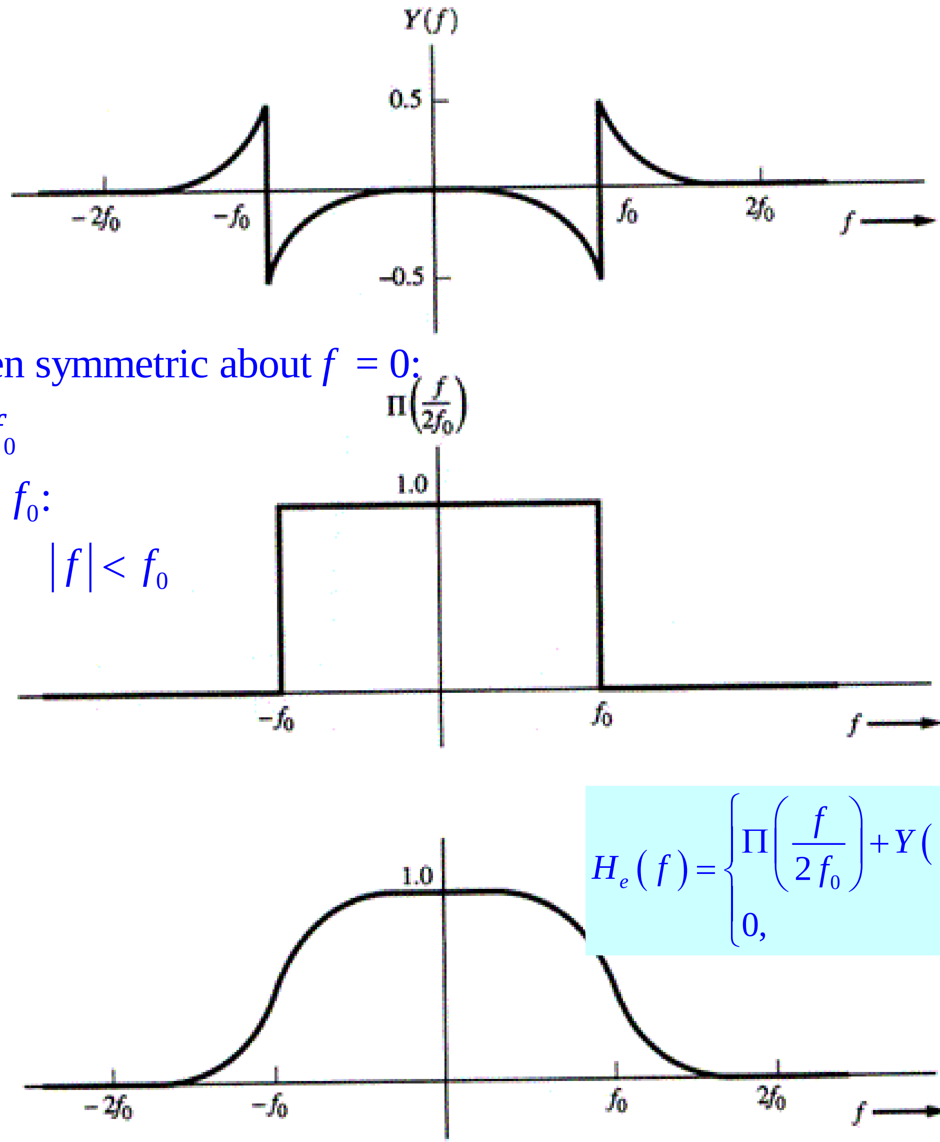


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Figure 3–27 Nyquist filter characteristic.